AN APPARENT DISCREPANCY BETWEEN IMPEDANCE MISMATCH FACTORS FOR NEAR-FIELD AND FAR-FIELD MEASUREMENTS

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ABSTRACT

In making accurate measurements of antenna gain one must correct for the impedance mismatches between (1) the signal generator and transmitting antenna, (2) between the receiving power sensor and the receiving antenna and (3) between the signal generator and receiving power sensor. This is true for both far-field gain measurements and near-field gain measurements. It has recently come to our attention that there is a lack of clarity as to the form the mismatch factor should take when correcting near-field measured data. We show that a different form of impedance mismatch factor is to be used with the voltage domain equations of near-field than has been used with the power domain Friis transmission equation.

Keywords: Gain Measurements, Near-Field Gain Measurements, Impedance Mismatch Correction

1.0 Introduction

It has long been recognized that in making accurate measurements of antenna gain one must correct for the impedance mismatches between (1) the signal generator and transmitting antenna, (2) between the receiving power sensor and the receiving antenna and (3) between the signal generator and receiving power sensor. This is true for both far-field gain measurements [1,2] and near-field gain measurements [3]. It has recently come to our attention that there is a lack of clarity as to the form the mismatch factor should take when correcting near-field measured data.

In the far field the applicable equation is the Friis transmission formula, which is a power domain expression. When the gain is determined in the far-field, by measurement of the range insertion loss, a mismatch factor M^{Power} should be used to adjust the measured power ratio according to the formula

$$M^{Power} = \frac{\left|1-\Gamma_G\Gamma_T\right|^2\left|1-\Gamma_R\Gamma_L\right|^2}{\left(1-\left|\Gamma_T\right|^2\right)\left(1-\left|\Gamma_R\right|^2\right)\left|1-\Gamma_G\Gamma_L\right|^2} \quad , \qquad (1)$$

where the Γ's are the complex reflection coefficients of the generator(G), load(L) representing a microwave power sensor, a transmitting antenna(T) and a receiving antenna (R). [5]

In the near field, in the particular case of planar near-field scanning, the applicable equation is the PNF transmission equation, which is a voltage domain expression. When the voltage normalization for gain is determined by measurement of the range insertion loss with a voltage receiver, a mismatch factor $M^{Voltage}$ is acknowledged to be necessary [4]. It adjusts the measured complex voltage ratio of "Range-Inserted" voltage to "Range Short-Circuited" voltage to yield the desired ratio of voltage phasors, $b_0'(\boldsymbol{P}_0)/F'a_0$, including a factor F' that appears in the PNF transmission equation, [6]. The proper formula for the voltage mismatch factor is

$$M^{Voltage} = \frac{(1-\Gamma_G\Gamma_T)(1-\Gamma_p{}'\Gamma_L)}{(1-\Gamma_G\Gamma_L)} \quad , \qquad (2)$$

where the Γ's are the same as above and Γ_p' is the complex reflection coefficient of the (receiving) probe.

One would perhaps expect the voltage mismatch factor to be the square root of the power mismatch factor and be a precise analog of the power mismatch factor. If so, comparing these two formulas, one would conclude that there is a discrepancy between the use of voltage versus power quantities. However, there is not an error at the root cause of the discrepancy. Rather there is a difference between the underlying theoretical quantities.

Resolution of the difficulty lies in using care to define the power quantities and the voltage quantities. In this paper we give a detailed derivation of the voltage mismatch factor and we show how we have resolved the apparent discrepancy.

In his preface, Kerns' terms the measurement of range insertion loss a "transfer normalization measurement" [3]. In the voltage domain it is a measurement of an insertion factor; in the power domain an insertion loss.

Newell et. al. have addressed this topic in Reference [4] from the point of view of the power domain expressions. The results here are in agreement with those authors.

1.0 The Procedure for Measurement of Range Insertion Loss

The antenna range is treated as a microwave two-port with the port of the transmitting antenna and the port of the receiving antenna serving as the ports. To make measurement of the insertion loss of this two-port, first a reference level must be established by making a "Through" measurement; this is accomplished by connecting the output of the signal generator with the input to the receiver and establishing a reference level. Please see Figure 1 where a "Through" measurement using a network analyzer is illustrated.

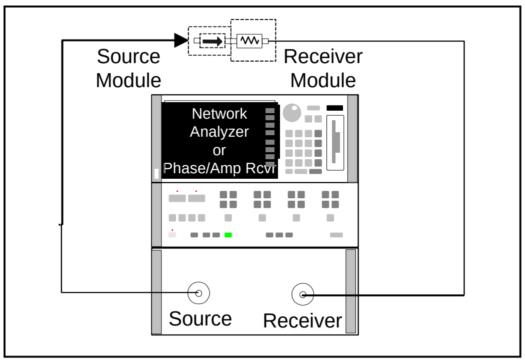

Figure 1 - Range in "Through" or "Short Circuited" Condition for Measurement of Incident Phasor a_0.

Second, the output from the signal generator is connected to the transmitting test antenna and the input to the receiver is connected to the output port of the receiving antenna. Please see Figure 2 where a schematic of this measurement is illustrated. The apparent insertion loss of the range is then the ratio of the "Range-Inserted" measurement to the "Range Short-Circuited" or "Through" measurement; however the ratio of the receiver readings must be adjusted to account for the impedances of the signal generator, the receiver, the transmitting and receiving antennas.

In the next section we show how this can be analyzed to determine the proper factor to apply to the ratio of the measured receiver voltages. This voltage mismatch factor $M^{Voltage}$ is defined for near-field scanning by the equation

$$\frac{b_0'}{a_0 F'} \equiv M^{Voltage} \frac{V_{Rcvr}^{RngInsrtd}}{V_{Rcvr}^{ShrtCct}} \quad . \tag{3}$$

The mismatch factor times the ratio of measured voltages yields the desired ratio of the theoretical voltage quantities. In eqn (3) F' is a factor used throughout Kerns' book [3] and defined as

$$F' \equiv \left(\frac{1}{1-\Gamma_p'\Gamma_L}\right) \quad , \tag{4}$$

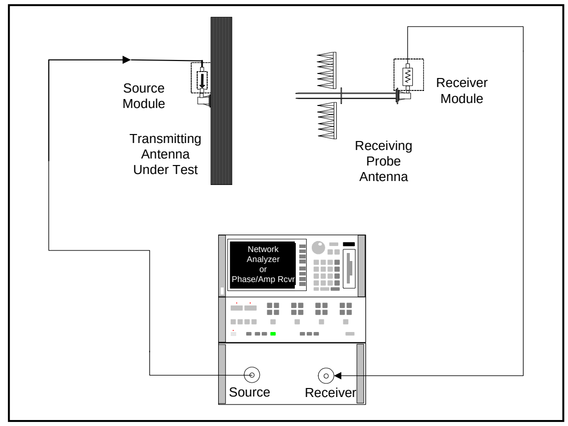

Figure 2 - Schematic of Planar Near-Field Range with AUT in Transmit as Set Up for Measurement of the Received Phasor b_0'.

where Γ_p' is the reflection coefficient of the probe antenna and Γ_L is the reflection coefficient of the load representing the receiver module.

2.0 Derivation of the Voltage Mismatch Factor

The voltage mismatch factor is derived straightforwardly by considering the analysis of each connection in terms of incident and reflected traveling waves at the reference plane of the connected pair. The classic case of a source or generator connected to a load is described in Section 6.2 of Kerns [3]. That section is reproduced here as Appendix B for convenient reference. Note equation 6.2-3. This case serves as a model for our analyses.

2.1 "Through" or "Short-Circuited" Connection

First we analyze the "Through" or "Short-Circuited" connection. Please refer to Figure 3. Directly from Kerns' equation 6.2-3, (see Appendix B), we find

$$a_0 = \frac{b_G}{1-\Gamma_G\Gamma_L} \quad . \tag{5}$$

From Appendix A we find the receiver's voltage response in the short-circuited condition to be proportional to a_0 via the factor $V_0(1+\Gamma_L)$ and therefore it can be written as

$$V_{Rcvr}^{ShrtCct} = \frac{V_0(1+\Gamma_L)b_G}{1-\Gamma_G\Gamma_L} \quad . \tag{6}$$

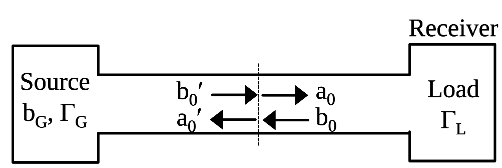

Figure 3 - Range in *"Short-Circuited" or "Through"* Configuration

2.2 The "Range-Inserted" Connections

We now examine the two connections associated with the "Range-Inserted" condition. First consider the connection of the source of radiation to the transmitting antenna; in a planar near-field setup this will be the antenna-under-test. This connection is illustrated in Figure 4.

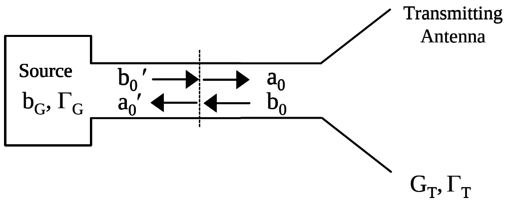

Figure 4 - At the Transmitting Antenna - Range in "*Range-Inserted*" Configuration

Directly from Kerns eqn 6.2-3, (Appendix B) we find

$$a_0 = \frac{b_G}{1-\Gamma_G\Gamma_T} \quad . \tag{7}$$

Second we examine the connection of the receiver input to the receiving antenna. This is illustrated in Figure 5.

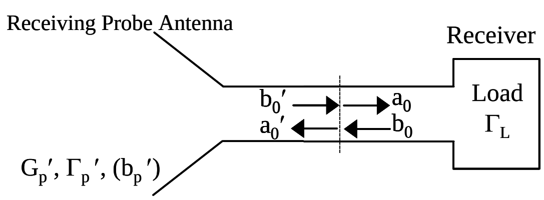

Figure 5 - At the Receiving Antenna - Range in "*Range Inserted*" Configuration

Again we can apply Kerns' eqn 6.2-3 using the voltage generated by the illuminated probe b_p' in place of b_G.

$$b_0' = \frac{b_p'}{1-\Gamma_p'\Gamma_L} \quad . \tag{8}$$

Note that from eqn (8), and in view of eqn (4), the following relationship holds

$$b_p' = b_0'/F' \quad . \tag{9}$$

The voltage response of the receiver is written as

$$V_{RcvAnt}^{RngInsrtd} = V_0(1+\Gamma_L)b_0' \quad . \tag{10}$$

The near-field ratio of interest is the phasor ratio (b_0'/a_0) and we can take the ratio of equations (7) and (10) to obtain this quantity

$$\frac{b_0'}{a_0} = \frac{V_{RcvAnt}^{RngInsrtd}}{V_0(1+\Gamma_L)}\frac{(1-\Gamma_G\Gamma_T)}{b_G} \quad . \tag{11}$$

This equation applies to the "Range-Inserted" configuration.

2.3 Ratio of "Range-Inserted" to "Range Short-Circuited" Configurations

Assuming that the quantity b_G is the same under both the "Short-Circuited" and "Range-Inserted" conditions, one may combine equations (6) and (11) to find that

$$\frac{b_0'}{a_0} = \frac{V_{RcvAnt}^{RngInsrtd}(1-\Gamma_G\Gamma_T)}{V_{Rcvr}^{ShrtCct}(1-\Gamma_G\Gamma_L)} \quad . \tag{12}$$

Note that this equation is equivalent to Newell's eqn. (14) in Reference [4].

Since in (planar) near-field theory it is the quantity b_0'/a_0F' that is of interest, we write

$$\frac{b_0'}{a_0F'} = \frac{(1-\Gamma_p'\Gamma_L)(1-\Gamma_G\Gamma_T)}{(1-\Gamma_G\Gamma_L)}\frac{V_{RcvAnt}^{RngInsrtd}}{V_{Rcvr}^{ShrtCct}} \quad . \tag{13}$$

From this equation we can identify clearly the voltage mismatch factor quoted earlier in eqn (2). It is the factor that we use to obtain the desired theoretical voltage quantity (b_0'/a_0F') from the ratio of measured receiver readings. This can be compared to the classic power mismatch factor quoted in eqn (1) above. These are not precise analogs of one another. There are two additional terms in the power mismatch factor of eqn (1) that correspond to adjustments for available power from the generator and from the receiving antenna, and to power accepted by (or delivered to) the transmitting antenna.

3. Reconciliation of the Difference Between the Voltage and Power Mismatch Factors

To reconcile the voltage and power mismatch factors we show that they are consistent one to the other. To see this we consider a simple thought experiment for the case of a near-field probe antenna located first in the near field and then in the far field of a transmitting test antenna.

<u>Near-Field Case</u> As we have just shown in Section 2, the Voltage Mismatch Factor takes the form of eqn (2) for this near-field case; and, eqn (13) gives us the correction to be applied to the ratio of voltages.

Far-Field Case Now let the probe antenna approach the far field of the transmit antenna. As the separation distance becomes large, the PNF transmission equation reduces to the voltage equivalent of the Friis transmission equation. [3, p.85] We may apply the Friis transmission equation and with it, the power mismatch factor, eqn (1):

$$\frac{P_{RcvPrb}^{Avail}{}'}{P_{XmtAnt}^{Accptd}} = \frac{|1-\Gamma_G\Gamma_T|^2|1-\Gamma_p{}'\Gamma_L|^2}{(1-|\Gamma_T|^2)(1-|\Gamma_p{}'|^2)|1-\Gamma_G\Gamma_L|^2}\frac{P_{Rcvr}^{RngInsrtd}}{P_{Rcvr}^{ShrtCct}} \quad . \tag{14}$$

Note that this relates (a) Available Power at the port of the probe in ratio to the Accepted Power at the port of the transmitting antenna and (b) the ratio of measured power quantities indicated by a power meter or receiver.

Using Kerns' Equation 6.2-4, Appendix B we find in terms of the amplitude a_0 incident upon the transmit antenna, that the accepted power is given by

$$P_{XmtAnt}^{Accptd} = (1-|\Gamma_T|^2)\tfrac{1}{2}|a_0|^2 \quad . \tag{15}$$

In terms of the amplitude b_0' incident from the probe, the available power (from the receiving probe) is given by eqn 6.2-5 of Appendix B, and making use of eqn (8), we find

$$P_{RcvPrb}^{Avail}{}' = \frac{\tfrac{1}{2}|b_p{}'|^2}{1-|\Gamma_p{}'|^2} = \frac{|1-\Gamma_p{}'\Gamma_L|^2}{1-|\Gamma_p{}'|^2}\tfrac{1}{2}|b_0{}'|^2 \quad . \tag{16}$$

Forming the ratio of these two equations gives

$$\frac{P_{RcvPrb}^{Avail}{}'}{P_{XmtAnt}^{Accptd}} = \frac{|1-\Gamma_p{}'\Gamma_L|^2}{(1-|\Gamma_p{}'|^2)(1-|\Gamma_T|^2)}\frac{\tfrac{1}{2}|b_0{}'|^2}{\tfrac{1}{2}|a_0|^2} \quad . \tag{17}$$

Equating the right sides of equations (14) and (17) reveals

$$\frac{|1-\Gamma_G\Gamma_T|^2}{|1-\Gamma_G\Gamma_L|^2}\frac{P_{Rcvr}^{RngInsrtd}}{P_{Rcvr}^{ShrtCct}} = \frac{\tfrac{1}{2}|b_0{}'|^2}{\tfrac{1}{2}|a_0|^2} \quad ; \tag{18}$$

and, including the impedance factor $|F'|^2$, eqn (9) in both sides of (18), gives us the result we seek; namely

$$\left|\frac{b_0{}'}{a_0F'}\right|^2 = \left|\frac{(1-\Gamma_p{}'\Gamma_L)(1-\Gamma_G\Gamma_T)}{1-\Gamma_G\Gamma_L}\right|^2\frac{P_{Rcvr}^{RngInsrtd}}{P_{Rcvr}^{ShrtCct}} \quad . \tag{19}$$

This is the power-domain equivalent of the voltage expression derived earlier, eqn (13). Thus the power and voltage mismatch factors are in fact equivalent !

4. Comments on Apparent Discrepancy

Examination of eqn (17) provides the key to understanding why there appears to be a discrepancy between the power and voltage mismatch factors. Notice that all of the quantities in (17) are theoretical quantities, and one does not have a simple relationship of the form

$$\frac{P_{RcvPrb}^{Incident}}{P_{XmtAnt}^{Incident}} \text{ " = " } \frac{\tfrac{1}{2}|b_0{}'|^2}{\tfrac{1}{2}|a_0|^2} \quad ,$$

as might be expected intuitively. From eqns (16) and (17), one can see that the power quantities at the transmitter and at the receiver are not related similarly to the respective voltage quantities. This "non-parallelism" between the power and voltage domains is what gives rise to the apparent discrepancy between the voltage and the power mismatch factors of eqns (1) and (2).

5. Computation of the Far Field

If the far electric field of an antenna is the quantity to be determined from a planar near-field scanning measurement, one usually employs the asymptotic expression of Kerns [3], page 73, eqn 1.6-1,

$$\boldsymbol{E(r)} \approx -i\gamma \boldsymbol{s}_{q0}(\boldsymbol{R}k/r)a_0\frac{e^{ikr}}{r} \tag{20}$$

to compute $\boldsymbol{E}(\boldsymbol{r})$ from the complete vectorial scattering matrix $\boldsymbol{s}_{q0}$. The complete scattering matrix is determined from a pair of coupling products $D'(\boldsymbol{K})$, $D''(\boldsymbol{K})$ each of which is obtained from expressions of the form

$$D(\boldsymbol{K}) \equiv \frac{e^{-i\gamma d}}{4\pi^2}\left[\frac{b_0{}'(\boldsymbol{P_0})}{F'a_0}\right]\int\frac{b_0{}'(\boldsymbol{P})}{b_0{}'(\boldsymbol{P_0})}e^{-i\boldsymbol{K}\cdot\boldsymbol{P}}d\boldsymbol{P} \tag{21}$$

See Kerns' [3] eqn 3.1-3, p.87. Note the normalization factor in brackets which is given by the measured voltage ratio of eqn (13). To obtain the far electric field, it is the voltage mismatch factor of (13) that is used to renormalize the measured data ratio under the integral.

Recall from Kerns, the relationship between gain and the complete vectorial scattering characteristic is

$$\mathbf{G}(\boldsymbol{K}) = \frac{4\pi Y_0\gamma^2|\boldsymbol{s}_{q0}(\boldsymbol{K})|^2}{\eta_0(1-|\Gamma_T|^2)} \quad ; \tag{22}$$

a "square" appears in the entry for $\boldsymbol{s}_{q0}(\boldsymbol{K})$ since gain is a power quantity; this has the effect of squaring the voltage mismatch factor when gain is to be computed. Furthermore, when gain is computed from the scattering matrix as in eqn (21), an additional factor of $(1-|\Gamma_T|^2)$ enters the denominator.

Recall also from Kerns the relationship between the effective area and the complete vectorial receiving characteristic for the probe.

$$\boldsymbol{\sigma}_q{}'(\boldsymbol{K}) = \frac{4\pi^2\eta_0|\boldsymbol{s}_{0q}{}'|^2}{Y_0(1-|\Gamma_p{}'|^2)} \quad . \tag{23}$$

As shown by Newell et.al. [4], when one accounts for the probe through the use of its effective area or gain, then yet another factor of $(1-|\Gamma_p{}'|^2)$ enters the denominator.

The net effect of computing gain – a power domain quantity – from the voltage domain of planar near-field in terms of the gain of the probe antenna, is to go over to the use of the five-term power domain mismatch factor from the three-term voltage domain factor of eqn (13) [4].

One often finds near-field practitioners, in anticipation of arriving at far-field power domain quantities, performing all of the impedance-related corrections in a single five-term step [4,8]. This elegant approach, however, can then lead to confusion in understanding the individual origins of the five terms.

Summary

The three-term voltage mismatch correction of eqn (2) converts the measured voltage ratio into the theoretical near-field voltage ratio b_0'/a_0F' which is used to determine $\boldsymbol{s}_{q0}(\boldsymbol{K})$ and the far electric field.

The five-term power mismatch factor of eqn(1) converts the measured range insertion loss power ratio into the theoretical Friis ratio $P^{Avail}_{RcvAnt}/P^{Accptd}_{XmtAnt}$. It's square root is also used to determine the gain function G($\boldsymbol{K}$), as in [4] and [8].

To avoid errors, care must be employed not to misapply these factors. Proper usage is required for accurate determination of gain.

8. REFERENCES

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[5] Reference 2, p.109.
[6] Reference 3, p.87.
[7] Reference 1, p.2-2.

[8] "Validation Standards", p.16, P. Miller, 2004 AMTA Europe Short Course Notes,

9. ACKNOWLEDGMENT

The author wishes to acknowledge the very helpful discussions with Dr. Carl Stubenrauch.

Appendix A
Equation for Modeling a Voltage Receiver

Please refer to Figure A1 below where we illustrate a circuit model for a microwave voltage receiver.

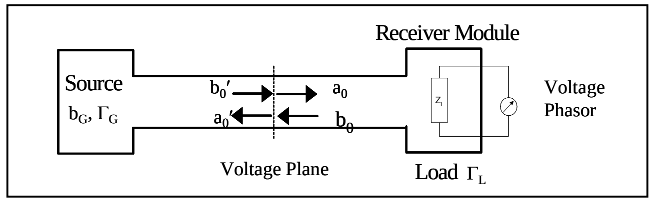

Figure A1 – Circuit Model of a Voltage Receiver

We show a simple case of a source of microwave excitation -- a generator – connected to an imperfect load that represents a the input to a microwave receiver. The load absorbs some of the microwave power incident and rejects the remainder due to the impedance mismatch of the load to the line.

Kerns [3, p.21] has shown that the voltage and current at the reference plane can be computed from the expressions

$$a_0 + b_0 = v_0 \qquad a_0 - b_0 = \varsigma^0 i_0 , \tag{A1}$$

where ζ^0 is the characteristic impedance of the waveguide. Following Kerns' description of source and load (Appendix B) we relate the phasors as follows:

$$\begin{aligned} b_0 &= \Gamma_L a_0 & b_0' &= \Gamma_G a_0' + b_G \\ b_0' &= a_0 & a_0' &= b_0 \end{aligned} \tag{A2}$$

We take the receiver's phasor readout to be proportional to v_0:

$$V_{Rcvr} = V_0 v_0 \tag{A3}$$

Here V_0 is a complex constant of proportionality that characterizes the gain and phase characteristic of this particular relative phase/amplitude receiver. After accounting for the effect of joining the source to the load we see from Kerns, eqn 6.2-3 (Appendix B) that

$$a_0 = \frac{b_G}{1 - \Gamma_G \Gamma_L} \tag{A4}$$

We can now compute the phasor voltage registered by the receiver:

$$V_{Rcvr} = V_0 v_0 = V_0 (a_0 + b_0) = V_0 (1 + \Gamma_L) a_0 \tag{A5}$$

The voltage read out by the receiver is proportional to the incident voltage a_0 via an impedance factor and a proportionality constant V_0 characteristic of the receiver.

Appendix B
Impedance Mismatch Equations from Kerns
[NOTE: In the body of the text we take η_0=1.]

6.2 Source and Load

To start with a simple and important case, we suppose that a one-port load is to be connected to a one-port source (or "generator") and that we wish to find the power delivered to the load (fig. 4). The load and the source are characterized by the equations

$$b_0 = \Gamma_L a_0, \qquad b_0' = \Gamma_G a_0' + b_G. \tag{6.2–1}$$

The equation on the left is a one-dimensional version of (5.3–3), and the one on the right comes directly from (5.7–2). [We adhere to the common convention of denoting reflection coefficients of one-ports (or equivalent one-ports) by Γ's.] The joining equations are

$$b_0' = a_0, \quad a_0' = b_0 \tag{6.2–2}$$

as follows from (5.7–5) and (5.3–1) (and as might be expected). Solving the system of (1) and (2) for b_0' or a_0, we obtain

$$a_0 = \frac{b_G}{1 - \Gamma_G\Gamma_L}. \tag{6.2–3}$$

The expression in the denominator is of a type characteristic of scattering equations: it can be interpreted in terms of a summation of waves multiply reflected between the source and the load.

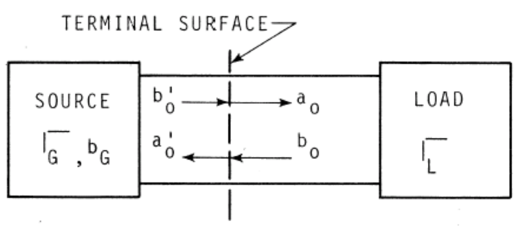

FIGURE 4. *Source and load.*

From (5.4–2), applied in the present case and under the present conventions, the net (time-average) power delivered to the load is

$$P_L = \frac{1}{2}\eta_0 |a_0|^2 (1 - |\Gamma_L|^2) = \frac{1}{2}\eta_0 |b_G|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_G\Gamma_L|^2}. \tag{6.2–4}$$

If, in particular, the load is a conjugate match to the generator ($\Gamma_L = \overline{\Gamma}_G$), then

$$P_L = P_A = \frac{1}{2}\eta_0 |b_G|^2 \frac{1}{1 - |\Gamma_G|^2}, \tag{6.2–5}$$

where P_A is termed the available power of the generator and is the maximum power that can be delivered by the given generator to a passive load. (Maximum power is not delivered to a reflectionless load unless the generator itself is reflectionless). The delivered power can be expressed in terms of the available power and a mismatch factor in the following ways:

$$P_L = P_A \frac{(1 - |\Gamma_G|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_G\Gamma_L|^2} \tag{6.2–6a}$$

$$= P_A \left[1 - \left|\frac{\Gamma_L - \overline{\Gamma}_G}{1 - \Gamma_G\Gamma_L}\right|^2\right]. \tag{6.2–6b}$$

This result recurs frequently, usually in the form of (6a). The second form shows clearly the maximum property of P_A.