Near-Field Antenna Measurement Theory II
Cylindrical
Overview

- Cylindrical coordinate systems
- Brief summary of rigorous derivation of transmission equation
- Development of transmission equation using measurement approach
- Comparison to planar transmission equation
- Translation of centers for probe receiving coefficients
- Far-field quantities
- Probe correction
- Probe coefficients from far-field pattern
- Sample measurements and probe correction data
Cylindrical Coordinates \((\rho, \phi, z)\)

Spherical Coordinates \((r, \theta, \phi)\)
AUT And AUT-Centered Probe Coordinate Systems
Schematic Of AUT, Probe And Cylinder

- $C_0 = \text{AUT-Centered and Measurement Coordinate System}$
- $C_0' = \text{AUT-centered Probe Coordinate System}$
- $C_1 = \text{Probe-Centered Coordinate System}$
Near-Field Cylindrical Range
Modal Expansion of Electric Field

\[
\vec{E}(\rho, \phi, z) = \sum_{n=-\infty}^{\infty} \int \left[ B_n^1(\gamma) \tilde{M}_{n\gamma}^{(1)} + B_n^2(\gamma) \tilde{N}_{n\gamma}^{(1)} \right] d\gamma
\]

\[
\tilde{M}_{n\gamma}^{(1)} = \left[ \frac{in}{\rho} H_n^{(1)}(\kappa \rho) \hat{\rho} - \kappa H_n^{(1)'}(\kappa \rho) \hat{\phi} \right] e^{in\phi} e^{i\gamma z}
\]

\[
\tilde{N}_{n\gamma}^{(1)} = \frac{1}{k} \left[ i\gamma \kappa H_n^{(1)'}(\kappa \rho) \hat{\rho} - \frac{n\gamma}{\rho} H_n^{(1)}(\kappa \rho) \hat{\phi} + \kappa^2 H_n^{(1)}(\kappa \rho) \hat{z} \right] e^{in\phi} e^{i\gamma z}
\]
To Calculate Electric Field Anywhere

- Find the Cylindrical mode coefficients $B_n^s(\gamma)$

- Evaluate the Hankel Functions and derivatives at the radius and $z$ specified by the product $\kappa \rho = \rho \sqrt{k^2 - \gamma^2}$

- Sum over all the modal indices and integrate over all values of $\gamma$

$$\vec{E}(\rho,\phi,z) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ B_n^1(\gamma) \vec{M}_{n\gamma}^{(1)} + B_n^2(\gamma) \vec{N}_{n\gamma}^{(1)} \right] d\gamma$$
Far Electric Field

\[ \vec{E}(r, \phi, \theta) = \frac{-2k \sin \theta e^{ikr}}{r} \sum_{n=-\infty}^{\infty} (-i)^n \left[ B_n^1(k \cos(\theta))\hat{\phi} - i B_n^2(k \cos(\theta))\hat{\theta} \right] e^{in\phi} \]

Does not require calculation of Hankel functions

Requires only the cylindrical mode coefficients \( B_n^s(\gamma) \)
Cylindrical Mode Coefficients

Since Far-field, Near-field, Gain and Polarization ratios can be found from the cylindrical mode coefficients, determining $B_n^s(\gamma)$ for a given antenna is the goal of the near-field measurements.
Cylindrical Waves And Notation

Amplitude of each cylindrical wave is specified by the coefficient $B_n^s(\gamma)$ and does not vary with any of the cylindrical coordinates ($\rho, \phi, z$).
Derivation of the Transmission Equation

- Express the antenna fields in cylindrical coordinates using vector mode functions.
- Write the scattering matrix for the antenna and probe in their own coordinate systems.
- Using field expressions in each coordinate system, derive the joining equations.
- Use joining equations and scattering matrix for each antenna to derive the transmission equation.
- Solve the transmission equation for unknown antenna cylindrical mode coefficients.
\( \gamma \) specifies a “direction of propagation” since the phase of the cylindrical wave in the z-direction is given by \( e^{i\gamma z} \).
The s-index in $B_n^s(\gamma)$ specifies the polarization of the wave since $s=1$ modes produce $\phi$-component (horizontal) far-fields and $s=2$ modes produce $\theta$-component (vertical) far-fields.
Cylindrical Mode Phase Patterns

The n-index in $B_n^s(\gamma)$ specifies the phase variation in phi through the factor $e^{in\phi}$ in definition of M and N.

Phase for $n = 1$

Phase for $n = 5$
Features Of Scattering Matrix Approach

- Does not require evaluation of Hankel functions
- Transmission equation valid in near and far-field
  - Only approximation is multiple reflections neglected and finite scan dimension in Z.
- Provides for probe correction of arbitrary probe
- Results in efficient data processing using the FFT
Cylindrical Scattering Matrix Schematic

AUT

Probe

\[ b_n^s(\gamma) \]

\[ a_n^s(\gamma) \]

\[ b'_n^s(\gamma) \]

\[ a'_n^s(\gamma) \]
Single Cylindrical Wave, $\gamma = 0, n = 0$

Linear Polarization

**Probe at** $z_0 = 0, \phi_0 = 0$
Transmission Equation Development

Single cylindrical wave, $\gamma = 0$, $n = 0$, linear polarization

**AUT Equations**

\[ b_0^2(0) = T_0^2(0) a_0 \]

**Probe Equations**

\[ a_0^2(0) = b_0^2(0) = T_0^2(0) a_0 \]

\[ b_0'(0,0) = R_0'^2(0) a_0^2(0) \]

\[ b_0'(0,0) = F' a_0 R_0'^2(0) T_0^2(0) \]

\[ F' = \frac{1}{1 - \Gamma'_i \Gamma'_0} \]
Single Cylindrical Wave, $\gamma = 0$, $n = 0$,

Two Polarizations

$\mathbf{b}_0^2(0)$

$\mathbf{b}_0^1(0)$

Probe

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Transmission Equation Development

Single cylindrical wave, $\gamma = 0$, $n = 0$, two polarizations

**AUT Equations**

\[
\begin{align*}
    b^2_0(0) &= T^2_0(0)a_0 \\
    b^1_0(0) &= T^1_0(0)a_0
\end{align*}
\]

**Probe Equations**

\[
\begin{align*}
    a^s_0(0) &= b^s_0(0) = T^s_0(0)a_0 \\
    b'_0(0,0) &= \sum_{s=1}^{2} R'^s_0(0)a'^s_0(0)
\end{align*}
\]

\[
\begin{align*}
    b'_0(0,0) &= F'a_0 \sum_{s=1}^{2} R'^s_0(0)T^s_0(0)
\end{align*}
\]
Two cylindrical waves, $\gamma = 0$

Two polarizations, two values of $n$
### Transmission Equation Development

Two cylindrical waves, $\gamma = 0$, two polarizations, two values of $n$

<table>
<thead>
<tr>
<th>AUT Equations</th>
<th>Probe Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1^s(0) = T_1^s(0)a_0$</td>
<td>$a_n^s(0) = b_n^s(0) = T_n^s(0)a_0$</td>
</tr>
<tr>
<td>$b_4^s(0) = T_4^s(0)a_0$</td>
<td>$b_0'(0,0) = \sum_{n=1,4} \sum_{s=1}^2 R_n^s(0)a_n^s(0)$</td>
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</tbody>
</table>

$$b_0'(0,0) = F'a_0 \sum_{n=1,4} \sum_{s=1}^2 R_n^s(0)T_n^s(0)$$
Spectrum Of Cylindrical Waves

Different values of $\gamma$, $s$ and $n$

$\mathbf{b}_3^2(\gamma_1)$

$\mathbf{b}_1^1(\gamma_1)$

$\mathbf{b}_3^1(\gamma_1)$

$\mathbf{b}_4^2(0)$

$\mathbf{b}_1^1(0)$

$\mathbf{b}_4^1(0)$

$\mathbf{b}_5^2(\gamma_2)$

$\mathbf{b}_1^2(0)$

$\mathbf{b}_0'(0,0)$
Transmission Equation Development

Spectrum of cylindrical waves, different values of $\gamma$, $s$ and $n$

AUT Equations

$$b_n^s(\gamma) = T_n^s(\gamma) a_0$$

Probe Equations

$$a_n'(\gamma) = b_n^s(\gamma) = T_n^s(\gamma) a_0$$

$$b'_0(0,0) = \int \sum_{n=-\infty}^{\infty} \sum_{s=1}^{2} R_n'(\gamma) a_n'(\gamma) \, d\gamma$$

$$b'_0(0,0) = F a_0 \int \sum_{n=-\infty}^{\infty} \sum_{s=1}^{2} R_n'(\gamma) T_n^s(\gamma) \, d\gamma$$
Probe Moved On Cylinder

\[ b'_0(\phi_0, z_0) \]

\( a_0 \)

\( \text{AUT} \)
Transmission Equation Development

Spectrum of cylindrical waves, different values of $\gamma$, $s$ and $n$ probe moved to $(\phi_0, z_0)$

$$b_n^s(\gamma) = T_n^s(\gamma) a_0$$

$$a_n^{'s}(\gamma) = b_n^s(\gamma) = T_n^s(\gamma) a_0$$

$$b_0'(\phi_0, z_0) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{s=1}^{2} R_n^s(\gamma) a_n^{'s}(\gamma) e^{i\phi_0} e^{i\gamma z_0} d\gamma$$

$$b_0'(\phi_0, z_0) = F' a_0 \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{s=1}^{2} R_n^s(\gamma) T_n^s(\gamma) e^{i\phi_0} e^{i\gamma z_0} d\gamma$$
Measurements With Two Probes

First probe usually has same polarization as AUT

\[ b'_0(\phi_0, z_0) = F' a_0 \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{s=1}^{2} R_n^s(\gamma) T_n^s(\gamma) \ e^{i n \phi_0} e^{i \gamma z_0} \ d\gamma \]

Second probe usually is cross polarized to AUT

\[ b''_0(\phi_0, z_0) = F' a_0 \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{s=1}^{2} R''_n^s(\gamma) T_n^s(\gamma) \ e^{i n \phi_0} e^{i \gamma z_0} \ d\gamma \]
Inversion Of Transmission Equations

Using Fourier series for $n$ and Fourier integral for $\gamma$, for first probe data

\[ I'_n(\gamma) = \sum_{s=1}^{2} R'_n(\gamma) T'_n(\gamma) \]

\[ = \frac{1}{4\pi^2 a_0} \int_{-\infty}^{\infty} \int_{0}^{2\pi} b'_0(\phi_0, z_0) e^{-im\phi_0} e^{-i\gamma z_0} d\phi_0 dz_0 \]
Inversion Of Transmission Equations

For second probe data

\[
I''_n(\gamma) = \sum_{s=1}^{2} R''_n(\gamma) T^n_s(\gamma)
\]

\[
= \frac{1}{4\pi^2 a_0} \int_{-\infty}^{\infty} \int_{0}^{2\pi} b''_0(\phi_0, z_0) e^{-i\phi_0} e^{-i\gamma z_0} d\phi_0 dz_0
\]
Data Point Spacing And Maximum n

Band limits of the AUT pattern in the \( \theta \)-direction define a data point spacing in \( z \) like the planar case

\[
\delta_z \leq \frac{\lambda}{2}
\]

Due to the exponential decrease in the reactive cylindrical modes, the maximum \( n \) value is

\[
n_{\max} \leq k a \sin(\theta), \quad \text{and} \quad \Delta \phi \leq \frac{\lambda}{2a} \quad \text{radians}
\]

\( a \equiv \text{MREin NSI Software} \)

Due to these band limits, the integration is replaced by summation without approximation and the FFT is used to calculate the I’s.
Cylindrical Near-field Sampling Criteria

Note that index $n_{max}$ is therefore driven by the size and mounting offset of the AUT being considered.

The above sampling criteria are valid for the probe exterior to the reactive near-field of the AUT.
Probe Correction

\[ I'_n(\gamma) = \sum_{s=1}^{2} R'^s_n(\gamma)T^s_n(\gamma) = R'^1_n(\gamma)T^1_n(\gamma) + R'^2_n(\gamma)T^2_n(\gamma) \]

\[ I''_n(\gamma) = \sum_{s=1}^{2} R''^s_n(\gamma)T^s_n(\gamma) = R''^1_n(\gamma)T^1_n(\gamma) + R''^2_n(\gamma)T^2_n(\gamma) \]
Probe Correction Equations

Using concise notation and deleting explicit reference to $\gamma$ and $n$

$$T^1 = \frac{I' + I''}{R'^1 + R''^2 \frac{R'}{R''}}$$

$$T^2 = \frac{I'' + I'}{R''^2 + R'^1 \frac{R''}{R'}}$$
Far Electric Field

\[ \vec{E}(r,\phi,\theta) = -\frac{2k \sin\theta e^{ikr}}{r} \sum_{n=-\infty}^{\infty} (-i)^n \left[ \frac{B_n^1(k \cos(\theta))}{k} \hat{\phi} - i B_n^2(k \cos(\theta)) \hat{\theta} \right] e^{in\phi} \]

Does not require calculation of Hankel functions
Requirements only the cylindrical mode coefficients \( B_n^s(\gamma) \)
Cylindrical Coefficients And Transmitting Function

\[ B_n^s(\gamma) = b_n^s(\gamma) = T_n^s(\gamma) a_0 \]

Therefore solving for \( T^s \) will give far-field, gain, and polarization ratios.
Cylindrical And Planar Probe Correction Equations

\[ T^1 = \frac{I' + I''}{R'^1} \left( \frac{1}{R''} \right) \frac{1}{1 - \frac{R''}{R'}} \]

\[ T^2 = \frac{I''}{R'^2} \left( \frac{1}{R'} \right) \frac{1}{1 - \frac{R''}{R'}} \]

\[ t_A = \frac{D_A}{s_A'} - \frac{D_E}{s_E' \rho_s'} \]

\[ t_E = \frac{D_E}{s_E''} - \frac{D_A}{s_A'' \rho_s''} \]
Sample Probe Correction Theta Cut

Cylindrical Near-Field Probe Correction
Main Component, Elevation Cut

- Amplitude (dB)
- Elevation (deg)

No Probe Correction
OEWG Probe Correction
Difference
Sample Probe Correction Phi Cut

Cylindrical Near-Field Probe Correction
Azimuth Cut, Main Component

- No Probe Correction
- OEWG Probe Correction
- Difference

Amplitude (dB) vs. Azimuth (deg)
References
