Practical Issues in Advanced Antenna Pattern Comparison

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Abstract

This paper addresses some of the practical considerations and numerical consequences of using the Advanced Antenna Pattern Comparison (AAPC) method to improve the accuracy of antenna measurements in compact ranges. Two main issues are of particular importance:

1. Appropriateness of circle-fitting algorithm results to the measured data.
2. Ambiguous circles due to the distribution of data.

These issues deal specifically with Kasa's circle-fitting procedure—an essential part of the AAPC method—and provides useful checks for conditions commonly met with the use of this technique. We also briefly address the single interfering wave case that the AAPC method tacitly assumes.

1 Introduction

The advanced antenna pattern comparison method [1][2] is a numerical technique to improve measurement accuracy of high-performance antennas in compact ranges. It is superior to the traditional APC method [3] because both amplitude and phase data are used to estimate the error vector, in turn reducing the number of measurements required (to a minimum of three). The error vector is calculated from a circle-fitting technique based on some minimization criterion, typically least-squares.

Since the circle-fitting technique is a critical element of the AAPC method, it shall be investigated in more detail here. In particular, we seek to make noteworthy those elements of the circle-fitting technique which contribute most to the accuracy (or inaccuracy) of the AAPC method. It is our intention to derive error bounds where appropriate to analyze the applicability of the technique to the fitted data.

Finally, we turn our attention to the single interfering wave paradigm that the AAPC method tacitly assumes. In effect, we seek to briefly address those elements of the single interfering wave assumption that are critical for the AAPC method.

It is the ultimate purpose of this paper to present a unified numerical treatment of some practical issues associated with the AAPC method. To that end, the author hopes that the approach presented here—with its inherent level of detail—is used when analyzing any numerical method for improving measurement accuracy of high-performance antennas in compact ranges.

2 Numerical Consequences of Kasa's Algorithm

A least-squares error criterion for circle fitting is

\[ \sum_{i=1}^{N} (R_i - R)^2 = \min \] (2.1)

where

\[ R_i = \sqrt{(x_i - A)^2 + (y_i - B)^2} \] (2.2)

\((x_i, y_i)\) represent the x-y coordinates of the ith data point, \(N \geq 3\) is the number of data points, and \((A, B)\) the coordinates of the center, as shown in Fig. 1.

Kasa realized that error criterion defined by (2.1) was difficult to handle analytically and proposed a modified least-squares criterion [4]

\[ \sum_{i=1}^{N} (R_i^2 - R^2)^2 = \min \] (2.3)

where \(R_i\) is given by (2.2).
Let us define

\[ R_i = R + \Delta R_i \quad \text{(2.4)} \]

where \( \Delta R_i \) is the ith error, i.e., \( \Delta R_i = R_i - R \). Let us now factor (2.3) as

\[ \sum_{i=1}^{N} (R_i + R)(R_i - R) = \sum_{i=1}^{N} (4R^2\Delta R_i^2 + 4R\Delta R_i R_i + \Delta R_i^2) \]

If \( \Delta R_i \) is small so that \( R_i \approx R \), then Kasa's algorithm results in the error

\[ \sum_{i=1}^{N} 4R^2\Delta R_i^2 = 4R^2 \sum_{i=1}^{N} \Delta R_i^2 \quad \text{(2.5)} \]

Kasa's procedure therefore approximates the original least-squares criterion of (2.1) by a small error term.

Now let us form the relative error bound as

\[ \left| \frac{R_i - R}{R} \right| \leq \frac{\Delta R_i}{R} \quad 0 \leq \alpha < 1 \quad \text{(2.11)} \]

We therefore have

\[ |\Delta R_i|^2 = |\alpha R_i|^2 \leq \alpha^2 R_i^2, |\Delta R_i|^2 = |\alpha R_i|^2 \leq \alpha^2 R_i^2 \quad \text{(2.12)} \]

for any \( i \). The worst case in the above is equality. Including the equality and rearranging (2.9) we get

\[ 4R^2 \sum_{i=1}^{N} \alpha^2 R_i^2 + \sum_{i=1}^{N} \alpha^2 R_i^2 \leq \frac{4R^2}{M} \sum \Delta R_i^2 \quad \text{(2.13)} \]

We have \( N \) summations so \( \sum \alpha^2 R_i^2 = N \alpha^2 R_i^2 \), and \( \sum \alpha^2 R_i^2 = N \alpha^2 R_i^2 \). Therefore,

\[ 4N \alpha^2 R_i^2 + N \alpha^2 R_i^2 \leq \frac{4R^2}{M} \sum \Delta R_i^2 \]

or

\[ N \alpha^2 R_i^2 (4 + \alpha) \leq \frac{4R^2}{M} \sum \Delta R_i^2 \quad \text{(2.14)} \]

If \( \alpha < 4 \), then \( 4 + \alpha = 4 \) and

\[ \alpha^2 \leq \frac{1}{N} \sum \Delta R_i^2 \quad \text{(2.15)} \]

Now we make note of the fact that

\[ \sum_{i=1}^{N} \frac{a_i b_i}{c_i} = \frac{1}{N} \sum_{i=1}^{N} a_i b_i, \quad a_i, b_i \geq 0 \quad \text{(2.17)} \]

since \( b \) is a constant. We may therefore rewrite (2.16) as

\[ \alpha^2 \leq \frac{1}{N} \sum \frac{\Delta R_i^2}{R_i^2} \quad \text{(2.18)} \]

However,

\[ \frac{\Delta R_i^2}{R_i^2} = \left( \frac{R_i - R}{R} \right)^2 = \left( \frac{R_i - R}{R} \right)^2 \leq \alpha^2 \quad \text{(2.19)} \]

for all \( i \), in turn implying,

\[ \alpha^2 \leq \frac{1}{M N} \sum \left( \frac{\Delta R_i^2}{R_i^2} \right) \leq \frac{1}{M N} \sum \Delta R_i^2 \quad \text{(2.20)} \]

Since we have \( N \) summations, \( \sum \alpha^2 = N \alpha^2 \) and (2.20) becomes

\[ \alpha^2 \leq \frac{1}{M N} \sum \alpha^2 \quad \text{(2.21)} \]
or finally
\[ a = \max \left| \frac{R_i - R}{R} \right| \leq \frac{1}{M} \]  
(2.22)

Eq. (2.22) represents a check for the "closeness" of Kasa's approach to the original least-squares criterion. \( R \) is calculated from the Kasa procedure and then we use (2.22) for each \( R_i \) to make sure we are not violating some predefined bound. For example, if we wanted an order of magnitude difference between the square error and the higher order error terms in (2.8), we would make \( M = 10 \), forcing \(|\Delta R_i/R| \leq 0.1\).

3 Ambiguous Circles

Data points that lie almost on a line or are crowded together in close proximity can result in ambiguous circles as shown in Fig. 2. In this case, small variations in the data points will result in large deviations in the center and radius of the fitted circle. This is a different problem than Kasa's sensitivity analysis [4].

![Figure 2.](image)

Kasa's algorithm forms a linear equation (using Kasa's notation)
\[ E = DQ \]
(3.1)
where [4]
\[ D = \left( \begin{array}{ccc} 2 \sum x_i & 2 \sum y_i & N \\ 2 \sum x_i x_i & 2 \sum x_i y_i & \sum x_i \\ 2 \sum x_i y_i & 2 \sum y_i y_i & \sum y_i \end{array} \right) \]
(3.2)
\[ E = \left( \begin{array}{c} \sum (x^2 + y^2) \\ \sum (x^2 + y^2) \\ \sum (x y + y^2) \end{array} \right) \]
(3.3)
\[ Q = \left( \begin{array}{c} A \\ B \\ C \end{array} \right) \]
(3.4)
The radius is calculated from
\[ R^2 = A^2 + B^2 + C = Q^T Q - (C^2 - C) \]
(3.5)
where \( ^T \) is the transpose operator.

A close investigation of (3.2) will reveal that if this matrix is ill-conditioned, then large errors will result in the solution to
\[ Q = D^{-1} E \]
(3.6)
if \( x_i \approx x_{i2} \) and \( y_i \) is arbitrary, then row 2 becomes a linear combination of row 1. Similarly, if \( y_i \approx y_{i2} \) and \( x_i \) is arbitrary, row 3 is a linear combination of row 1. If \( x_i \approx y_i \) the data points are crowded together then rows 2 and 3 are linearly dependent. We will now derive a check for this condition.

Assume we have a solution
\[ \hat{Q} = Q + \epsilon Q \]
(3.7)
Let us determine the residual error as
\[ r = DQ - E \]
(3.8)
Solving this equation gives
\[ r = D \epsilon Q \]
(3.9)
so
\[ \epsilon Q = D^{-1} r \]
(3.10)
Taking the supremum norm we get
\[ ||Q|| = ||D^{-1} r|| \leq ||D^{-1}|| ||r|| \]
(3.11)
where for a vector \( \epsilon \)
\[ ||\epsilon|| = \max_i |\epsilon_i| \]
and for a matrix \( A \)
\[ ||A|| = \max_{i,j} |a_{ij}| \]
From (3.1) we have
\[ ||E|| = ||DQ|| \leq ||D|| ||Q|| \]
(3.12)
If we form the relative error we have
\[ \frac{||Q||}{||Q||} \leq ||D|| \]
(3.13)
We see that the relative error is proportional to the condition number \( K(D) \) of the Kasa matrix, i.e.,
\[ K(D) = ||D|| ||D^{-1}|| \]
(3.14)

A large condition number implies a large error for \( Q \)—or actually, the calculated \( \hat{Q} \). Since the center \((A, B)\) is obtained directly from \( Q \) and the square radius \( R^2 \) is proportional to \( Q^T Q \), large errors as measured by the error bound of (3.13)—and more specifically, the condition number \( K(D) \) in (3.14)—give an indication of the distribution of the data and the potential for ambiguous circles.
4 Single Interfering Wave Paradigm

There are many extraneous field sources in a compact range, namely: extraneous field components generated by reflections and scattering of the incident field, fields generated by leakage of the range antenna system, and mutual coupling between the range antenna and the antenna under test (AUT).

In the AAPC method the main field—or direct signal—is represented by the complex vector $E_d$. The vector sum of the extraneous field components is represented by the complex vector $E_e$. Nord and Vokurka [1] noted that "a theoretical evaluation of the error is prevented due to the very complicated diffraction problem of finding the total interference pattern between $E_d$ and $E_e"$. They then made the assumption that the frequency and polarization of $E_d$ and $E_e$ were identical. In addition, they made the assumption that only one extraneous source was present and "both incident fields behave like plane waves." While their experimental results seem to confirm the validity of the single wave assumption, we will briefly consider the consequences of the single wave assumption.

Before we proceed, let us recall that the AAPC method makes use of both magnitude and phase data to reconstruct the error vector, and often results in substantially less measurement samples than the traditional APC method. This reduction in samples has statistical consequences if the single wave assumption was not true. Indeed, other assumptions about the extraneous sources would have to be made, i.e., independence of the error sources, normal (or uniform) distribution of the error sources (both magnitude and phase), etc. If there is little information about the statistical nature of the error sources, then another approach must be taken to assure that the single interfering source is a valid assumption. We now present one possible approach for determining if the single interfering wave assumption is valid.

Let us first denote the complex vector $E_i$ as an error vector composed of the sum of the different errors present in the compact range. Following the development presented in §2, we would require

$$|e_i| \geq \sum_{i=1}^{K} |e_i| |i \neq p$$

or

$$|e_i| \geq M \sum_{i=1}^{K} |e_i| \quad i \neq p, M \geq 1$$

(4.1)

for the $p$th (largest) error source in the $K$ total extraneous sources. The problem has now been reduced to one of determining the largest source of error in the compact range and establishing that (4.1) holds. If this is the case, then $e_i$ is the single interfering wave and the AAPC method would successfully account for it. It is not possible to isolate the single largest error source, then the AAPC method's results may be misleading, as it depends on the nature of the single interfering wave for a proper estimate of the error.

5 Conclusion

The AAPC method is a promising numerical technique for improving the measurement accuracy of high-performance antennas in compact ranges. In order to use the AAPC method successfully, however, it is important to be aware of its shortcomings, and to assess whether the method is providing reliable results for the application at hand.

We have attempted to bring to light those practical issues that are critical in the use of the AAPC method, namely: Appropriateness of circle-fitting algorithm to the measured data and ambiguous circles due to the distribution of data. We also briefly considered the single interfering wave assumption of the AAPC method.

For the circle-fitting case, we first investigated the condition under which Kasa's algorithm breaks down from a true least-squares criterion. We next accounted for the distribution of data in ambiguous circles for the error estimate and the conclusion drawn from the AAPC method must be discarded.

Finally, we have briefly addressed the single interfering wave assumption of the AAPC method. We have attempted to bring this issue to light and have formulated a check for the validity of the assumption based on the single error being greater than the sum of all the other errors. While this approach is logical, its effectiveness at addressing the single interfering wave problem is still to be thoroughly investigated.

References


