ERROR ANALYSIS OF CIRCULAR-POLARIZATION COMPONENTS SYNTHESIZED FROM LINEARLY POLARIZED MEASUREMENTS

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ABSTRACT

A usual way of performing pattern-measurements on circularly polarized antennas is by measuring the linear components of the field and mathematically converting those to the left-hand and right-hand circular components. These synthesized circular components are sensitive for a number of factors: The exact orthogonality of the measured linear components, the measurement-accuracy of both phase and amplitude of the measured linear components, the polarization-pureness (or the accuracy of the description of the polarization-characteristics) of the probe, etc. This paper analyzes these factors, using a computer-model. An indication on the requirements to be imposed on the measurement-equipment is provided.

Keywords: Antenna Measurements, Measurement errors.

1 Introduction

Consider a probe-antenna with arbitrary polarization characteristics. Let's attach a right-hand rectangular coordinate-system to this antenna, where Z is in the direction of an incoming wave, and X and Y are (more or less arbitrary, but fixed and orthogonal) the vertical and horizontal axes of the probe-antenna. We can describe the response of this antenna to an incoming electrical plane wave as if the antenna consists of two orthogonal antennas, each responding to a perfectly linear and mutually orthogonal polarized wave:

$$R = R_X \cdot E_X + R_Y \cdot E_Y , \qquad (1)$$

where R_X and R_Y are the (complex) responses to the X- and Y-polarized electric fields (i.e. parallel to the reference axis), which are characterized by complex numbers E_X and E_Y.

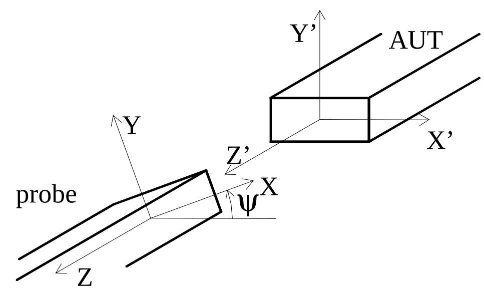

Figure 1: Arrangement of antennas during test and definition of coordinates.

When this probe-antenna is rotated around its axis by an angle ψ, the overall response of this antenna to the previously described wave is obtained from eq. 2.69 – 2.71 from [1] and inspection of the directions of the vectors in Figure 1 as a function of angle ψ:

$$\begin{aligned} R = {} & E_{X'} \cdot (R_X \cdot \cos(\psi) - R_Y \cdot \sin(\psi)) + \\ & + E_{Y'} \cdot (R_X \cdot \sin(\psi) + R_Y \cdot \cos(\psi)) \end{aligned} , \qquad (2)$$

where $E_{X'}$ and $E_{Y'}$ indicate the fields to which the antenna is responding, not necessarily parallel to the reference-axes of the probe-antenna and for instance being transmitted by an actual AUT.

A frequently used technique for characterizing a generally polarized wave uses the measurement of two orthogonal components. For this measurement an essentially linearly polarized probe is used. However, the full descriptions of the responses of the two measurements are:

$$\begin{aligned} R_1 = {} & (E_{X'} \cdot (R_X \cdot \cos(\psi_1) - R_Y \cdot \sin(\psi_1)) + \\ & + E_{Y'} \cdot (R_X \cdot \sin(\psi_1) + R_Y \cdot \cos(\psi_1))) \cdot e^{j \cdot k_0 \cdot z_1} \\ R_2 = {} & (E_{X'} \cdot (R_X \cdot \cos(\psi_2) - R_Y \cdot \sin(\psi_2)) + \\ & + E_{Y'} \cdot (R_X \cdot \sin(\psi_2) + R_Y \cdot \cos(\psi_2))) \cdot e^{j \cdot k_0 \cdot z_2} \end{aligned} \qquad (3)$$

The index 1,2 indicate the two (more or less) orthogonal measurements, k_0 indicates the wavenumber, z_1 and z_2 indicate the distances between the probing antenna and the antenna under test. In this formula the distance between probe and antenna under test is assumed to be so large that a (small) variation of this distance results in a negligible amplitude-variation.
In the ideal case the probe is rotated 90° between measurement 1 and 2, the response to X-polarized signals is zero, and there is no displacement between the measurements. Therefore:

$$\psi_1 + 90° = \psi_2 \qquad R_X = 0 \qquad z_1 = z_2 \tag{4}$$

When the Circularly Polarized (CP) components are to be calculated from the linearly measured components, the following formulas are applied (combination of [2, eq. 1,2,3]):

$$R_R = \frac{R_1 + j \cdot R_2}{\sqrt{2}} \qquad R_L = \frac{R_1 - j \cdot R_2}{\sqrt{2}} \tag{5}$$

Combining (3) and (5) will give the calculated right-hand and left-hand circular components in terms of actual parameters (X- and Y-polarized E-fields from the AUT, co- and cross-pol sensitivity of the probing-antenna, rotation-angles and displacements). As such, this enables us to characterize the error in the calculated parameters due to errors in the measurement-parameters. It is easily verified that perfect measurement-parameters (as in (4)) will give perfect calculated CP-components for a CP-polarized wave.

To describe the polarization of a circularly polarized antenna the Polarization sense, Axial Ratio (AR), and tilt angle (τ) are calculated during post processing.
In the remainder of this paper the behavior of the errors in AR and τ will be analyzed. Causes of error considered here are the ones which may occur using the method of measuring the polarization of an electric field by rotating the probe antenna. These errors are non-zero (and non-compensated) cross-polarization of the probe, imperfect rotation of the probe and axial displacement of the probe (which has more or less the same effects as phase-errors due to cable-flexing or rotary-joint movement).

2. Error calculation of AR and τ

The parameters one is usually interested in (axial ratio and tilt-angle) are easily expressed in terms of the complex circular polarization ratio:

$$\rho_c = \frac{R_L}{R_R} \tag{6}$$

$$AR = \frac{1+|\rho_c|}{1-|\rho_c|} \tag{7}$$

$$\tau = \frac{\arg(\rho_c)}{2} \tag{8}$$

With all steps in the measurement-process characterized by the formulas above, we are now capable to simulate the effects of positioning and probe-calibration errors on the parameters as calculated. Therefore the obtained equations were implemented in a computer-model, of which calculation-results are shown here.
The steps in the simulation are as follows:

Step 1: Calculate actual horizontal Electric field component from a defined axial ratio ('true axial ratio'), a defined tilt-angle ('true tilt-angle') and a fixed vertical Electric field component.

↓

Step 2: Calculate the responses R_1 and R_2 of the probe (using eq. 3) when applying a defined probe positioning / cross-polar error (one source of error at a time).

↓

Step 3: Calculate the CP components (using eq. 5) from the simulated measured linear components resulting from step 2.

↓

Step 4: Calculate the simulated measured axial ratio and tilt-angle (using eq. 6-8) from the CP components obtained in step 3.

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Step 5: Normalize simulated measured axial ratio and tilt-angle from step 4 to the true axial ratio and tilt-angle from step 1 and display graphically.

2.1 The effect of positioning errors on AR and τ

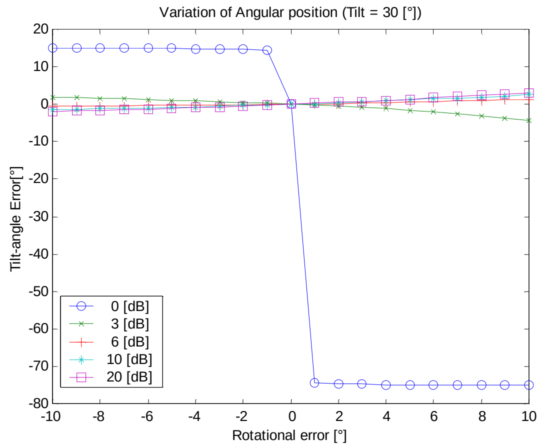

Figure 2: Tilt-angle error vs probe rotational error, at various true axial ratios and a fixed true tilt-angle.

Figure 2 shows the simulated measured tilt-angle as a function of the error in probe-rotation. The true tilt-angle is fixed to 30° and the different curves are for different true axial ratios (0, 3, 6, 10, 20 dB). It is observed that the error in the measured tilt-angle due to an error in rotating the probing antenna increases with decreasing true axial ratio.

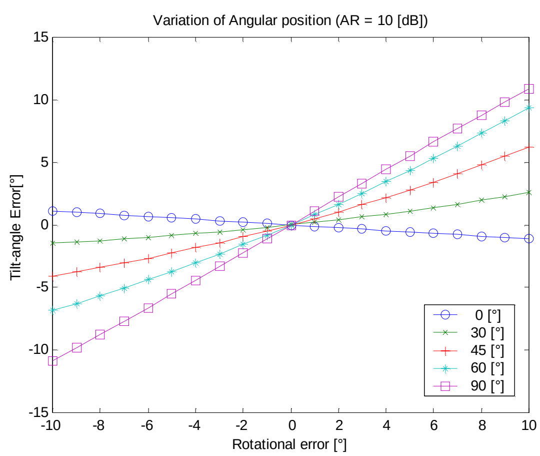

Figure 3: Tilt-angle error vs probe rotational error, at various true tilt-angles and a fixed true axial ratio.

Figure 3 also shows the simulated measured tilt-angle as a function of the error in probe-rotation except that here the true axial ratio is fixed to 10 dB and the different curves are for different true tilt-angles (0°, 30°, 45°, 60°, 90°). Inspection of this graph shows that the tilt-angle error due to an error in probe-rotation increases with increasing true tilt-angle.

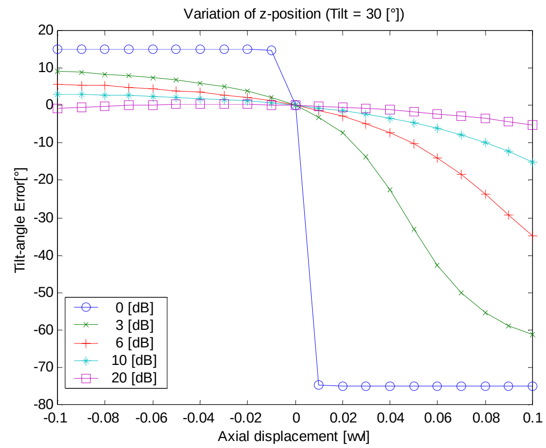

Figure 4: Tilt-angle error vs axial displacement, at various true axial ratios and a fixed true tilt-angle.

Figure 4 and Figure 5 show the simulated measured tilt-angle as a function of the variation in distance between AUT and probe, between the two orthogonal measurements. In Figure 4 the true tilt-angle is fixed and the true axial ratio is varied between curves, while in Figure 5 the true axial ratio is fixed and the true tilt-angle is varied. It is observed that an error in axial position (distance between AUT and probe) has an increasing influence on the tilt-angle error with decreasing true axial ratio or a true tilt-angle differing from 45°.

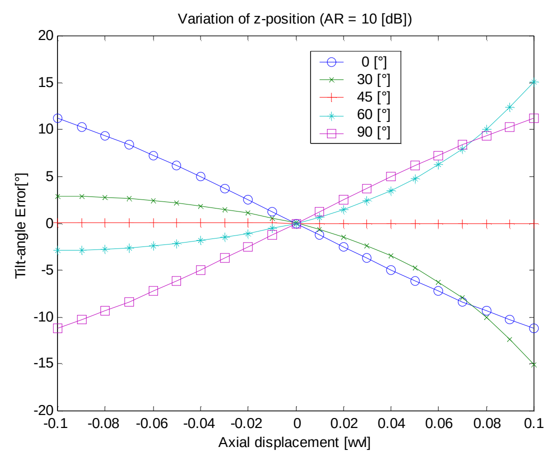

Figure 5: Tilt-angle error vs axial displacement, at various true tilt-angles and a fixed true axial ratio.

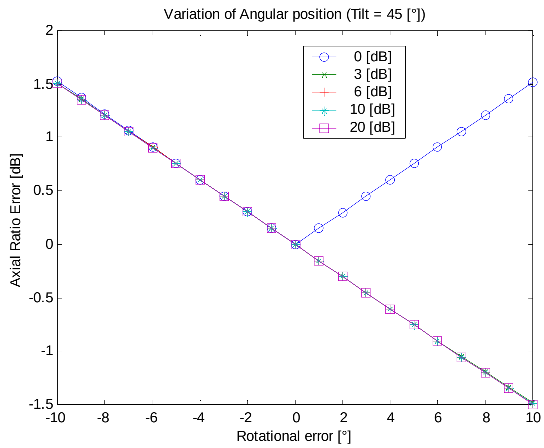

Figure 6: Axial ratio error vs probe rotational error, at various true axial ratios and a fixed true tilt-angle.

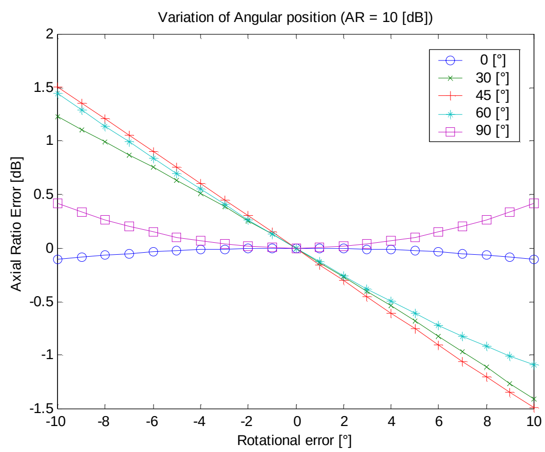

Figure 7: Axial ratio error vs probe rotational error, at various true tilt-angles and a fixed true axial ratio.

In Figure 6 and Figure 7 the axial ratio error as a function of the error in angle ψ (rotational error) for respectively a range of true axial ratios and a range of true tilt-angles is displayed. The tendencies noted indicate that the measured axial ratio becomes increasingly sensitive for errors in probe-rotation when the true tilt-angle moves away from 45°.

For the displayed range of rotational error, the true axial ratio appears not to influence the sensitivity of the measured axial ratio to errors in the probes rotational angle, apart from direction change for the 0dB true axial ratio curve. For rotational errors larger than displayed in this graph, also the curves for larger true axial ratios will eventually show this bend, caused by a local minimum in measured (CP) cross-polarization content at that rotational error. The location of this bend depends on the true axial ratio.

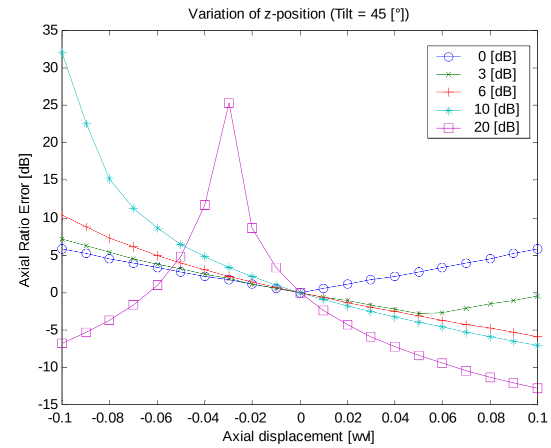

Figure 8: Axial ratio error vs axial displacement, at various true axial ratios and a fixed true tilt-angle.

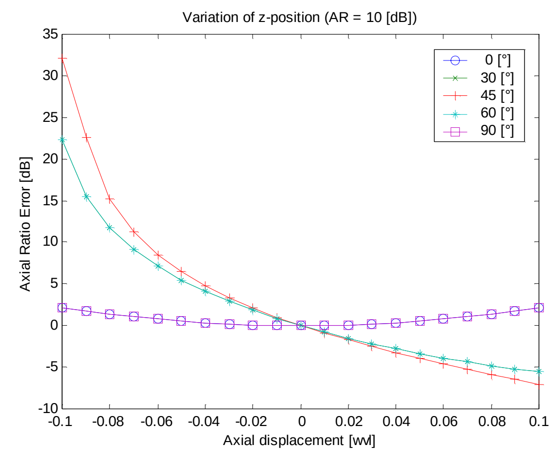

Figure 9: Axial ratio error vs axial displacement, at various true tilt-angles and a fixed true axial ratio.

Figure 8 and Figure 9 show the axial ratio error as a function of the change in distance between AUT and probe (axial displacement) between the two orthogonal measurements for respectively a range of true axial ratios and a range of true tilt-angles. The sensitivity of the axial ratio error for axial displacement shows to increase for increasing axial ratios and tilt-angles approaching 45°.

2.2 The effect of probe cross-polar errors on AR and τ

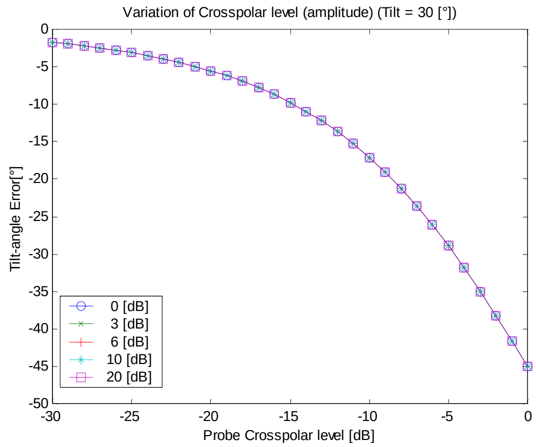

Figure 10: Tilt-angle error vs probe cross-polar amplitude (x-pol phase = 15°), at various true axial ratios and a fixed true tilt-angle.

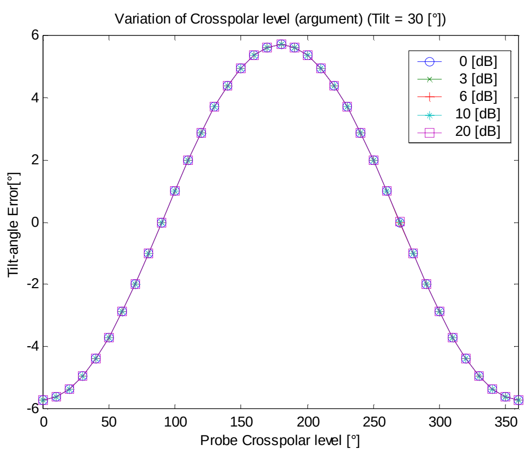

Figure 11: Tilt-angle error vs probe cross-polar amplitude (x-pol amp = -20 dB), at various true axial ratios and a fixed true tilt-angle.

Figure 10 and Figure 11 show the tilt-angle error as a function of respectively the probe's cross-polar amplitude (with fixed phase, chosen to give nearly maximum error) and cross-polar phase (with fixed amplitude), for a range of true axial ratios. The sensitivity of the tilt-angle error appears to be independent of the true axial ratio or true tilt-angle (therefore dependence on true tilt-angle not shown here). From Figure 11 it is observed that the maximum error in measured tilt-angle due to probe cross-polarization occurs when this component is in phase or in counter-phase with the main-polarization. Figure 10 shows that this error increases with increasing cross-polar amplitude.

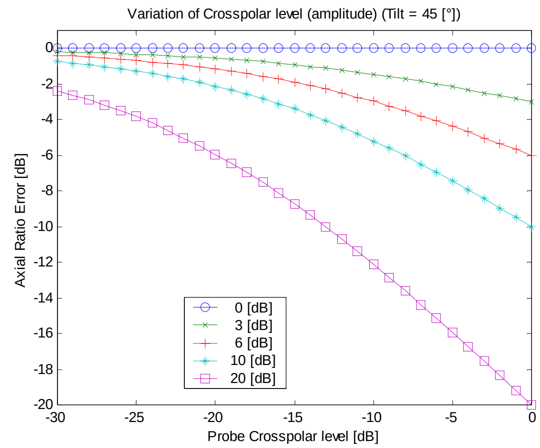

Figure 12: Axial ratio error vs probe cross-polar amplitude (x-pol phase = 90°), at various true axial ratios and a fixed true tilt-angle.

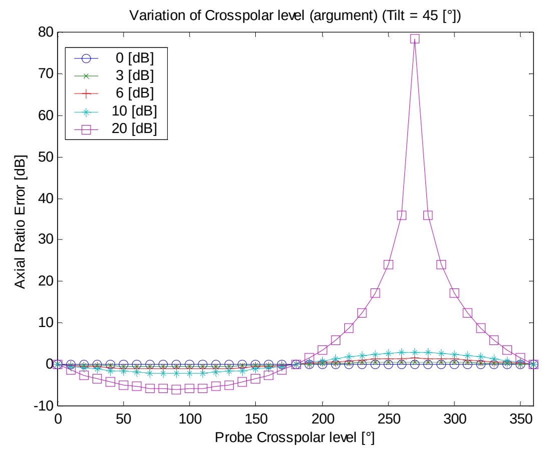

Figure 13: Axial ratio error vs probe cross-polar amplitude (x-pol amp = -20 dB), at various true axial ratios and a fixed true tilt-angle.

Finally, Figure 12 and Figure 13 show the axial ratio error as a function of respectively the probe's cross-polar amplitude (with fixed phase) and cross-polar phase (with fixed amplitude), for a range of true axial ratios. Here it is observed that the influence of the cross-polar component of the probe on the axial ratio error increases with increasing true axial ratio, especially when the cross-polar component is in quadrature with the co-polar component. (The axial ratio error due to the probe's cross-polar level does not depend on the true tilt-angle, and is not show here.)

AR_{true} [dB]	τ_{true} [°]	Δz = 0.01 λ		Δψ = 1°		Xpol = -20 dB, 90°	
		ΔAR [dB]	Δτ [°]	ΔAR [dB]	Δτ [°]	ΔAR [dB]	Δτ [°]
0	0	0.54	NA	0.15	NA	0.00	NA
10	0	0.02	1.26	0.00	0.11	2.12	0
10	30	0.79	0.68	0.13	0.20	2.12	0
10	45	0.91	0	0.15	0.51	2.12	0
10	60	0.79	0.68	0.13	0.82	2.12	0
10	90	0.02	1.26	0.00	1.11	2.12	0
20	0	0.02	0.36	0.00	0.01	5.93	0
20	30	2.11	0.21	0.13	0.25	5.93	0
20	45	2.40	0	0.15	0.51	5.93	0
20	60	2.11	0.21	0.13	0.76	5.93	0
20	90	0.02	0.36	0.00	1.01	5.93	0

Table 1: Numerical values for errors (absolute values) in axial ratio and tilt-angle due to axial displacement, rotational error and probe cross-polarization.

For reference, some values for the axial ratio error and tilt-angle error are given in Table 1. It is observed that both axial displacement and probe cross-polarization may cause significant errors in measured axial ratio.

3 Conclusion

From the data presented here it can be concluded that the method of linear-component measurement to obtain the axial ratio is especially sensitive for the phase-error (axial displacement) and for the probe's cross-polarization. This cross-polarization-content can be measured and compensated for. The phase-error (axial displacement) has to be kept to a minimum by a carefully designed polarization-rotator and proper RF-cabling and rotary joints.

4. REFERENCES

[1] Balanis, Constantine A., "Antenna Theory, analysis and design", John Wiley & Sons, 1982

[2] Newell, Allen C., "Improved Polarization Measurements Using a Modified Three-Antenna Technique", IEEE Transactions on Antennas and Propagation, Vol. 36, No. 6, June 1988, pp. 852-4