MEASUREMENT OF EIRP AND ANTENNA RESPONSE FOR ACTIVE ANTENNAS
WITH SPHERICAL NEAR-FIELD SCANNING

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ABSTRACT

A measurement technique for Effective Isotropic Radiated Power (EIRP) using planar near-field scanning has been demonstrated earlier. In this paper I show how we at MI Technologies have implemented using the spherical near-field method the measurement of EIRP and a vector phasor quantity analogous to Effective Area that we call Antenna Receive Response. This technique is applicable to all antennas, including active antennas.

Keywords: Antenna Measurement, EIRP Measurement, Effective Area Measurement, Measurement of Active Aperture Antennas

1.0 Introduction

With active aperture antennas one does not necessarily have access to a single port of the antenna under test. Often with active aperture antennas, such a port does not exist. This fact imposes restrictions on the measurement of EIRP and Effective Area or Antenna-Response-Upon-Receive. The antenna must be measured with the range itself appropriately calibrated. Carrying out the appropriate measurement steps is then straightforward using a far-field or compact range. On a near-field range, however, the solution is less obvious.

Similarly, active antennas consisting of many T/R modules receiving coherently can pose a different type of measurement problem. A conventional array receiving antenna consisting of a single port fed via a combining network from an aperture of receiving elements (possibly augmented with individual low noise amplifiers) may be characterized by the quantity classically defined as effective area. (Effective Area is the ratio of received power available at the port of the antenna to the incident radiation power flux density in power per unit area.)

An active antenna formed of separate heterodyne receivers phase locked to an internal reference may not have a single port into which all of the signals are combined. Often, combining upon receive is done digitally. In this case it is appropriate to focus on the antenna’s effective area as a measure of the antenna’s effectiveness in receiving an incoming wave.

Measurement of EIRP using planar near-field has been demonstrated earlier [1]. In this paper I show how MI Technologies has realized the measurement of EIRP and also Effective Area using the spherical near-field method.

![Figure 1 Schematic of Active Aperture Antenna in Receive Mode](image)

2.0 Active Aperture Antennas

Figure 1 illustrates the block diagram of an active aperture antenna. It is shown here in the receive mode with the combining done digitally. Clearly there is not a single port at which a combined RF analog signal appears. Therefore the conventional definition of Effective Aperture is not useful in describing the action of the receiving antenna. Rather we define a quantity we call Antenna Receive Response. It is the digital level after combining that appears as the antenna’s response to a plane wave of known strength. It is a vector-phasor quantity in which an amplitude and a phase is associated with each of two far-field polarization components.

In the transmit mode of the active antenna, each transmitter generates an amplified outgoing high level signal from an input phase reference signal distributed to...
each transmitter from a common source. The total transmitted power is in general unrelated to the amplitude of the reference signal. Therefore the conventional definition of gain is not useful in describing the action of the antenna. Rather we focus on Effective Isotropic Radiated Power or EIRP to quantify the strength of the transmitted wave.

3.0 Far-Field Measurement of EIRP for Active Aperture Antennas

Consider the antenna of Figure 1 transmitting a wave into the far field where its strength is to be measured. Classically, the quantity of interest with the antenna in transmit mode would be the Radiation Intensity. Gain is defined as the ratio of the peak radiation intensity to the quantity $P_o/4\pi R^2$ where $R$ is the distance of the measuring antenna away from the transmitting antenna. The transmitting antenna illuminates the measuring antenna with a certain radiation power density $S$. The power received by the measuring antenna, whose gain and therefore effective area is known, is the product of the effective area and the area power density $S$.

The area power density $S$ of the transmitted wave and the radiation intensity $\Phi$ of the transmitted wave are related by

$$S = \Phi / R^2$$

A useful way of expressing the EIRP is as $4\pi \Phi$. Thus we can write the EIRP as

$$4\pi \Phi = 4\pi R^2 P / A_{\text{eff}}$$

The quantities on the right side are known for the far-field range. Thus we measure the far-field EIRP from knowledge of the range geometry, the known calibrated measuring antenna and a measurement of received power.

4.0 Near-Field Measurement of EIRP for Active Aperture Antennas

To measure EIRP for the transmitting antenna using spherical near-field we can expect to follow the same approach – i.e. to calculate it from a knowledge of the range geometry, a calibrated probe antenna and a measurement of received power at the port of the probe. The spherical near-field (SNF) transform computes the Normalized Far-Field voltage response of an elemental Hertzian dipole as a function of $\phi$ and $\theta$ – i.e. as a function of position on the radiation sphere.

The theta- and phi- components of the electric field of the test antenna, $E_\theta$ and $E_\phi$, are related to the Hertzian dipole voltage response $w^e$, sampled at two values of the polarization angle $\chi$ -- 0 degrees (0 radians) and 90 degrees ($\pi/2$ radians):

$$w^e(A;0,\theta,\phi) = \frac{\sqrt{6\pi\eta}}{2k} E_\theta(A,\theta,\phi)$$

$$w^e(A;\frac{\pi}{2},\theta,\phi) = \frac{\sqrt{6\pi\eta}}{2k} E_\phi(A,\theta,\phi)$$

where

$A$ = radius of the measurement sphere

$\eta$ = admittance of free space

$k = 2\pi/\lambda$

$\lambda$ = wavelength of the radiation

The units of the voltage quantities $w$, and later $v$, and of the spherical modal expansion coefficients are [watts]. See Hansen [2], (3.45 and 3.46) p. 77.

The normalized far field of the test antenna $W^e$ is defined as the voltage response of an electric dipole probe to a transmitting test antenna normalized by the factors $kA$ and $e^{ikA}$, and extrapolated to the far field - $kA \to \infty$

$$W^e(\chi;\theta,\phi) \equiv \lim_{kA \to \infty} \left[ W^e(A;\chi,\theta,\phi) \frac{kA}{e^{ikA}} \right]$$

See Hansen [2], (3.50) p. 78. The units of $w^e$ are [watts]$^{1/2}$ See Hansen [2], p.27, paragraph 2. Because the quantity $kA$ is dimensionless, the units of $W^e$ are also [watts]$^{1/2}$.

The far-field output of the TICRA software is related to the normalized far-field defined by Hansen by the relation

$$W_{\text{TICRA}} = \frac{2\sqrt{2}}{\sqrt{3}} W^e$$

This is because the TICRA software utilizes a very useful --but fictitious -- dipole probe that is a Hertzian dipole with a gain of 4 –i.e. 6dB, whereas Hansen’s dipole is a simple Hertzian dipole with a gain of 3/2 or 1.76 dB. The TICRA 6 dB dipole has a value of the probe receiving $R_{\text{sun}}$ characteristic [2] of 1.15 (i.e. 2/$\sqrt{3}$) whereas the Hertzian dipole has a value of $R_{\text{sun}}$ of 0.707 (i.e. 1/$\sqrt{2}$).

In the context of SNF theory, one can write an expression for the EIRP of an antenna identical in form to the usual classical EM expression:
\[ 4\pi \Phi_{snf} = \frac{\eta}{2} \left| E_{\theta}^2 + E_{\phi}^2 \right| R^2 \]

where \( R \) is a far-field distance \( \rightarrow \infty \).

With substitutions from the SNF equations above, one finds the simple result that the EIRP is given by

\[ 4\pi \Phi_{snf} = \frac{1}{2} \left| W_{T|copol} \right|^2 \]

where again Hansen’s impedance convention applies.

This SNF result is premised upon the assumption that the input to the SNF transform is expressed in absolute voltage units, not the relative antenna measurement receiver readings expressed in dB. To correct for the arbitrary units of the receiver a normalization of the relative receiver data must be made. The calibration factor derives from a receiver calibration using a power meter with the SNF probe located in a region of strong near-field energy, desirably at the values of \( \theta \) and \( \phi \) where the near-field peak is found.

Calibration Factor = \{ \[ P_{NFcop}(\chi) / \left| w_{NFcop}(\chi) \right|^2 \] \} \]

Here \( w_{NFcop} \) is the near-field response of the probe. This factor has the effect of “dividing out” the arbitrary receiver units and “multiplying in” the units in which the power meter quotes its reading. Since the SNF transform is linear, the calibration factor can applied either before or after the transform is taken. Note that this equation is linear power units such as mW, not in dBm. The EIRP can be expressed in logarithmic units such as dBW, once the calibration factor is applied.

### 3.0 Steps to Measure EIRP

1. Acquire a scan of near-field data, \( w_p(ANF; \chi, \theta, \phi) \) by reading the receiver versus \( \theta, \phi \) for both ports of the probe antenna:

\( w_p = w_p(ANF; \chi=0^\circ, 90^\circ; \theta, \phi) \)

\( ANF = \) Near Field Scan Radius

\( \chi = \) “polarization angle” or “probe port”

\( = 0 \) (0 degrees) or port 1,

\( = \pi/2 \) (90 degrees) or port 2.

\( \theta = \) Step Angle, \( 0^\circ \leq \theta \leq 120^\circ \)

\( \phi = \) Scan Angle, \( -180^\circ \leq \phi \leq +180^\circ \)

2. Identify the near-field peak in the acquired data file:

\( w_{NFpk} = w_p(ANF; \chi = 0^\circ \) or \( 90^\circ; \theta_{NFpk}, \phi_{NFpk}) \)

3. Measure the probe response for the stronger co-polarized port, expressed in Watts with the probe located at the position of the same near-field peak as in 2, using a power meter attached to the probe.

\[ P_{NF} = P_{NF}(ANF; \chi = 0^\circ \) or \( 90^\circ; \theta_{NFpk}, \phi_{NFpk}) \]

4. Run the SNF transform (without any normalization such as for directivity), and find the peak in the resulting output data set corresponding to the normalized far field defined by Hansen [2] as

\[ W_{FFpk} = \lim_{kA \to \infty} \left( \frac{kA}{e} \right) \]

5. Evaluate EIRP for the peak far-field point by applying the calibration factor developed in steps 2 and 3 above.

### 5.0 Far-Field Measurement of Effective Area for Active Aperture Antennas

Consider the antenna of Figure 1 receiving a planar wave of known electric field strength generated by a far-field source. We would like to measure the sensitivity of the antenna system to this wave. Classically, the quantity of interest with the antenna in receive mode would be the Effective Area of the receiving antenna. Effective Area is defined as the ratio of the peak received power \( P_R \) at the port of the antenna relative to the area power density of the planar wavefront.

\[ A_{eff} = \frac{P_R}{S} \]

Assume \( R \) is the distance of the source antenna away from the receiving antenna. The source antenna illuminates the receiving antenna with a certain radiation area power density \( S \). The radiation power density due to the source antenna is given by

\[ S = G P_T / (4\pi R)^2 \]

Where \( P_T \) is the power accepted by the transmitting source antenna from the signal generator and \( G \) is the gain of the source antenna, which is assumed to be known.

The power \( P_R \) received at the port of the receiving antenna whose effective area is not yet known is the product of that effective area and the area power density

\[ A_{eff} = \frac{4\pi R^2 P_R}{G S} \]

S. Thus the unknown effective area is given by

\[ A_{eff} = \frac{4\pi R^2 P_R}{G S} \]

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This text represents the document's content in natural text format, ensuring all mathematical expressions and equations are accurately transcribed and formatted for readability. The document is centered around the measurement of Effective Area for Active Aperture Antennas, detailing the steps to measure the Effective Near Field (ENF) through the use of a Near-Field Scanner (SNF) and applying a calibration factor. The document also emphasizes the importance of understanding the relationship between near-field and far-field measurements, particularly in the context of antenna design and analysis.
The quantities on the right side are known for the far-field range. Thus we measure the far-field Effective Area from (1) knowledge of the range geometry, (2) the transmitting characteristic of the known calibrated source antenna and (3) measurements of both transmitted and received power levels.

6.0 Near-Field Measurement of Antenna Response or Effective Area for Active Aperture Antennas

To measure the receiving characteristic for the active aperture receiving antenna using spherical near-field we can expect to follow the same approach – i.e. to calculate it from a knowledge of the range geometry, a calibrated probe antenna and a measurement of transmitted power levels at the port of the probe and the received signal at the port of the test antenna.

As can be seen from Figure 1, if the antenna is a active aperture type of antenna and if array combining is performed digitally, there is not a single received microwave signal. Rather the received signal exists only in digital form. It may be rendered in arbitrary antenna units.

We define the Receive Response $R_{rev}$ of the active aperture test antenna to be the Received Voltage $w_{t}^{FF}$, here written in [Watts]$^{½}$ according to the Hansen convention, in ratio to the magnitude of the Incident Electric Field Strength $E_o$ of the illuminating planar wavefront, written here in [Volts-per-meter] as is done by Hansen [2].

$$R_{rev} = w_{t}^{FF}/E_o$$

This is a voltage quantity as is appropriate for near-field theory, not a power quantity such as the classical quantity Effective Area is.

Since the test antenna is receiving, and the “probe antenna” is transmitting, the usual form of the SNF transmission equation is not directly applicable. Rather one starts out by making use of an alternate form that is written in terms of adjoint quantities for the transmitting and receiving characteristics of the probe and test antenna. See Hansen’s equations 3.17 and 3.20 [2]. When the adjoint quantities are employed, the SNF transmission equation still retains its form. This expression is appropriate for the probe antenna at any distance outside the test antenna’s minimum sphere. It calculates $w_t$ the response of the test antenna to an electric dipole.

Hansen [2] also develops on pp.110-120 a form of the transmission equation that utilizes the concept of the infinitely remote probe antenna. Combining these two approaches, one can calculate the response of the test antenna to a transmitting probe antenna such as an electric dipole at an “infinite” distance away. – i.e. in both the near field and the far field. In particular one can assume the infinitely remote probe antenna is a Herzian dipole.

Another approach to calculating Antenna Response is to calculate the test antenna’s response to a planar wave, $w_t^{PW}$, using adjoint probe coefficients that correspond a “planar wave generator used in receive mode.”

These two approaches yield the same result

$$W_t = W_{t}^{PW}$$

provided that the following equation holds:

$$\nu_p = \frac{2 \sqrt{\eta}}{k} \left( \frac{2 \pi}{3} i E_o \right)$$

where

$E_o$ is the strength of the illuminating electric field

in [volts/m]$^{½}$

$\nu_p$ is the input voltage at the port of the “transmitting” probe antenna

$i$ is the square root of $-1$, with the -i$t$ time convention

The form of the spherical near-field (SNF) transmission equation used to derive our expression for Receive Response in fact computes the voltage response of the test antenna to an infinitely remote elemental Herzian dipole as a function of $\varphi$ and $\theta$ -- i.e. as a function of its position on the radiation sphere.

When the TICRA probe parameters are included in the consideration, one finds that

$$R_{rev} = \frac{W_{t}^{PW}}{E_o} = \frac{\sqrt{\pi \eta}}{ki} \mathcal{W}^{TICRA}$$

In principle, then we can compute the antenna’s receive response using the TICRA spherical near-field to far-field transform and the relationship above. An explicit expression illustrating the working equation is

$$R_{rev} = \frac{w_{t}^{FF}/E_o}{(-j)(\nu_p)} = \left( \frac{\sqrt{\pi \eta}}{k} \right) \left( \frac{-1}{\sqrt{2P_o}} \right)$$

$$\mathcal{W}^{TICRA}(0, \theta_{FFpk}, \phi_{FFpk}) e_\theta +$$

$$+ \mathcal{W}^{TICRA}(\pi/2, \theta_{FFpk}, \phi_{FFpk}) e_\phi$$

This expression differs from the previous one in the following manner:
The time convention has been changed to $+j\omega$

(2) the antenna response is represented as a vector with a $\theta$- and a $\phi$- component

(3) the factor $1/\sqrt{2P_p}$ has been added to account for the proper power normalization of the relative near-field receiver readings on the input side of the transform.

Further corrections to this expression may be needed to account for the impedance convention and a calibration factor for the receiver in the active antenna.

There is a power-based analog of the voltage Receive Response. Recall the definition of Receive Response. Taking the square of both sides of this result gives

$$(R_{rcv})^* \cdot (R_{rcv}) = \pi \eta \left[ \frac{1}{2} \left| W_{TICRA}^{Rcv}(\theta, \phi_{FFpk}) \right|^2 P_p + \frac{1}{2} \left| W_{TICRA}^{Rcv}(\pi/2, \theta_{FFpk}, \phi_{FFpk}) \right|^2 \right]$$

The quantity on the left can be rewritten in the Hansen impedance convention as

$$\frac{(R_{rcv})^* \cdot (R_{rcv})}{\eta} = \frac{1}{2} (|w_{FF}|)^2 = \frac{P^R_{rcv}}{\eta \left| E_{in} \right|^2 S_{illum}}$$

where $P^R_{rcv} = \text{Power of the signal received by the antenna under test}$

and $S_{illum} = \text{Power flux density of the illuminating planar wave.}$

Thus

$$\frac{P^R_{rcv}}{S_{illum}} = \frac{\lambda^2}{4\pi} \left[ \frac{1}{2} \left| W_{TICRA}^{Rcv}(\theta, \phi_{FFpk}) \right|^2 + \frac{1}{2} \left| W_{TICRA}^{Rcv}(\pi/2, \theta_{FFpk}, \phi_{FFpk}) \right|^2 \right] P_p$$

Recall that the definition of effective area for the antenna under test would be

$$A_{t}^{eff} = \frac{P^R_{rcv}}{S_{illum}}$$

Therefore we can also write that

$$A_{t}^{eff} = \frac{\lambda^2}{4\pi} \left[ \frac{1}{2} \left| W_{TICRA}^{Rcv}(\theta, \phi_{FFpk}) \right|^2 + \frac{1}{2} \left| W_{TICRA}^{Rcv}(\pi/2, \theta_{FFpk}, \phi_{FFpk}) \right|^2 \right] P_p$$

If the test antenna were reciprocal, and in view of the classic conversion factor of $\lambda^2/4\pi$, the equivalent gain, measured upon the receive mode, would be written

$$G_{t}^{Rcv} = \frac{1}{P_p} \left[ \frac{1}{2} \left| W_{TICRA}^{Rcv}(\theta, \phi_{FFpk}) \right|^2 + \frac{1}{2} \left| W_{TICRA}^{Rcv}(\pi/2, \theta_{FFpk}, \phi_{FFpk}) \right|^2 \right]$$

Note that $A_{t}^{eff}$ and $G_{t}^{Rcv}$ are power-based quantities, whereas $W_{TICRA}$ is a voltage quantity. However, the numerator of this expression is a power quantity and is equal to the total power in both far-field components, as the Hansen impedance convention [2] still applies. The dimensions of this expression are of course unitless.

### 7.0 Steps to Measure Receive Response

1. Acquire a scan of near-field data $w_{t}^{NF}$ by reading the receiver as a function of $\theta$ and $\phi$ for each port of the probe antenna.

$$w_t^{NF} = w_d(A_{nF} ; \chi = 0^\circ \text{ & } 90^\circ ; \theta, \phi)$$

2. The common port of the probe antenna is excited by a voltage signal $v_{p}^{FF}$. The common port of the probe antenna is excited by a voltage signal $v_{p}^{FF}$.

3. Measure the input power to the probe by use of a power meter to obtain $P_{NF}^{p}$.

4. Run the TICRA SNF transform without directivity normalization to obtain the normalized far field

$$W_{TICRA}^{NF} = \lim_{k \to \infty} \left( \frac{kA}{e^{i\theta \cdot \phi}} W_{t}(\chi \cdot \theta, \phi) \right)$$

5. Find the far-field peak in the normalized far field $W_{FFpk}^{NF} = W_{TICRA}^{NF}(\chi \cdot \theta_{FFpk} \phi_{FFpk})$

Evaluate Receive Response from the peak far-field point by multiplying by $(\sqrt{\pi} \sqrt{\eta}) \left( \frac{1}{k} \right) \left( \frac{1}{2P_{NF}^{p}} \right)^{1/2}$

### 8.0 Verification of Software for EIRP Measurement

We have verified the consistency of the EIRP subroutine in the case of a special edition of the MI-3000 software by the procedure described below.

1. First we measured the gain of a 24 inch dish antenna at 13 GHz by the method of range insertion loss and spherical near-field scanning. The result neglecting the impedance mismatch correction was 32.2 dB.

2. Second we used a power meter to measure the amount of power from the signal source impinging upon the port of the dish antenna; the result was +8.5 dBm.

3. The EIRP is computed as the product of Gain and Input Power, or +40.7 dBm, which is +10.7 dBW.

4. We then measured the EIRP by successively measuring the comparison between the receiver reading and the power meter reading at the port of the near-field.
probe at a point on the SNF pattern corresponding to the near-field peak and ran the EIRP analysis
5. The result was +10.27 dBW. The comparison to step 3. is within approximately 0.4 dB and was deemed acceptable in view of the omission of the impedance mismatch correction and other significant details. The figure below illustrates the connections.

Figure 2. Connections for EIRP Measurement

9.0 Verification of Software for Receive Response

We have verified the consistency of the Receive Response subroutine in MI-3000 special software by the method described below.

1. First we measured the gain of the 24 inch dish antenna at 13 GHz by the method of range insertion loss and spherical near-field scanning. The result neglecting the impedance mismatch correction was 32.2 dB (Same result as used for EIRP).

2. Second we used a power meter to calibrate the MI-1797 receiver by comparing at the near-field peak, the value out of the test antenna read by the receiver to the value read by the power meter. The comparison was

<table>
<thead>
<tr>
<th>MI-1797 Receiver Reading</th>
<th>+9.56 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Meter Reading</td>
<td>-27.4 dBm (57.4 dBW)</td>
</tr>
</tbody>
</table>

This calibration is for reference here. On other systems, the active antenna receiver may already be calibrated.

3. Then we proceed to measured the Receive Response of the dish. First we measured the power of the signal impinging upon the probe antenna port from the signal source. The result was +4.5 dBm or -25.5 dBW.

4. To complete the measurement of the Receive Response a spherical near-field scan was taken and the far-field transformed result was obtained (without any normalization for directivity.) The peak far-field value was +73.1 dB-[V^2].

5. The Receive Response in terms of Gain Upon Receive is given by the impedance-adjusted formula

\[
G_{T_{rcv}} = \frac{1}{P_p} \left[ \frac{1}{Z_{rcv}} \left| \mathcal{W}_{TICRA}(0, \theta_{FFpk}, \phi_{FFpk}) \right|^2 \right] + \frac{1}{P_p} \left[ \mathcal{W}_{TICRA}(\pi/2, \theta_{FFpk}, \phi_{FFpk}) \right]^2
\]

Presuming that the impedance of the antenna port is 50 Ohms, so that 10log(1 / Z_{rcv}) = -17.0 dB-Mho, we find for the nominal result

10log( G_{T_{rcv}} ) = +73.1 dB-[V^2] - (-25.5 dBW) - (-17.0 dB-Mho) = +81.6 dB.

To further compare it to the gain of the antenna obtained in step 1 above, the result must be adjusted for the calibration of the MI-1797 receiver. The receiver reading used to compute the far-field and the absolute power meter reading in step 2 above differ by a factor of -57.4 dBW -9.56 dB = -67.0 dB-Mho. Again, we are presuming an impedance for the AUT of 50 Ohms. The factor by which the input to the transform should be adjusted is thus

\[
(Z_{rcv})_{PNF}^{AUT} = \frac{1}{P_p} \left| \mathcal{W}_{TICRA}(0, \theta_{NFpk}, \phi_{NFpk}) \right|^2 = 17.0 \text{ dB-Ohm} + (-67 \text{ dBW}).
\]

The adjusted value of \( G_{T_{rcv}}^{PNF} \) is thus +81.6 dB +16.9897 dB-Ohm – 67 dB-Mho = (81.6 + 17.0 - 67) dB or \( G_{T_{rcv}} = 31.6 \text{ dB}. \)

To within the accuracy of these non-impedance – corrected measurements, this agrees with the value of 32.2 dB measured in step 1. (These are only nominal values of measured gain used for the purpose of this software checkout.) Note that the connections between the antenna pair and the measurement system are “range-reversed” from the connection shown above for EIRP.

10. Summary

For the first time known to the author, EIRP and Receive Response techniques capable of operating on active aperture antennas have been devised and implemented using spherical near-field scanning.

9. REFERENCES


9. ACKNOWLEDGMENTS

I wish to express my appreciation to Graeme Hedgecock of Alenia Marconi Systems for informative discussions leading to the ideas contained in this paper. Also appreciation to my colleagues at MI Technologies who have helped in defining the solutions presented here.