Near-Field Antenna Measurement
Theory III
Spherical
Overview

- Spherical Coordinate Systems
- Spherical Modes
- Development Of Transmission Equation
- Comparison To Planar And Cylindrical
- Translation Of Centers For Probe Receiving Coefficients
- Far-field Quantities
- Probe Correction
- Sample Data
Spherical Measurement Schematic

Showing definition of three angular variables $\theta, \phi, \chi$. 
E-Field And Spherical Waves

\[ \vec{E}(r, \theta, \phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ Q^{1mn} \vec{M}^{nm}(\theta) + Q^{2mn} \vec{N}^{nm}(\theta) \right] e^{im\phi} \]

\( \vec{M}^{nm} \) and \( \vec{N}^{nm} \) are the spherical wave functions which define the amplitude and phase of the spectrum of waves that will represent any field.
Development of Transmission Equation

- Scattering matrix for AUT using spherical modes
- Scattering matrix for probe using AUT coordinate spherical modes
- With probe at $\theta$ and $\phi = 0$ develop transmission equation for:
  - Single mode, single polarization
  - Single mode, dual polarization
  - Spectrum of modes
- Rotate probe to $(\theta, \phi, \chi)$
Spherical Near-field Scanning

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Spherical Transmission Equation

\[ W^a(\chi, \phi, \theta) = \sum_{m} \sum_{n} \sum_{\mu} \left( \sum_{s=1}^{2} P_{s}^{\mu n a} Q_{s m n} \right) e^{i m \phi} \ d_{m \mu}^{(n)}(\theta) \ e^{i \mu \chi} \]

where \( W^a(\chi, \phi, \theta) = \frac{b'_0(\chi, \phi, \theta, a)(1 - \Gamma \Gamma')}{a_0} \)

= measured data on sphere of radius a
Definition Of Indexes

\( p^{s\mu na} \) = Probe spherical mode coefficients with respect to measurement sphere of radius \( a \).

\( Q^{smn} \) = AUT spherical mode coefficients for position and orientation of AUT in measurement sphere.

- \( s \) = polarization index = 1,2 ; 1= TE modes, 2=TM modes
- \( m \) = azimuthal (\( \phi \)) index for AUT,
- \( \mu \) = azimuthal (\( \phi \)) index for probe.
- \( n \) = polar angle (\( \theta \)) index for both probe and AUT
Transmission Equation Comparison

\[ b'(x, y, d) = F'a_0 \int \int \left[ \tilde{s}'_{02}(\vec{K}) \cdot \tilde{t}_{10}(\vec{K}) \right] e^{i\gamma d} e^{i(k_x x + k_y y)} \, dk_x \, dk_y \]

\[ b'(\phi_0, z_0) = F'a_0 \int \sum_{n=-\infty}^{\infty} \left[ \sum_{s=1}^{2} R'^s_s(\gamma) T^n_s(\gamma) \right] e^{i\phi_0} e^{i\gamma z_0} \, d\gamma \]

\[ W^a(\chi, \phi, \theta) = \sum \sum \sum \sum \left[ \sum_{s=1}^{2} P^{s\mu n a} Q^{s m n} \right] e^{i\phi} d^{(n)}_{m\mu}(\theta) e^{i\mu \chi} \]
Far-field And Gain Equations

\[ \vec{E}_{\lim r \to \infty} (r,\theta,\phi) = \frac{a_0 e^{ikr}}{kr} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} (-1)^{n+1-s} \left[ Q^{1mn} \vec{M}^{nm} (\theta) + Q^{2mn} \vec{N}^{nm} (\theta) \right] \]

\[ G(\theta,\phi) = \frac{4\pi\eta_0}{k^2 \left( 1 - |\Gamma'_0|^2 \right)^2} \left| \sum_{n=1}^{\infty} \sum_{m=-n}^{n} (-1)^{s-n} \left[ Q^{1mn} \vec{M}^{nm} (\theta) + Q^{2mn} \vec{N}^{nm} (\theta) \right] \right|^2 \]
Solution Of Transmission Equation

- Inversion of Transmission Equation to solve for the coupling product
- Probe Correction to solve for AUT Transmitting coefficients
- Calculation of Far-field, Gain, Polarization
Inversion Of Transmission Equation

Using the Orthogonality Relationship

\[
\frac{\delta_{nn'} \delta_{mm'} \delta_{\mu\mu'}}{(2n + 1)} = \frac{1}{8\pi^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \left( \sum_{m,n,\mu} e^{im\phi} d^{(n)}_{m\mu}(\theta) e^{i\mu\chi} \right) \\
\times \left( e^{-im'\phi} d^{(n')}_{m'\mu'}(\theta) e^{i\mu'\chi} \right) \sin \theta \, d\theta \, d\phi \, d\chi
\]
Inversion Of Transmission Equation

\[ I^{m', \mu', n', a} = \frac{1}{8\pi^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} W^{a}(\chi, \theta, \phi) \left( e^{-im\phi} d^{(n')}_{m', \mu'}(\theta) e^{i\mu\chi} \right) \times \sin\theta \, d\theta \, d\phi \, d\chi \]

Where,

\[ I^{m', \mu', n', a} = P^{1\mu', n', a} Q^{1m', n'} + P^{2\mu', n', a} Q^{2m', n'} \]
Problems With Spherical Calculations

1. For arbitrary probe, measurements must be taken for small increments of probe rotation $\Delta \chi$, requiring scanning over sphere many times and increasing computations.

2. Integration in $\theta$ with its evaluation of the complicated function $d_{m \mu}^{(n)}(\theta)$ is time consuming and uses approximate numerical integration.

3. Calculation of gain and far-field require evaluation of complicated $M$ and $N$ functions.
Innovations For Practical Processing

Use of Special Probe

If the probe’s far-field pattern is symmetric for rotation about the axis of the probe, the probe’s spherical mode expansion will only contain coefficients for $\mu = \pm 1$

$$P^{s\mu nA} = 0 \text{ for } \mu \neq \pm 1$$

This can be accomplished with circular waveguide probe fed with a single mode rectangular waveguide.
Example Of Circular Waveguide

From a design by Stubenrauch, NIST
Use of special probe then only requires data for $\chi = 0$ and $\chi = 90$ degrees. The data from these two measurements are generally referred to as “the $\phi$-component data and the $\theta$-component data” even though the probe is not perfectly polarized.

The “integration” in $\chi$ is unnecessary, and two equations result.
2- Integration in $\theta$ and evaluation of $d_{m'\pm1}^{(n')}(\theta)$ function.

Express the results of the $\phi$-integration and the $d_{m'\pm1}^{(n')}(\theta)$ function as Fourier series. The integration in $\theta$ then reduces to a matrix multiplication plus an FFT.

The coefficients of the matrix obtained from the Fourier series representation of $d_{m'\pm1}^{(n')}(\theta)$ are obtained efficiently from recursion relations.
Calculation of Far-field Using Transmission Equation

Far-field is the field an ideal dipole would measure at a large distance from the AUT. Once the AUT spherical coefficients are found they are substituted into the Transmission Equation with ideal dipole coefficients for the probe, and the response calculated.
Calculation of Far-field using Transmission Equation

\[ E_{\theta,\phi}(\phi, \theta) = \sum_{m} \sum_{n} \sum_{\mu} \left[ \sum_{s=1}^{2} D^{sn\infty} Q^{smn} \right] e^{im\phi} d^{(n)}_{m}(\theta) \]

where \( D^{sn\infty} \) = Ideal dipole coefficients for large sphere
Mode Cutoff

For minimum sphere radius \( r_0 = \text{MRE} \)

Maximum n value \( \leq kr_0 \)  NSI uses \( kr_0 + 10 \)

Minimum angular spacing \( \leq \frac{\lambda}{2r_0} \) Radians
Mode Cutoff Example

Spherical mode cutoff

$k_r = 100$

Mode index $n$

20 log $|h_0(100)/h_n(100)|$
Translation Of Centers For Probe

\[ P^{s\mu na} = \sum_{\sigma, \nu} P^{\sigma \mu \nu} C^{sn}_{\sigma \mu \nu} (a) \]

where, \( s, \mu, \) and \( n \) are indices in AUT coordinate system
\( \sigma, \mu, \) and \( \nu \) are indices in Probe coordinate system.
Steps In Probe Correction

- Measure or calculate the far-field pattern of the probe

- Input this data to the spherical program and calculate the probe spherical coefficients for its coordinate system

- Use the translation of centers equations to obtain the probe coefficients in the measurement coordinates

- Use the translated coefficients in the probe correction
Rotator Schematic
Examples of Spherical Systems

NSI Model 700S-50
Spherical NF
Antenna Weights to 136 kg; 2m diameter

Matra Marconi
Spherical NF
Used for satellite antenna testing
Spherical Near-field Range
Sample Spherical Near-field Data

Near-Field Amplitude on Spherical Surface
X-Band Slot Array, f= 9.338 GHZ, Phi Component
Sample Spherical Near-field Data

Near-Field Phase on Spherical Surface
X-Band Slot Array, f = 9.338 GHZ, Phi Component
Sample Spherical Near-field Data

Near-Field Amplitude on Spherical Surface
X-Band Slot Array, f= 9.338 GHz, Theta Component
Sample Spherical Near-field Data

Near-Field Phase on Spherical Surface
X-Band Slot Array, f= 9.338 GHZ, Theta Component

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Spherical Probe Correction

Effect of Probe correction on Spherical Measurements
Open Ended Waveguide Probe, X-Band

Amplitude (dB)

Elevation (deg)

Probe Correction
No Probe Correction

Difference

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Spherical Near-field Data

Near Field Amplitude Data for X-Band Slot Array

Amplitude (dB) vs. Theta (deg)
Comparing Planar FF Pattern with Spherical FF Pattern without Probe Correction. OEWG Probe.
Dipole Antenna on Spherical Range
Dipole Far-Field Pattern Results

Far-field Amplitude for Dipole Measured on Spherical Range