A SIMPLE ANALYSIS OF NEAR-FIELD BORESIGHT ERROR REQUIREMENTS

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ABSTRACT

The need to measure the boresight pointing direction of radar antennas to a high degree of accuracy yields a requirement for excellent positioning accuracy on near-field antenna ranges. Evaluation of this requirement can be accomplished by a full and complete sensitivity analysis.
Alternatively, to gain an understanding of the effects of errors more simply, one can approach the question of accuracy required in the setup, by use of a physical model and straightforward physical reasoning. The approach starts with the assumptions of a collimated wave with planar phase fronts and the premise that the boresight direction of such a sum beam is along the normal to the phase fronts. A sensitivity analysis of the simple trigonometric boresight relationship between mechanical boresight and phase front normal, shows how accurate the receiver and the positioner must be to achieve a given boresight determination. Such an approach has been known for many years as it regards planar scanning; and, the results are known to be applicable.

In this paper this consideration is extended to spherical scanners to arrive at estimates of the mechanical positioner accuracies and electrical receiver accuracies needed to make boresight measurements of radar antennas with spherical near-field ranges.

Keywords: Antenna Measurements, Near-Field Scanning, Boresight Measurements, Error Analysis

1.0 Introduction

The need to measure the boresight pointing direction of radar antennas to a high degree of accuracy yields a requirement for excellent positioning accuracy on near-field antenna ranges and excellent RF measurement accuracy as well. This paper describes a method of determining the positioning accuracy required of a spherical near-field scanner that will be used to measure the boresight error of antenna patterns – specifically sum-type patterns.

Evaluation of the positioning accuracy requirement can be accomplished by a proper sensitivity analysis: That analysis would use a numerical near-field model of the antenna, a numerical model of the imperfections in the apparatus, and a numerical near-field-to-far-field transform algorithm. To carry out such an approach, the antenna model and the model of the apparatus, must produce a map of the discrepancies in the near-field data brought about by specific modeled errors, and then the transform must calculate the effects of these errors upon the far-field pattern. This formal numerical approach, though tedious to pursue, can permit one to learn how accurate the near-field scanner must be and how accurate the receiver must be. Please see Figure 1 for a schematic block diagram summarizing this process.

This type of a formal procedure, however, requires that an electromagnetic model be available for the antenna in question, that the model be proven reliable, and that the model be capable of running on an available computer. To avoid the often lengthy process of carrying out such a numerical study, one can utilize in the case of planar near-field measurements the simple trigonometry of the relationship between mechanical boresight and phase front normal.

2.0 Planar Near-Field Boresight Equation

Refer to the schematic of Figure 2 to understand the simple relationship between the boresight direction of the outgoing planar wave emitted by an aperture antenna and the mechanical boresight axis of the antenna structure, please. Here the schematic of a near-field boresight measurement is illustrated. The antenna aperture is shown in plan view; and, the planar wavefronts that it emits are shown traveling outward at an angle θ with respect to the normal to the surface of the aperture itself. This aperture plane might be the surface that is defined by the elements of a phased array or the rim of a symmetric paraboloidal dish antenna. The normal to the aperture surface can be thought of as defining the mechanical boresight direction of the antenna.

The angle θ, between the phase front normal and the normal to the aperture, we designate as the beam steering angle. This is also equal to the angle between the line forming the phase front closest to the antenna and the face of the aperture. In the figure is also shown a line indicating the line of travel of a near-field probe as it traverses parallel to the antenna face in making a near-field scanning measurement. In traversing the face of the antenna, the probe will cut through several phase fronts, each one separated from the previous one by an equal distance, say 360 phase degrees or one wavelength, λ. The quantity φ is the number of radians of phase shift that the receiver detects at the port of the probe as the probe travels a distance x from the point labeled O to the opposite end of the aperture. The relationship between the beamsteering angle, the distance traveled by the probe, the wavelength, and the phase shift, is evident from the triangle formed by the heavy lines in the diagram:

$$\phi = x \sin\theta \frac{2\pi}{\lambda}$$

It is useful to note that alternatively,

$$\frac{2\pi}{\lambda} = \frac{360^o}{\lambda}$$

This simple relationship is the beam steering equation for aperture antennas. It can be used as a near-field equation for planar scanning to determine the boresight angle θ of the sum beam being formed from the phase shift φ measured by the probe and the distance x traveled by the probe. This is a well-known result. [1]

As an example of how this equation is used to determine the beam steering angle, suppose that the wavelength is 4 inches (corresponding to approximately 3 GHz), suppose that the receiver sees a phase shift of 3600 phase degrees and suppose that the corresponding amount of probe travel is 100 inches. Then the beam steering angle will be $\sin^{-1}(3600^o/(100''/4'')*360^o)$ or 23.6 degrees.

3.0 Planar Near-Field Boresight Sensitivity Equation

From the boresight equation one can obtain additional insight into the sensitivity of a planar near field system to measurement errors in the determination of boresight. To see how this works, take the boresight equation and after rearranging,

$$\sin\theta = \frac{\phi}{x} \frac{\lambda}{2\pi}$$

differentiate both sides with respect to the key variables θ. φ, and x:

$$\cos\theta \; d\theta = \frac{\lambda}{2\pi} \left[\frac{d\phi}{x} - \frac{\phi \cdot dx}{x^2} \right]$$

Using the boresight equation itself, this can be rewritten as

$$\cos\theta \; d\theta = \sin\theta \left[\frac{d\phi}{\phi} - \frac{dx}{x} \right]$$

and restated as

$$\frac{d\theta}{\tan\theta} = \left[\frac{d\phi}{\phi} - \frac{dx}{x} \right]$$

This is now in the standard form to serve as the basis for a sensitivity analysis. Since the signs of the uncertainties are almost always unknown and since the errors in φ and x are independent, the errors are combined as RSS root-sum of squares:

$$\left[\frac{d\theta}{\tan\theta} \right]^2 = \left[\left(\frac{d\phi}{\phi}\right)^2 + \left(\frac{dx}{x}\right)^2 \right]$$

We will now show how this equation for the sensitivity of the beam steering angle to measured phase and measured istance quantities can be used to estimate the boresight accuracy of a planar scanner.

4.0 Planar Near-Field Boresight Sensitivity Analysis Example

Suppose that we anticipate measuring an antenna whose dimension in the plane where accurate measurement of boresight is important is 102 inches and whose operating frequency is 3.0 GHz, where the wavelength is 3.93 inches. Further assume that we wish to impose a boresight measurement accuracy of 0.25 mR, for a beam steering angle of 45 degrees.

First, $\tan 45^o = 1$ so the equation we will use becomes.

$$[\ d\theta\ ]^2 = [\ (\frac{d\phi}{\phi})^2 + (\frac{dx}{x})^2\ ]$$

Now the total phase shift across the scan plane will be given by the first equation above

$$\phi = 102\ in\ \frac{1}{\sqrt{2}}\ \frac{2\pi}{3.93 in}$$
$$= 115.7 Rad = 6630\, phase \deg$$

Now we can estimate an error budget that will be required to meet the 0.25 mR of boresight uncertainty. To begin we assume equal contributions to $(d\theta)^2$ from the electrical & mechanical errors, $d\varphi$ and dx respectively.

For the electrical quantity we require that

$$(\frac{d\phi}{\phi}) = \frac{1}{\sqrt{2}}\ [\ 0.25\ mRad\ ]$$

Which gives

$$d\phi = 6630\, phase \deg\ [\,0.707\,]\ [\ 0.25 \cdot 10^{-3} Rad]$$
$$= 1.2\, phase \deg$$

for the accuracy required of the receiver or the receiving subsystem in measuring the phase.

For the mechanical quantity we require that

$$(\frac{dx}{x}) = \frac{1}{\sqrt{2}}\ [\,0.25\ mRad\ ]$$

which in turn gives

$$dx = 102\ in\ [\,0.707\,][\,0.25 \cdot 10^{-3} Rad\,]$$
$$= 0.018\ in$$

These values are reasonable, given the experience in using planar near-field scanning to determine the boresight direction of the beams of phased array search radar

It turns out that the boresight sensitivity equation is not directly applicable for steering angles at normal to the aperture where θ is about zero. Instead we revert back to an approximate version of the beamsteering equation and differentiate it versus φ alone since the uncertainty in position x is far less important

$$\phi = x\ \sin\theta\ \frac{2\pi}{\lambda} \approx x\ \theta\ \frac{2\pi}{\lambda}$$
$$\theta = \phi\ \frac{\lambda}{2\pi}\ \frac{1}{x}$$

Differentiating both sides gives

$$d\phi = \frac{1}{\sqrt{2}}\ d\theta\ (x)\ \frac{2\pi}{\lambda}$$

and inserting the numbers of the example gives

$$= \frac{1}{\sqrt{2}}\ [\,0.25\, mRad\ ][\ 102\ in][\frac{360 \deg}{3.93 in}]$$
$$= 1.7\, phase \deg$$

The equivalent equation for position shows the difficulty at aperture normal, where θ≈ 0:

$$dx = x\ \frac{d\theta}{\theta}$$

5.0 Spherical Near-Field Boresight Equation

To understand the general arrangement of a spherical near-field scanning geometry, please refer to Figure 3. There an aperture antenna is shown inside a spherical near-field scanning surface on which a probe antenna is to move in theta and phi, the spherical direction angles. This diagram shows only one scan dimension of the scanning system; one can think of the circle depicted as a great circle plane containing the North Pole. The aperture antenna is in general the same as the antenna of the planar near-field example but rather than choosing a non-zero scan angle of the beam relative to the aperture, the aperture is shown at a canted angle θ_{bs} relative to the vertical z-axis of the coordinate system. This angle represents the boresight direction of the antenna aperture but the canting angle alone produces the generality we desire in this example.

The same planar wavefronts as in the planar near-field case are shown exiting the spherical surface at an angle θ_{rel} measured relative to the radius vector drawn from the center of the circle. There are two boundary lines at angles θ_1 and θ_2 whose positions are defined by the margins of the collimated wave as the margins intersect the circle of the scanning sphere.

A chord of the circle is defined as the line that joins the two points of intersection of the two boundary lines with the circle. This chord, together with the phase front that intersects the meeting of the boundary line for θ_1 with the sphere, and a segment of the boundary line for θ_2 are shown in heavy dark ink to form a triangle that is analogous to the one studied earlier for planar scanning. This will form the trigonometric basis for the spherical near-field boresight equation.

The angles noted in Figure 3 are related by the following simple equations:

$$q_{bs} = q_{ref} + q_{rel}$$

$$q_{ref} = \frac{1}{2}[q_1 + q_2]$$

$$q_{coll} = [q_2 - q_1]$$

The angle θ_{ref} is the angle at which is found the center of the collimated wave as it exits the sphere. The angle θ_{rel} is the angle at which the collimated wave is travelling as measured with respect to a radius vector that is the bisector of the chord described above. And again, θ_{bs} is the angle of boresight of the far-field beam corresponding to the collimated wavefront. And, θ_{coll} is the apparent width measured in angle of the collimated wave as it exits the sphere.

With these definitions we can now write down the equation by which boresight can be determined directly from spherical near-field scanning data:

$$f = 2R \sin\frac{q_{coll}}{2} \sin q_{rel} \frac{360^o}{l}$$

where, we have used the length of the chord $2R\sin(\theta_{coll}$) in place of the distance x in the planar near field version of the boresight equation.

6.0 Spherical Near-Field Boresight Sensitivity Equations and Example

We can proceed to analyze the spherical near-field boresight case by first examining the sensitivity of θ_{rel} to the angles θ_1 and $\theta_{2,}$ ignoring the cross-term in θ_1 and θ_2 which are independent variables.

$$q_{bs} = q_{ref} + q_{rel}$$

$$dq_{bs} = dq_{ref} + dq_{rel}$$

$$q_{ref} = \frac{1}{2}[q_1 + q_2]$$

$$\left(\frac{dq_{ref}}{q_{ref}}\right)^2 = \frac{1}{\frac{1}{4}[q_1 + q_2]^2} \times$$

$$\times \frac{1}{4}[dq_1^2 + dq_2^2]$$

$$= \frac{1}{[q_{ref}]^2}\left[\frac{dq_1^2 + dq_2^2}{4}\right]$$

$$\approx \frac{dq^2}{2[q_{ref}]^2}$$

$$dq^2 \approx dq_1^2 \approx dq_1^2$$

We write the boresight equation in a slightly different form using the length x of the chord

$$f = x \sin q_{rel} \frac{360^{o}}{l}$$

$$x = \frac{L}{\cos q_{rel}}$$

This has the exact same form as the planar case and permits us to employ the same equation for a sensitivity analysis:

$$\left[\frac{dq_{rel}}{\tan q_{rel}} \right]^2 = \left[\left(\frac{df}{f}\right)^2 + \left(\frac{dx}{x}\right)^2 \right]$$

We can also write x as function of θ_{coll} and obtain

$$x = 2R \sin \frac{q_{coll}}{2}$$

$$q_{coll} = [\, q_2 - q_1 \,]$$

$$\frac{dx}{x} = \frac{1}{\tan \frac{q_{coll}}{2}} [\, dq_2 - dq_1 \,]$$

$$dq^2 \approx dq_1^2 \approx dq_1^2$$

$$\left[\frac{dx}{x} \right]^2 = \left[\frac{1}{\tan \frac{q_{coll}}{2}} \right]^2 2 \,[\, dq^2 \,]$$

Spherical Near-Field Boresight Sensitivity Analysis Example

Here we take an example very much like the example for planar near-field:

θ_{ref} = Reference Angle = 45 degrees
tan(theta=45deg) = 1
θ_{bs}= Beam Steering Angle = 20 degrees
L= Dimension of Aperture = 2.6 m = 102 in
Wavelength = 3.93 in

We will be looking for an overall boresight accuracy of 0.25 mR.

First we allocate ¼ of the error budget to each of the angles θ_1 and θ_2

$$\frac{dq^2}{2\,[\, q_{ref} \,]^2} = \frac{[\, 0.25mRad \,]^2}{4}$$

$$dq^2 = 2\,[\, 45\deg \,]^2 \frac{[\, 0.25mRad \,]^2}{4}$$

$$= 0.008\deg$$

Now we proceed to handle the sensitivity to the angle θ_{rel} and the electrical phase φ. First we must compute the value of φ:

$$f = \frac{L}{\cos q_{rel}} \sin q_{rel} \frac{360^{O}}{l}$$

$$f = 102\ in \quad 0.364 \frac{360^{o}}{3.93\ in}$$

$$= 3401\ phasedegrees$$

The allocation for electrical phase accuracy we assume will be ¼ of the total boresight measurement error budget. We calculate the equivalent allowed phase measurement error as

$$(df)^2 = (f)^2 \frac{1}{2} \left[\frac{0.25mRad}{\tan q_{rel}} \right]^2$$

$$= [\, 3401\ phasedegrees \,]^2 \left[\frac{0.25mRad}{\sqrt{2}\ \ 0.364} \right]^2$$

$$= [\, 1.65\ phasedegrees \,]^2$$

Lastly we allocate ¼ of the error to the measurement of θ_{coll} via x:

$$\left[\frac{1}{\tan \frac{q_{coll}}{2}} \right]^2 2 \,[\, dq^2 \,] = \frac{1}{4} \left[\frac{0.25mRad}{\tan q_{rel}} \right]^2$$

This can be rewritten as

$$[\, dq^2 \,] = \frac{1}{8} \left[\frac{\tan \frac{q_{coll}}{2}}{\tan q_{rel}} \right]^2 [\, 0.25mRad \,]^2$$

Or evaluating gives the required measurement accuracy for the angular readout of the spherical scanner:

$$[\, dq^2 \,] = [\, 0.0051\deg \,]^2$$

7. Summary

We have reviewed the consideration for accuracy in the electrical phase and mechanical position readout for planar near-field scanning. An example has shown that for a 100 inch aperture antenna operating at 3 GHz, the positioning accuracy required o achieve a 0.25 mR boresight accuracy is of the order of 0.018 inches and 1.5 degrees of phase.

This consideration has been extended to the case of spherical near-field scanning where the phase accuracy required is of the order of 1.5 phase degrees and the mechanical accuracy of the order of 0.005 degrees.

The boresight or beam steering equation for planar near-field is given by:

$$f = x \sin q \frac{2p}{l}$$

Where the variables are defined in Figure 2.

And the boresight equation for spherical near-field is of the same form and when written in terms of angular variables is given by:

$$f = 2R \sin \frac{q_{coll}}{2} \sin q_{rel} \frac{360^{o}}{l}$$

where the variables are defined by the schematic in Figure 3.

*NOTE

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REFERENCES

[1] Carl Stubenrauch, Lecture Notes on Planar Near-Field Scanning, CSUN Antenna Measurement Short Course, 2001.

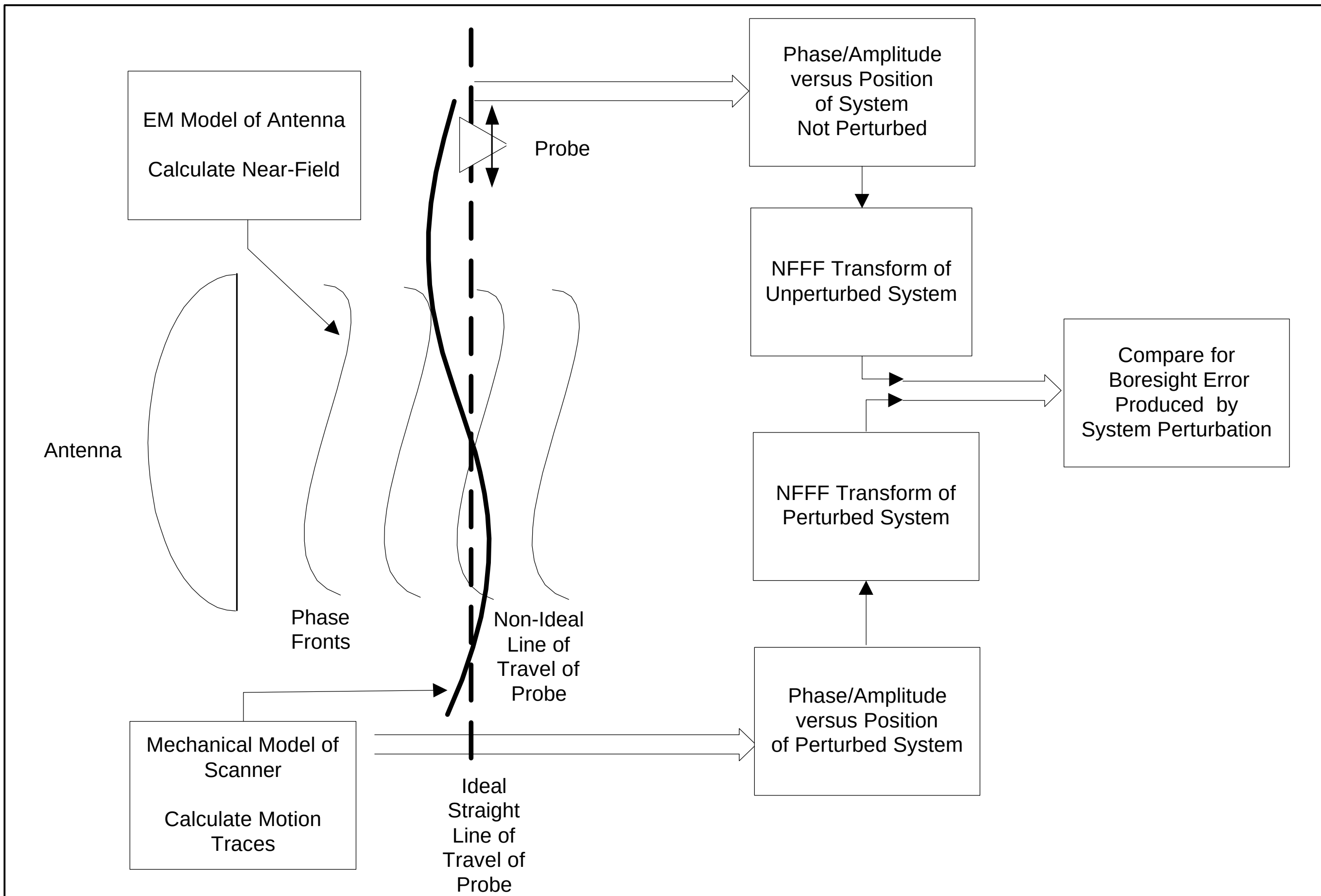

Figure 1 Schematic of Full and Complete Analysis of Boresight Measurement Error

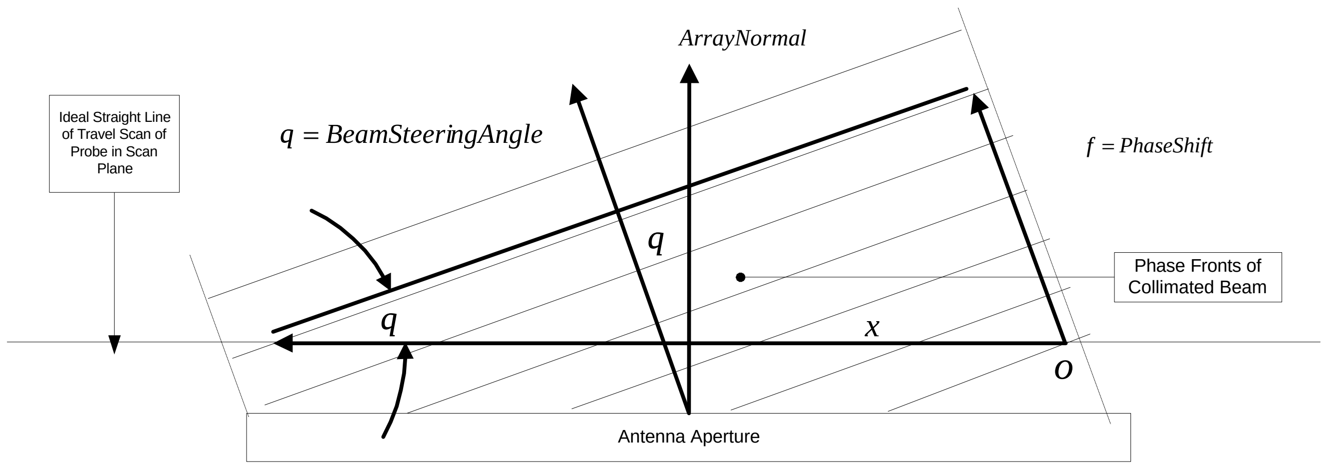

Figure 2 Schematic -- Near-Field Boresight Measurement with Planar Near-Field Scanner

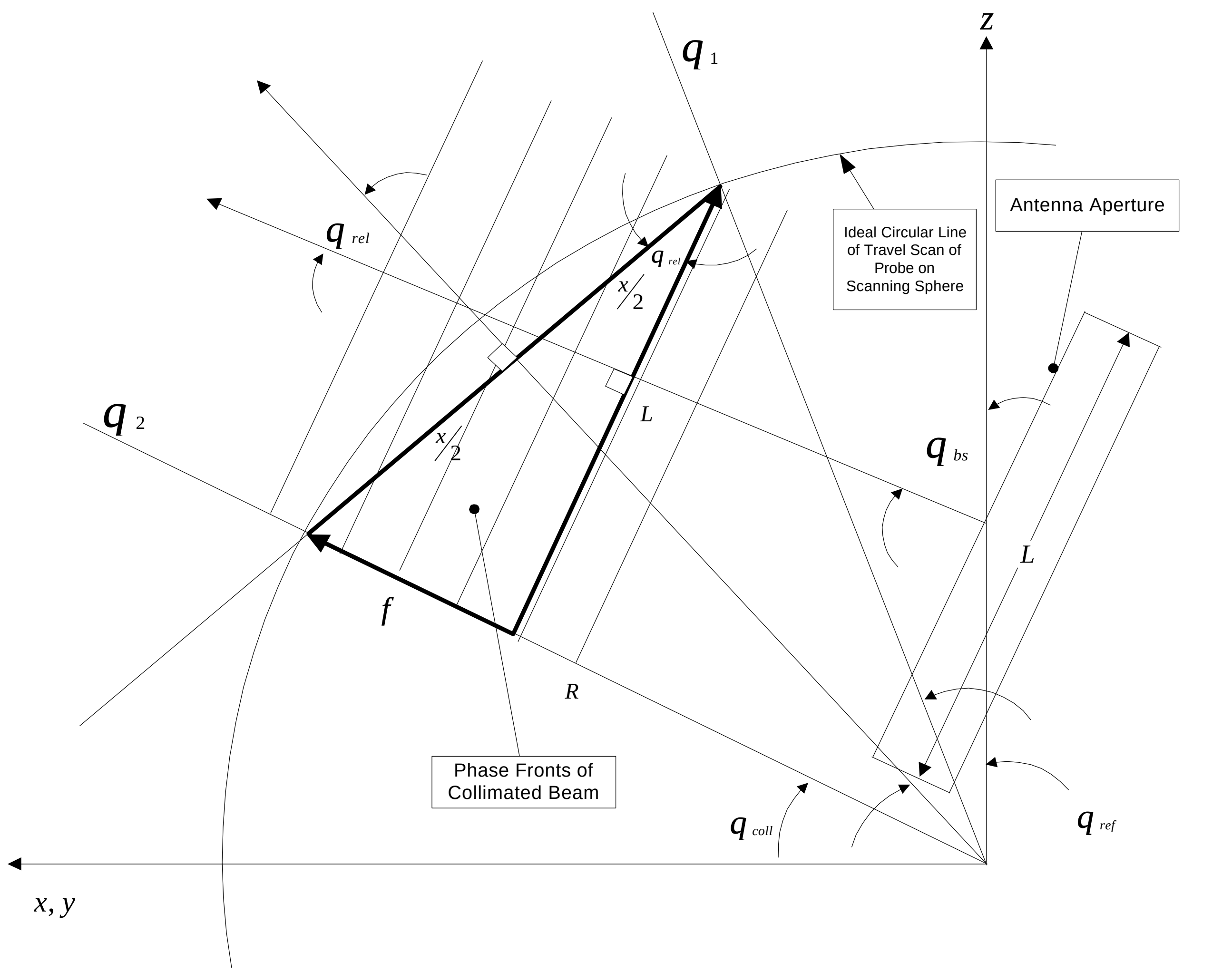

Figure 3 Schematic -- Near-Field Boresight Measurement with Spherical Near-Field Scanner