SPHERICAL COORDINATE SYSTEMS FOR DEFINING DIRECTIONS AND POLARIZATION COMPONENTS IN ANTENNA MEASUREMENTS

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ABSTRACT

The results of theoretical calculations or measurements for antennas are generally given in terms of the vector components of the radiated electric field as a function of direction or position. Both the vector components and the direction parameters must be defined with respect to a coordinate system fixed to the antenna. Along the principal planes there is no ambiguity about the terms such as vertical or horizontal component, but off the principal planes the definition of directions and vector components depends on how the spherical coordinate system is defined. This paper will define four different spherical coordinates that are commonly used in measurements and calculations, and propose a terminology that is useful to distinguish between them, and define the mathematical transformations between them. These concepts are essential when the results of different measurements or calculations are compared or when an antenna's orientation is changed. Both mathematical and graphical representations will be presented.
Keywords: Antenna, measurements, near-field, coordinate systems

1. INTRODUCTION

An antenna coordinate system is implicit in almost every antenna measurement. Terms such as pattern, main and cross-component, beam pointing and peak gain, imply the definition of directions and/or vector field components, which require a coordinate system. Reference is often made to **the** cross-component of an antenna as if there is a unique definition of such a quantity when in fact the cross-component as well as the main component will depend on which coordinate system is used and how it is oriented with respect to the antenna. Ambiguity and confusion can be avoided by using precise definitions and a terminology that distinguishes the different coordinates. Ludwig[1] proposed such a terminology, and it is widely used. It does not clearly define all the coordinate systems commonly used however, and the following discussion will propose additions to the Ludwig terminology.

2. THETA PHI SPHERICAL COORDINATES

We begin by defining the X-Y-Z axes of the antenna coordinate system such that the main beam is approximately along the Z-axis. The X- or Y-axes are defined approximately coincident with the major polarization axis as shown in Figure 1. The precise

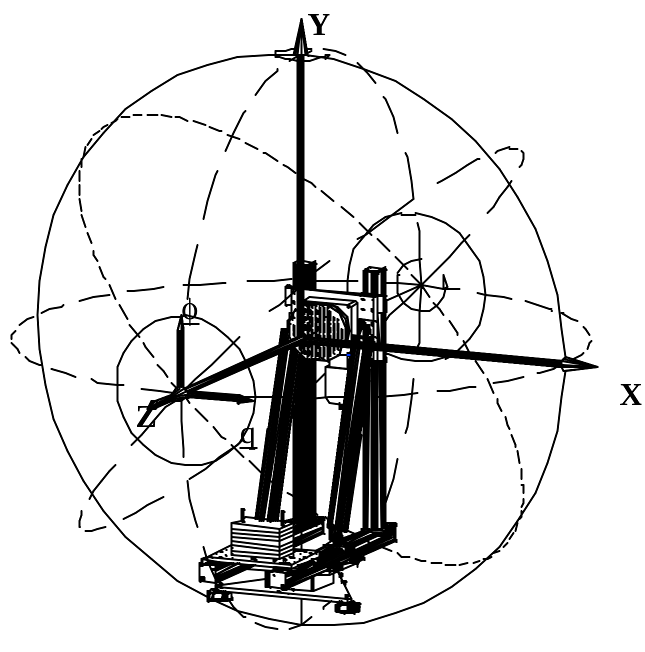

Figure 1** θ-ϕ **Spherical Coordinates

definition of the axes location and orientation must be accomplished by using fiducial marks, alignment mirrors or optical telescopes. Once this definition is accomplished, the X- Y- and Z-axes remain fixed to the antenna and do not change for any of the spherical coordinate systems that will be discussed.

The spherical coordinates shown in Figure 1 are the usual θ-ϕ coordinates with the Z-axis as the polar axis. The sphere surrounding the antenna should be much larger than the antenna since it represents the sphere on which the far-field vector components are measured or represented. For convenience it is shown as just enclosing the antenna. With this coordinate system, directions are specified by the angles (θ,ϕ) and vector components by the unit vectors q and **f.** The rotator system used for this coordinate system is the roll over azimuth positioner shown in Figure 1. In a far–field measurement, a vertically polarized source antenna will illuminate the AUT with a field that is polarized in the f-direction and the received signal will then correspond to the ϕ-component pattern. A horizontally polarized source will produce the θ-component pattern. It is important to note that when the antenna is rotated, the sphere defining the spherical coordinates and components stays fixed to the AUT and also rotates. The sphere is used to define the pattern radiated by the AUT as a function of coordinates fixed to the antenna. The sphere is not used to specify the directions the main beam points relative to axes fixed in space.

If the AUT is linearly polarized, the vectors q and f shown in Figure 1 are not the most appropriate for specifying the field vectors since in the region of the main beam the vectors change direction relative to the antenna as a function of ϕ. The orientation of the Z-axis relative to the antenna could be changed but there are some situations where the current definition is preferable, and it also makes it much easier to transform between the other spherical coordinates if we retain this definition.

3. LUDWIG-3 OR H-V VECTOR COMPONENTS

A modification of the θ-ϕ coordinates overcomes the polarization problem noted above, and is widely used in anechoic chamber measurements. The q and f unit vectors are rotated about the radial direction by the angle ϕ to obtain the vector components referred to as Ludwig-3 components. We have used the notation **h** and **v** to refer to these components since they define vectors that are approximately horizontal and vertical over most of the hemisphere. The resulting coordinates and vectors for this coordinate system are shown in Figure 2. In a far-field measurement using the Ludwig-3 components, the source antenna is rotated about its axis by the angle ϕ in synchronization with the AUT ϕ-rotation. If the magnitude and phase of two components in one system are known, the components in the other one are found from the following transformations.

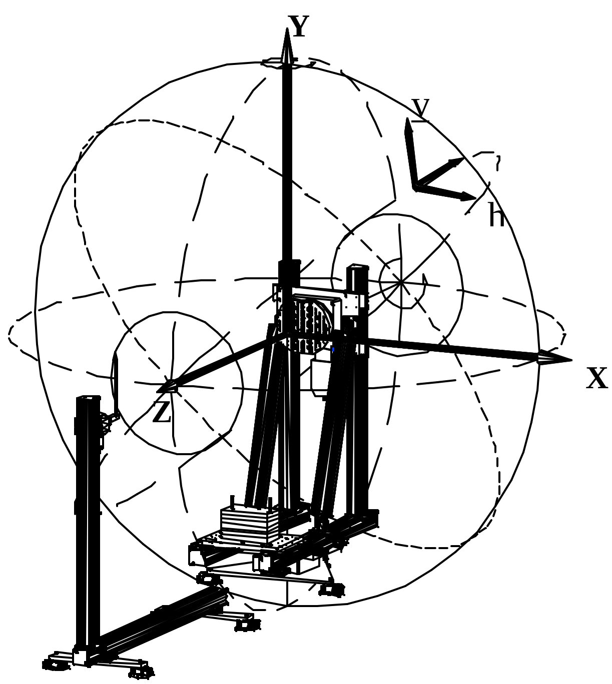

Figure 2 Ludwig-3 Or H-V vector components

$$E_h(q,f) = E_q(q,f)\cos f - E_f(q,f)\sin f$$
$$E_v(q,f) = E_q(q,f)\sin f + E_f(q,f)\cos f$$
(1)

Corresponding transformations will be shown for all the coordinate systems.

4. LUDWIG-2, AZIMUTH-ELEVATION COMPONENTS

There are actually two coordinate systems that come under the Ludwig-2 definition. They correspond to two different types of far-field rotators and the corresponding orientations of the polar axis of the spherical coordinates. The AZ-EL components are used with the Azimuth over Elevation rotator shown in Figure 3 and the polar axis in this case is coincident with the Y-axis.

With this rotator, the origin of the AUT coordinate system is not centered on the antenna. It is at the intersection of the two axes of rotation, which is inside the mechanical structure. This will not effect the far-field amplitude pattern, but will change the phase pattern.

The rotation of the sphere with the AUT for this coordinate system is apparent in Figure 4 where the AUT has been rotated in both Azimuth and Elevation. The point on the sphere with coordinates (A,E) is now

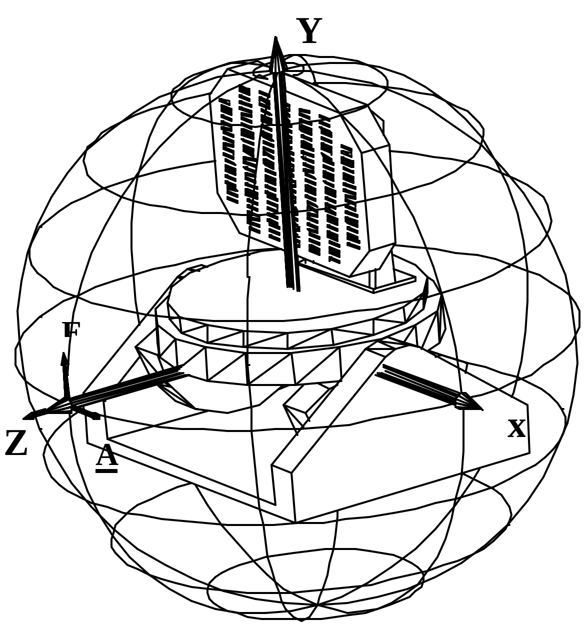

Figure 3 Az-El Coordinate System And Azimuth Over Elevation Rotato r.

along the line fro m the origin to the source antenna, and the **A** and **E**

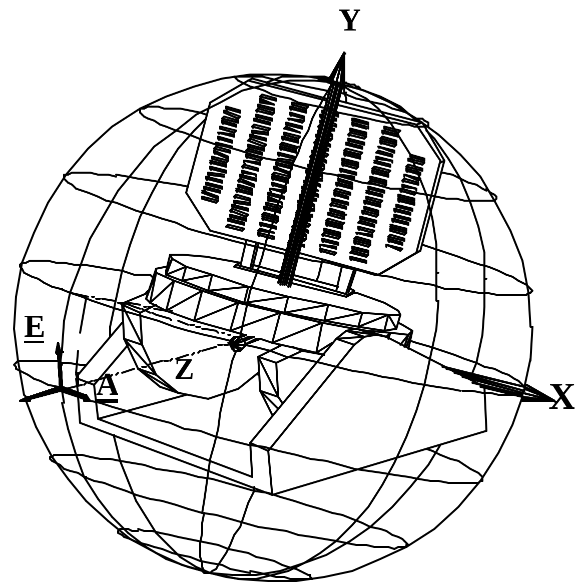

Figure 4 Rotated Azimuth Over Elevation Rotator.

components at this location will be measured by horizontally and vertically polarized source antennas respectively.

The transformations between θ-ϕ components and A-E components is given by,

$$E_A(A,E) = \frac{\cos\phi}{\cos E} E_\theta(\theta,\phi) - \frac{\cos\theta \sin\phi}{\cos E} E_\phi(\theta,\phi)$$

$$E_E(A,E) = \frac{\cos\theta \sin\phi}{\cos E} E_\theta(\theta,\phi) + \frac{\cos\phi}{\cos E} E_\phi(\theta,\phi)$$

(2)

where

$$\cos E = \sqrt{1-(\sin\theta \sin\phi)^2}\ .$$

(3)

5. LUDWIG-2 ALPHA-EPSILON COMPONENTS

The other Ludwig-2 components are associated with an elevation over azimuth rotator where the polar axis is coincident with the X-axis shown in Figure 5.

The transformations for these components are,

$$E_\alpha(\alpha,\varepsilon) = \frac{\cos\theta \cos\phi}{\cos\alpha} E_\theta(\theta,\phi) - \frac{\sin\phi}{\cos\alpha} E_\phi(\theta,\phi)$$

$$E_\varepsilon(\alpha,\varepsilon) = \frac{\sin\phi}{\cos\alpha} E_\theta(\theta,\phi) + \frac{\cos\theta \cos\phi}{\cos\alpha} E_\phi(\theta,\phi)$$

(4)

$$E_\alpha(\alpha,\varepsilon) = \frac{\cos A}{\cos\alpha} E_A(A,E) - \frac{\sin A Sin E}{\cos\alpha} E_E(A,E)$$

$$E_\varepsilon(\alpha,\varepsilon) = \frac{\sin A Sin E}{\cos\alpha} E_A(A,E) + \frac{\cos A}{\cos\alpha} E_E(A,E)$$

(5)

The angles in the different coordinate systems are related by,

$$\sin\theta \cos\phi = \cos E \sin A = \sin\alpha,$$
$$\sin\theta \sin\phi = \sin E = \cos\alpha \sin\varepsilon,$$
$$\cos\theta = \cos E \cos A = \cos\alpha \cos\varepsilon.$$

(6)

6. GENERAL CONCLUSIONS

Along the xz principal plane,

$$E_A(A,0) = E_\alpha(\alpha,0) = E_h(\theta,0\,or\,\pi) = \pm E_\theta(\theta,0\,or\,\pi)$$

$$E_E(A,0) = E_\varepsilon(\alpha,0) = E_v(\theta,0\,or\,\pi) = \pm E_\phi(\theta,0\,or\,\pi)$$

(7)

And along the yz principal plane,

$$E_A(0,E) = E_\alpha(0,\varepsilon) = E_h(\theta,\pm\frac{\pi}{2}) = \mp E_\phi(\theta,\pm\frac{\pi}{2})$$

$$E_E(0,E) = E_\varepsilon(0,\varepsilon) = E_v(\theta,\pm\frac{\pi}{2}) = \pm E_\theta(\theta,\pm\frac{\pi}{2})$$

(8)

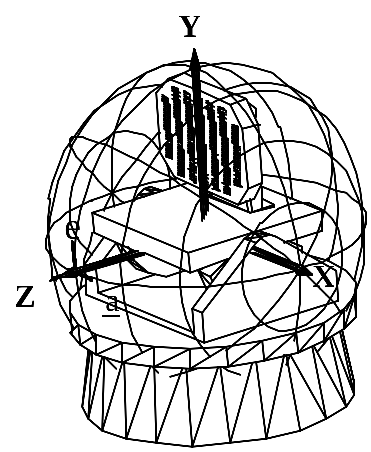

Figure 5** α-ε **Coordinate System With Associated Elevation Over Azimuth Rotator.

Therefore along the principal planes, there is little or no difference between the components, but as we move off the principal planes, the differences increase.

Various examples will be given to illustrate the use of the different coordinates and vector components and the potential problems that can arise if used incorrectly.

REFERENCES

[1] A. C. Ludwig, “The definition of cross polarization, IEEE Transactions on Antennas and Propagation. AP-21(1) pp. 116-119, January 1973.