AN EFFICIENT APPROACH FOR ESTIMATING THE DATA EXTERNAL TO THE MEASUREMENT REGION IN THE HELICOIDAL SCANNING

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Abstract

An effective approach for the estimation of the data external to the near-field measurement region in the helicoidal scan is developed in this work. It is based on the nonredundant sampling representations of the electromagnetic field and uses the singular value decomposition method for extrapolating the outside samples. It is so possible to reduce the inevitable truncation error affecting the near-field reconstruction, due to the finite dimension of the helix. Numerical examples assess the effectiveness of the proposed technique.

Introduction

Among the near-field–far-field (NF–FF) transformation techniques, that using the cylindrical scan is particularly attractive when considering antennas that concentrate the electromagnetic (EM) radiation in an angular region centred on the horizontal plane. An innovative cylindrical NF–FF transformation with helicoidal scanning (Fig. 1) has been recently proposed in [1] in order to reduce the time needed for data acquisition by means of continuous and synchronized movements of the probe and of the antenna under test (AUT). To this end, a sampling representation of the field over the helix has been developed by properly applying the theoretical results on the nonredundant representations of electromagnetic (EM) fields [2] and assuming the AUT enclosed in a sphere of radius a. Then, the choice of the helix (elevation) step equal to the sample spacing needed to interpolate the data along a generatrix allows one to obtain a two-dimensional optimal sampling interpolation (OSI) formula for reconstructing the field at any point on the cylinder. It is so possible to determine the NF data needed by the classical NF–FF transformation with cylindrical scanning [3]. However, since the measurement region is always finite, an inevitable truncation error affects the reconstruction in the zones close to the ends of such a region. Therefore, the reconstruction results to be accurate in a zone smaller than the measurement one and this implies a decrease of the angular region wherein an accurate FF reconstruction is obtained.

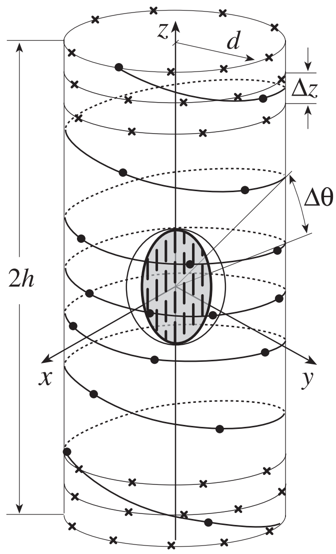

Fig. 1 - Helicoidal scanning.

This paper is devoted to the extrapolation of the NF samples external to the scanning region. The estimation of such data (otherwise equal to zero in the application of the OSI algorithm) allows one to reduce the truncation error in the zone close to the ends. The method is based on the nonredundant sampling representations of the EM field [2] and uses the Singular Value Decomposition (SVD) approach [4] to determine the outside samples. Although the reduction of the NF truncation error reflects in an extension of the angular region wherein an accurate FF reconstruction is attained, only the results relevant to the NF region are reported for brevity.

Nonredundant sampling representation of the EM field

Let us consider the field radiated by an AUT and observed on a cylinder of radius d in the NF region. According to [2], if the AUT is enclosed in a sphere of radius a and the helix is described by an analytical parameterization $\underline{r} = \underline{r}(\xi)$, the "reduced electric field" $\underline{F}(\xi) = \underline{E}(\underline{r}(\xi))\, e^{j\gamma(\xi)}$, $\gamma(\xi)$ being a phase function to be determined, can be closely approximated by a spatially bandlimited function. The related bandlimitation error becomes negligible as the bandwidth exceeds a critical value W_ξ [2], so that it can be effectively controlled by choosing a bandwidth equal to $\chi' W_\xi$, with $\chi' > 1$. A nonredundant sampling representation of the EM field on a helix with constant step $\Delta\theta$ can be obtained by using the following expressions for the phase function and parameterization [2]:

$$\gamma(s) = \frac{\beta}{2}\int_0^s \left[\max_{\underline{r}'} \hat{R}\cdot\hat{t} + \min_{\underline{r}'} \hat{R}\cdot\hat{t}\right] \mathrm{d}s \; ; \qquad \xi = \xi(s) = \frac{\beta}{2W_\xi}\int_0^s \left[\max_{\underline{r}'} \hat{R}\cdot\hat{t} - \min_{\underline{r}'} \hat{R}\cdot\hat{t}\right] \mathrm{d}s \tag{1}$$

where β is the wavenumber, s is the arclength of the helix, $\hat{t}$ is the unit vector tangent to it at the observation point P, $\underline{r}'$ denotes the source point, and $\hat{R}$ is the unit vector pointing from the source point to P. By imposing the passage of the helix through a fixed point of the generatrix at $\phi = 0$, the coordinates of P are given by: $x = d\,\cos(\varphi - \varphi_i)$, $y = d\,\sin(\varphi - \varphi_i)$, $z = d\,\cot\theta$, where φ is the angular parameter which describes the helix and $\theta = k\varphi$. Such a helix can be obtained as intersection of the cylinder with the line from the origin to a point which moves on a spiral wrapping the sphere of unit radius. The elevation step is equal to the sample spacing needed to interpolate the field along a cylinder generatrix according to [2]. Therefore, k is chosen such that the step of the helix, determined by two consecutive intersections (at φ and $\varphi + 2\pi$) with the considered generatrix, is equal to $\Delta\theta = 2\pi/(2M+1)$, with $M = \mathrm{Int}(\chi M') + 1$ and $M' = \mathrm{Int}(\chi'\beta a) + 1$, $\chi > 1$ being an oversampling factor. Since $\Delta\theta = 2\pi k$, it results $k = 1/(2M+1)$.

As can be easily seen, the extreme values of $\hat{R}\cdot\hat{t}$ are determined by considering the intersection of the plane defined by $\hat{t}$ and the unit vector $\hat{r}$ (pointing from the origin to P) with the cone with the vertex at P and the generatrices coincident with the tangents to the source ball. Denoting by $\hat{R}_{1,2}$ the related unit vectors and by ε the angle between $\hat{r}$ and $\hat{t}$ (Fig. 2), it results:

$$\left(\hat{R}_1 + \hat{R}_2\right)/2 = \hat{r}\sin\delta = \hat{r}\sqrt{1 - a^2/r^2} \tag{2}$$

$$\left(\hat{R}_1 - \hat{R}_2\right)\cdot\hat{t}/2 = \cos\delta\sin\varepsilon = (a/r)\sin\varepsilon \tag{3}$$

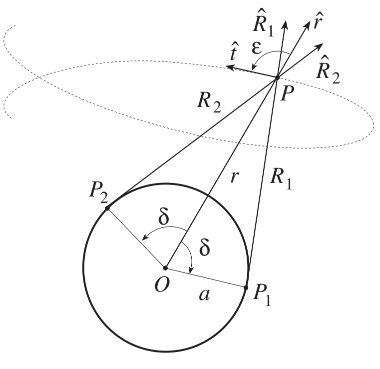

Fig. 2 - Geometry in $\hat{r}$, $\hat{t}$ plane.

Substituting (2) into the former relation in (1) and taking into account that $\mathrm{d}r = \hat{r}\cdot\hat{t}\,\mathrm{d}s$, we get:

$$\gamma = \beta\int_0^r \sqrt{1 - a^2/r^2}\,\mathrm{d}r = \beta\sqrt{r^2 - a^2} - \beta a\cos^{-1}\left(\frac{a}{r}\right) \tag{4}$$

On the other hand, by taking into account that

$$\mathrm{d}s = \left(d/\sin^2 k\varphi\right)\sqrt{k^2 + \sin^4 k\varphi}\;\mathrm{d}\varphi\,; \quad \hat{r}\cdot\hat{t} = -\left(k\cos k\varphi\right)\big/\sqrt{k^2 + \sin^4 k\varphi} \tag{5}$$

and substituting (3) into the second relation in (1), it results:

$$\xi = \frac{\beta a}{W_\xi}\int_0^\varphi \sqrt{k^2 + \sin^2 k\varphi'}\;\mathrm{d}\varphi' \tag{6}$$

That is to say, ξ is proportional to the curvilinear abscissa along the spiral wrapping the sphere of unit radius. Since such a spiral is a closed curve, it is convenient to choose W_ξ such that ξ covers a 2π range when the entire curve on the sphere is described. Accordingly

$$W_\xi = \frac{\beta a}{\pi}\int_0^{(2M+1)\pi} \sqrt{k^2 + \sin^2 k\varphi'}\;\mathrm{d}\varphi' \tag{7}$$

It must be stressed that, unlike the previous approach in [1], the helix originates now from a given point at the top of the cylinder, thus simplifying the realization of the scanning from the practical viewpoint.

According to these results, the OSI formula to reconstruct the field at any point of the helix is [2]:

$$F_{z,\phi}(\xi) = \sum_{n=n_0-q+1}^{n=n_0+q} F_{z,\phi}(\xi_n)\,\Omega_{N''}(\xi - \xi_n)\,D_N(\xi - \xi_n) \tag{8}$$

where $n_0 = \mathrm{Int}\left((\xi - \xi(\varphi_i))/\Delta\xi\right)$ is the index of the sample nearest (on the left) to the output point, $2q$ is the number of retained samples and $\xi_n = \xi(\varphi_i) + n\Delta\xi = \xi(\varphi_i) + 2\pi n/(2N+1)$, with $N = \mathrm{Int}(\chi N') + 1$ and $N' = \mathrm{Int}(\chi' W_\xi) + 1$. Moreover, $D_N(\xi)$ and $\Omega_{N''}(\xi)$ are the Dirichlet and Tschebyscheff Sampling functions [2] and $N'' = N - N'$. Above results can be used to evaluate the field at any point P on the cylinder from the samples on the helix. The first step is to determine the "intermediate" samples (intersection points of the helix with the generatrix passing through P) by using (8). Once these samples have been evaluated, because of the particular choice of $\Delta\theta$, the field can be reconstructed via the OSI expansion:

$$F_{z,\phi}(\theta,\phi) = \sum_{m=m_0-p+1}^{m_0+p} F_{z,\phi}(\theta_m) D_M(\theta-\theta_m)\,\Omega_{M''}(\theta-\theta_m) \tag{9}$$

where $F_{z,\phi}(\theta_m)$ are the intermediate samples, $\theta_m = \theta_m(\phi) = \theta(\varphi_i) + k\phi + m\Delta\theta = \theta_0 + m\Delta\theta$, $M'' = M - M'$, $m_0 = \mathrm{Int}((\theta-\theta_0)/\Delta\theta)$ and the other symbols have the same meaning as in (8).

Extrapolation algorithm

Let us now deal with the estimation of the NF samples external to the scanning region $-h \le z \le h$ on the measurement cylinder. In order to explain the methodology, let us consider the zone $0 \le z \le h$. Besides the regular samples acquired via the helicoidal scanning, let us assume the knowledge of the field components on J rings spaced at a fixed step Δz, from the top of the cylinder. On each of these rings, the extra samples are known at the points specified by $\phi_{n,j} = n\Delta\phi_j = 2n\pi/(2N_j+1)$, where

$$N_j = \mathrm{Int}(\chi N'_j) + 1 \;; \quad N'_j = \mathrm{Int}(\chi^* W_{\phi_j}) + 1 \;; \quad W_{\phi_j} = W_\phi(\theta_j) = \beta a \sin\theta_j \tag{10}$$

$$\chi^* = 1 + (\chi' - 1)\left[\sin\theta_j\right]^{-2/3} \;; \quad \theta_j = \tan^{-1}(d/(h - j\Delta z)) \;; \quad j = 1,\ldots,J \tag{11}$$

On each cylinder generatrix fixed by ϕ, the field components at the intersection points $P(\theta_j,\phi)$ with the extra rings can be evaluated via the following OSI expansion [2]

$$F_{z,\phi}(\theta_j) = F_{z,\phi}(\theta_j,\phi) = \sum_{n=n_0-q+1}^{n_0+q} F_{z,\phi}(\theta_j,\phi_{n,j})\,\Omega_{N''_j}(\phi-\phi_{n,j})\,D_{N_j}(\phi-\phi_{n,j}) \tag{12}$$

where $2q$ is the number of retained samples along ϕ, $n_0 = \mathrm{Int}(\phi/\Delta\phi_j)$, and $N''_j = N_j - N'_j$. Applying (9) to each of the points $P(\theta_j,\phi)$, just p unknown outside samples $F_{z,\phi}(\theta_m)$ are always involved, since the other can be reconstructed via (8). Accordingly, by centring the OSI formula (9) on the first known intermediate sample at θ_0, so that the index m assumes negative values for the external samples to be estimated, we get:

$$F_{z,\phi}(\theta_j) - \sum_{m=0}^{m_0+p} F_{z,\phi}(\theta_m) D_M(\theta_j-\theta_m)\Omega_{M''}(\theta_j-\theta_m) = \sum_{m=-\bar{p}}^{-1} F_{z,\phi}(\theta_m) D_M(\theta_j-\theta_m)\Omega_{M''}(\theta_j-\theta_m) \tag{13}$$

$$j = 1,\ldots,J$$

where $\bar{p} \le p$ is the number of external samples to be estimated. For each field component, these J equations can be rewritten in matrix form as $\underline{\underline{A}}\,\underline{x} = \underline{b}$, where $\underline{b}$ is the sequence of the known terms, $\underline{\underline{A}}$ is the $J \times \bar{p}$ matrix, whose elements $A_{jm} = D_M(\theta_j - \theta_m)\,\Omega_{M''}(\theta_j - \theta_m)$ are given by the weight functions in the considered OSI expansion and $\underline{x}$ is the sequence of the unknown outside samples $F_{z,\phi}(\theta_m)$, with $m = -\bar{p},\ldots,-1$. A solution, which is the best approximation in the least squares sense of the linear system (13), can be obtained by using the SVD technique. A quite similar procedure can be used for extrapolating the samples external to the lower end of the cylinder.

Once the outside samples relevant to the considered generatrix have been estimated, the field components at any point on it can be evaluated via the OSI expansion (9).

Numerical results

Some numerical tests assessing the effectiveness of the extrapolation process are shown. They refer to a uniform planar array of 0.8 λ spaced elementary Huygens sources polarized along the z axis and lying in an elliptical zone on the plane $y = 0$, with major and minor semi-axes equal to 16 λ and 10 λ (λ being the wavelength). The radius d of the cylinder is 20λ and its height $2h$ is 120λ. In the following we show only the results obtained for $z \ge 0$, quite analogous results have been achieved for $z \le 0$. According to the adopted sampling representation, $J = 4$ extra rings have been acquired and they are spaced at $\Delta z = 1.2\lambda$, from the top of the cylinder. In order to reduce the ill-conditioning of the matrix $\underline{\underline{A}}$, only the estimation of $\bar{p} = 4$ outside samples has been tackled. Moreover, we have assumed $p = 15$ in the extrapolation process of the outside samples, whereas $q = 11$ has been adopted both in (12) and in (8) to obtain the involved known samples. It must be stressed that the SVD is

applied to a small matrix with a negligible computational effort. Fig. 3 shows the amplitude of the NF z-component (the most significant one) on the cylinder generatrix at $\phi = 90°$. It has been reconstructed without using the extrapolation process and putting the outside samples equal to zero. As can be seen in Fig. 4, by using the here proposed estimation procedure, the reconstruction is very accurate not only in the whole measurement region, but also in a zone outside it. It is worthy to note that, in both cases, $p = 7$ has been adopted when applying (9) for the field reconstruction. In Fig. 5 a further reconstruction example on the generatrix at $\phi = 110°$ is reported. In order to assess more quantitatively the effectiveness of the approach, the maximum and mean-square reconstruction errors relevant to the NF z-component have been evaluated by comparing the exact field values and those reconstructed with and without the estimated outside samples. Fig. 6 shows such errors, normalized to the z-component maximum value. As can be seen, the errors evaluated by taking into account the estimated samples decrease quite rapidly until very low levels are reached. On the contrary, those obtained without considering them saturate to constant values, due to the truncation error present in the zones close to the cylinder ends.
It is useful to note that, in the considered example, the overall number of employed NF data is 26 365. In particular, the number of "extra" samples on the rings (at the ends of the scanning cylinder) is 920.

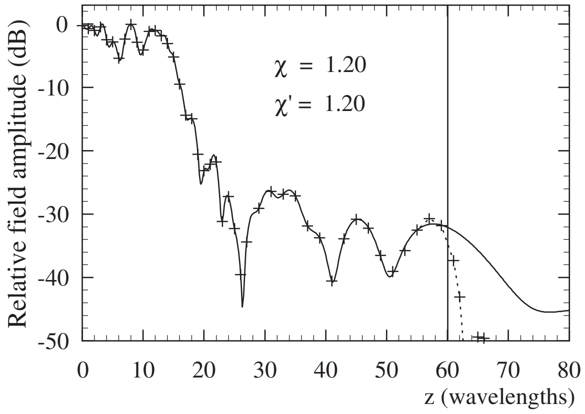

Fig.3 - Amplitude of the NF z-component at $\phi = 90°$. Solid line: exact. Crosses: reconstructed without estimated outside samples.

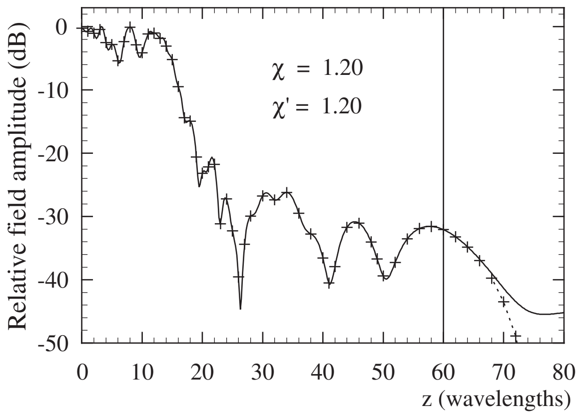

Fig.4 - Amplitude of the NF z-component at $\phi = 90°$. Solid line: exact. Crosses: reconstructed with estimated outside samples.

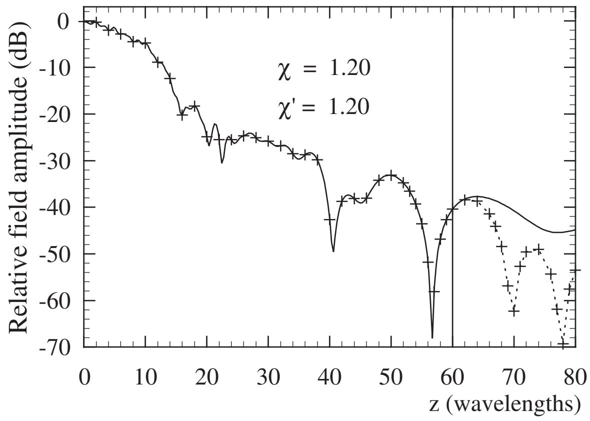

Fig.5 - Amplitude of the NF z-component at $\phi = 110°$. Solid line: exact. Crosses: reconstructed with estimated outside samples.

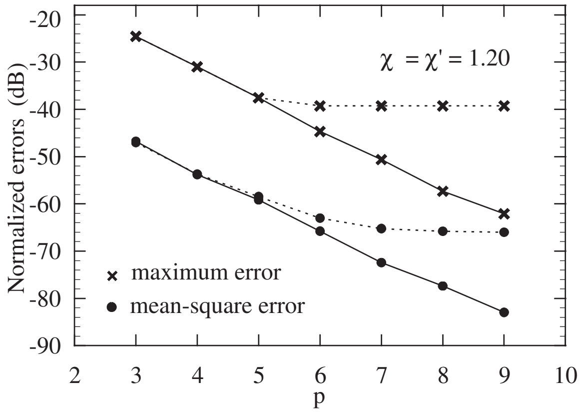

Fig.6 - Normalized reconstruction errors. Dashed line: without estimated outside samples. Solid line: with estimated outside samples.

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