ESTIMATING THE UNCERTAINTIES DUE TO POSITION ERRORS IN SPHERICAL NEAR-FIELD MEASUREMENTS

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ABSTRACT

Probe position errors, specifically the uncertainty in the theta and phi position of the probe on the measurement sphere, are one of the sources of error in the calculated far-field and hologram patterns derived from spherical near-field measurements. Until recently, we have relied on analytical result for planar position errors to provide a guideline for specifying the required accuracy of a spherical measurement system. This guideline is that the angular error should not result in translation along the arc of the minimum sphere of more than λ/100.

As a result of recent simulation and analysis, expressions have been derived that relate more specifically to spherical near-field measurements. Using the dimensions of the Antenna Under Test (AUT), its directivity, the radius of the sphere (the minimum sphere) enclosing all radiating surfaces and the frequency we can estimate the errors that will result from a given position error. These results can be used to specify and design a measurement system for a desired level of accuracy and to estimate the measurement uncertainty in a measurement system.

Keywords: Antenna measurements, Measurement errors, Spherical near-field.

1.0 Introduction

Equations have been derived for planar near-field measurements[1] that give an estimate of the error in the far-field due to X Y and Z position errors. These have been derived using the equations relating near-field to far-field transformations. The complexity of the spherical transformation equations prevents a similar derivation for this measurement type and so equations have been derived using a combination of reference to the planar results and empirical tests. The following describes the results of those tests.

2.0 Simulation Approach

A theta position error in spherical near-field measurements will produce errors in all three linear dimensions and since the Z-error has the largest effect, we begin by assuming that the theta position error equation will be similar to one of the following equations for the planar Z-error in peak gain and sidelobe respectively.

$$\Delta G_{dB}\left(\theta,\phi\right) \le \frac{43}{\sqrt{\eta}}\left(\frac{\delta_z(rms)}{\lambda}\right)^2 \cos^2\theta_B \, g\left(\theta,\phi\right) \quad (1)$$

$$\Delta P_{dB}\left(\theta,\phi\right) \le \frac{13.5\,\delta_z(\theta,\phi)}{\lambda} \cos\theta_B \, g\left(\theta,\phi\right) \quad (2)$$

where G is the antenna gain, P the far-field pattern amplitude λ the wavelength, η the aperture efficiency, δ_z the Z-position error and θ_B the direction of the main beam. $g(\theta,\phi)$ is the reciprocal of the sidelobe level in voltage not dB, for instance, for a –40 dB sidelobe, $g(\theta,\phi)$ is 100.

To derive a similar relation for the spherical case measured spherical near-field data for six antennas that represent the types generally measured was selected. The antennas are:

1- A Y-polarized dipole element at the origin.
2- A Y-polarized dipole offset from the origin.
3- A pyramidal horn.
4- A slotted planar array.
5- A small patch array mounted on a large model.
6- A reflector antenna.

A program was developed that could be used with the measurement software to access the measured data, calculate the derivatives of the amplitude and phase, $\dfrac{da}{d\theta}$ and $\dfrac{d\phi}{d\theta}$ at each point and calculate the change in the amplitude and phase at each position for different types of position errors.

$$\Delta a = \frac{da}{d\theta}\delta\theta \qquad \Delta\phi = \frac{d\phi}{d\theta}\delta\theta \tag{3}$$

Three types of errors were considered.

$$\begin{gathered}\text{A perodic error}\\ \delta\theta=\theta_{\max}\cos(2\pi\theta / Period)\\[1em] \text{A random error}\\ \delta\theta=\theta_{\max}\left[Random\left(\pm 1\right)\right]\\[1em] \text{A linear scale error}\\ \delta\theta=\theta_{\max} * \left(\theta / \theta\, span\right)\end{gathered} \tag{4}$$

These errors were applied to each data set for various values of $\theta_{\max}$ ranging from 0.01 degrees to 0.2 degree. In each case, the far-field patterns were compared with the unmodified data to determine the error signal level and the change in the far-field peak value due to the applied position error. The program also calculated four parameters related to the phase derivative since it was assumed that the phase error would have the major effect and a single parameter was needed that would characterize the derivative over the complete measurement region or over the full sphere. The four parameters were:

1- The average phase derivative over the measurement region.

$$\bar{D}_M = \frac{\sum_{i=1}^{N} \frac{d\phi}{d\theta_i}}{N} \tag{5}$$

$$N = \text{Total Number of Measurement Points}$$

2- The amplitude weighted average of the phase derivative over the measurement region.

$$\bar{D}_{WM} = \frac{\sum_{i=1}^{N} a_i \frac{d\phi}{d\theta_i}}{N} \tag{6}$$

$$a_i = \text{measured amplitude}$$

3- The average phase derivative over the full sphere.

$$\bar{D}_S = \frac{\sum_{i=1}^{N} \frac{d\phi}{d\theta_i}}{N_S}$$

$$N_S = \text{Number of points over complete sphere} \tag{7}$$

4- The amplitude weighted average of the phase derivative over the full sphere.

$$\bar{D}_{WS} = \frac{\sum_{i=1}^{N} a_i \frac{d\phi}{d\theta_i}}{N_S} \tag{8}$$

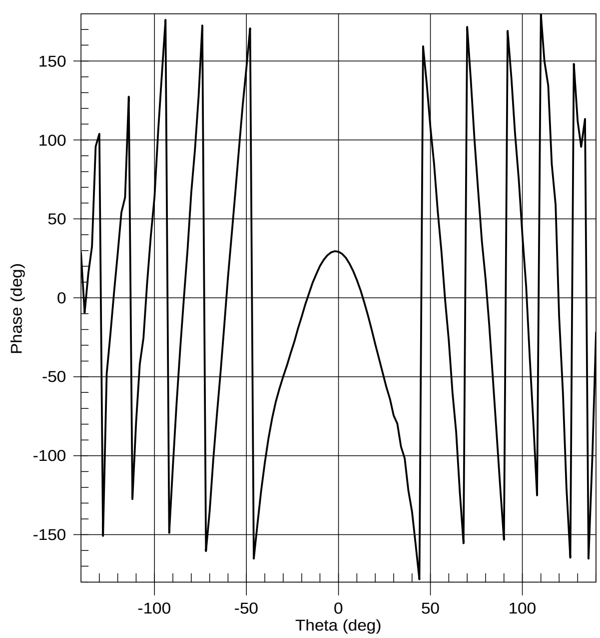

Figure 1 Measured near-field phase for pyramidal horn antenna at 9.5 GHz.

3.0 Simulation Results

Figure 1 shows a sample of the near-field phase for the horn antenna while Figures 2 shows the phase error that results from a periodic theta position error with a maximum value of 0.2 degrees and a period of 40 degrees. The resulting far-field patterns with and without the position error and the error signal level are shown in Figure 3.

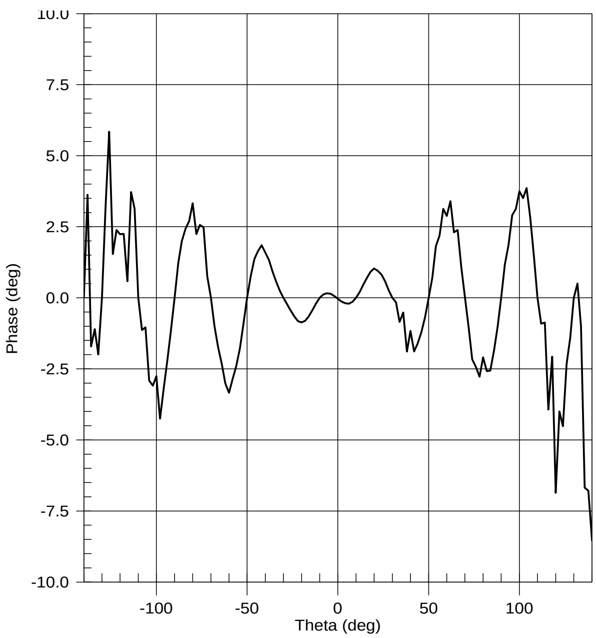

Figure 2 Phase error for pyramidal horn antenna due to periodic theta position error of 0.2 deg.

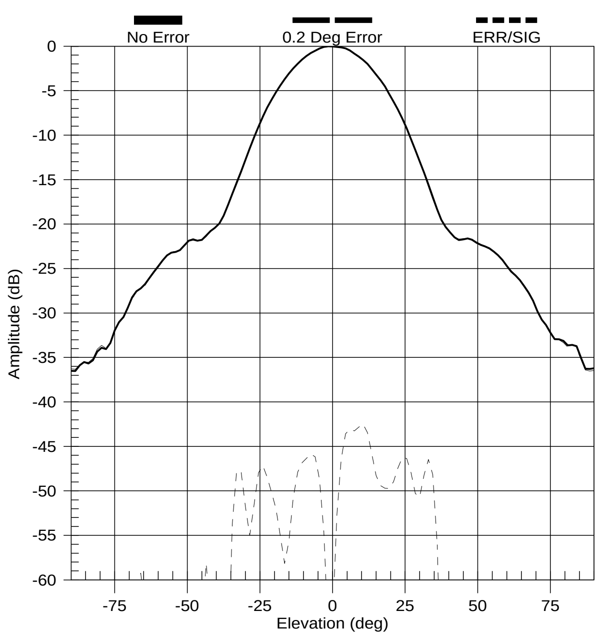

Figure 3 Error signal level in the far-field pattern due to periodic theta position error of 0.2 degree.

The peak error signal level was tabulated for each of the tests and was used to derive a general equation for all antennas and error types. For the example in Figure 3, the tabulated error was –45 dB.

The initial conclusions from the tests were:

1- The error signal level was proportional to the magnitude of the position error and not the square of the position error and so the derived equation would be similar to Equation 2 and not Equation 1.
2- The error signal was relatively insensitive to the error type. Periodic, random and scale errors produced error signal levels of approximately the same magnitude although the details of the error were different.
3- Periodic errors had a slightly larger effect than the other two and so this type was used to establish an upper bound equation that would cover all cases.

Replacing the Z-position error in EQ. (2) with the equivalent phase error, and assuming that the phase error in the spherical measurement will be proportional to the maximum position error and some measure of the phase derivative, the desired error equation is then assumed to have the form,

$$\begin{aligned} &\Delta P_{dB}(\theta,\phi) \le \alpha \frac{\theta_{\mathbf{max}} \bar{D}}{360} \\ &\left[\text{Sidelobes and Mainbeam}\right] \\ &\mathit{and} \\ &\frac{Err}{Sig} \le \frac{\alpha}{8.6} \frac{\theta_{\mathbf{max}} \bar{D}}{360} \end{aligned} \tag{9}$$

In Equation (9) $\bar{D}$ could be any of the average derivatives defined in Equations (5) – (8). The “best” one to use was determined by using the results of one error simulation for each antenna type and calculating the Err/Sig ratio and the four derivative averages for each simulation. These results along with the input maximum theta error were then used to solve for α in Equation (9). The “best” derivative parameter should give consistent results for the constant α in Equation (9) for all the antenna types. Table 1 summarizes the results of that calculation and demonstrates that the weighted average over the full sphere gives the most consistent results. When the average constant derived from this calculation is used to recalculate the Err/Sig level, the results are within 2 dB of the measured values for all antenna types.

Using the average of the constant calculated from the weighted average over the full sphere the equation to estimate the effect of the theta position error is then

$$\left.\frac{Err}{Sig}\right._{dB} \leq 20Log\left(0.075\,\theta_{\mathbf{max}}\bar{D}_{WS}\right) \qquad (10)$$

In Equation (10) and the following, $\theta_{\max}$ is in degrees. This equation can be used after measurements are completed to estimate the effect of theta position errors by calculating the average derivative over the full sphere using the measured data, but additional tools are needed to predict the error for a new antenna measurement before acquiring the data. To predict the error, we need some method of estimating the average derivative of the phase over the full sphere. From Table 1 this parameter varies greatly for different antenna types, frequencies and measurement conditions. In general $\bar{D}_{WS}$ will be largest for a small antenna like a dipole that is mounted far away from the center of the sphere operating at a high frequency. It will be smallest for a directive antenna mounted near the center of the sphere operating at a low frequency.

While it is difficult to predict the average of the phase derivative for a given antenna, the maximum phase slope can be predicted from the frequency and the radius of the minimum sphere that completely encloses the AUT. Given these two parameters, the maximum spacing of the measurements points in the theta direction is

$$\Delta\theta = \frac{2\pi}{2\left(\frac{2\pi}{\lambda}R\right)} = \frac{180}{\pi}\frac{\lambda}{2R} \text{ degrees} \qquad (11)$$

$$\textit{where } \text{R = radius of minimum sphere}$$

And from the sampling criteria, the phase cannot change by more than 180 degrees over this measurement interval. The maximum phase slope for a given frequency and R is

$$\frac{d\phi_{\max}}{d\theta} \leq \frac{2\pi R}{\lambda} \text{ Degrees Phase/Degree in theta} \qquad (12)$$

Using this maximum instead of the weighted average, would give an unrealistically large error estimate, and it would be desirable to have an approximate expression to estimate the weighted average of the phase derivative for any given antenna and measurement situation. The simulation has shown that for antennas at or near the origin, the ratio between the maximum phase change and the weighted average is not too dependent on the AUT. When the antenna is located away from the origin of the measurement sphere, the relationship depends on the antenna type and the distance from the origin. A simple and useful expression has been derived from the simulation that uses the maximum phase derivative, a constant independent of the AUT and the directivity of the AUT, to provide an estimate of the error level that can be used before measured data is available. Once measurements are performed, a more reliable error level can be obtained by calculating the weighted average derivative and using Equation (10). The approximate expressing is,

$$\left.\frac{Err}{Sig}\right._{dB} \approx 20Log\left(\frac{0.11R\,\theta_{\mathbf{max}}}{\lambda\sqrt{Dir}}\right) \qquad (13)$$

4.0 Summary

Simulation on measured spherical near-field data has resulted in expressions that can be used to estimate the error due to theta position errors. These can be used on measured data or to approximate the error for a given antenna and measurement situation.

5. REFERENCES

[1] A. C. Newell, Error analysis techniques for planar near-field measurements, IEEE Trans. Antennas & Propagation, AP-36, p. 581, 1988.

[2]J. E. Hansen, Spherical near-field antenna measurements, Chapter 6, Peter Peregrinus, Ltd, 1988.

[3]A. C. Newell, Range evaluation and uncertainty analysis of near-field and far-field antenna measurements, Part II, AMTA Short Course, Philadelphia, PA 15 Oct 2000.

[4]Doren W. Hess, An expanded approach to spherical near-field uncertainty analysis, AMTA Proceedings, Cleveland, OH, 2002.

AUT	Freq. (GHz)	Amp Err (Deg.)	ERR/SIG dB	Min Sphere Radius (m)	Measured Derivative Averages				Error Constant α Calculated Using				ERR/SIG dB
					Meas. Area		Full Sphere		Meas. Area		Full Sphere		Calculated
					Ave Slope	Wt Ave	Ave Slope	Wt Ave	Ave Slope	Wt Ave	Ave Slope	Wt Ave	Using Wt Ave α, Full Sphere
Dipole (Origin)	0.82	0.2	-47	0.21	1.5	0.41	1.5	0.4	33	119	33	173	-45
Dipole(Offset)	1.72	0.2	-30	0.40	9.9	2.5	9.9	2.5	49	196	49	196	-29
Horn	9.5	0.2	-45	0.25	13.9	0.41	10.8	0.32	6	212	8	272	-47
Slot Array	9.338	0.2	-42	0.36	15.5	1.36	7.8	0.68	8	90	16	181	-40
Patch/Model	15.6	0.2	-15	0.91	58.3	22.7	32.4	12.6	47	121	85	218	-15
24 " Dish	11.7	0.05	-46	0.38	38.8	2.1	19.4	1.06	8	148	16	293	-48
						Average Const =			25	148	34	222	

Table 1 Summary of results from error simulation.