AUTOMATED THREE-ANTENNA POLARIZATION MEASUREMENTS USING DIGITAL SIGNAL PROCESSING

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Abstract

In this paper we present a three-antenna measurement procedure which yields the polarization of an unknown antenna to an accuracy comparable to that of the improved method of Newell. The complete method is based on step-scan motion of the two polarization axes on which the antenna pairs are mounted. As a special case this step-scan procedure includes the usual single axis polarization pattern method of polarization measurement.

This three-antenna polarization measurement method can be readily automated and is carried out straightforwardly with the assistance of a minicomputer for data acquisition and data reduction. The data reduction method is based on conventional digital Fourier transform techniques and has the advantage of inherent noise rejection. It utilizes a large number of sample points which greatly overdetermine the parameters to be measured.

The method has been verified experimentally with measurements made on multiple overlapping sets of three antennas, as is conventional for this kind of procedure. The data are presented for broad-beam antennas of the type use as near field probe horns.

I. Introduction

The purpose of a polarization measurement is to determine the tilt angle, axial ratio and sense of the electric field polarization ellipse by means of a procedure carried out on an antenna measurement range. In general, these parameters are functions of direction, and therefore in general a polarization measurement of the field of an antenna must be made at each pattern direction of interest. Such solid angle measurements of polarization require knowledge of the polarization of the range antenna with which the measurement is made.

When the range antenna itself has unknown polarization characteristics, a three-antenna polarization measurement is performed to determine the polarization characteristics of the range antenna. Because of the complexity of the data reduction associated with a three-antenna polarization measurement procedure, such a procedure is usually limited to determining the polarization of an antenna in only one direction of the radiation pattern. By performing polarization pattern measurements on each of the three possible pairs of antennas in a three antenna set one is able to ascertain the axial ratio, tilt angle, and sense of the polarization ellipse for each of the three antennas. Usually, the polarization for the direction of the peak of the beam for each of the three antennas is measured. The only a-priori knowledge required is the approximate tilt angle (within 45°) of one of the antennas in the three antenna set and the fact that one of the antennas is reciprocal.

A number of different procedures have been devised in the past for performing three-antenna measurements of the complex polarization ratios of antennas. Newell and Kerns[l] and Joy and Paris[2] independently developed three-antenna phase/amplitude polarization measurement methods in the early 1970’s. Newell, Baird, and Wacker[3] described the NBS extrapolation technique for the measurement of polarization and gain in 1973 and gave some experimental results obtained using the method. All polarization measurement methods up to this time had a requirement for accurate measurement of phase. In fact, in certain unusual measurement situations, all the polarization information derives from the phase measurement.

In 1975(4] Newell recognized that a three-antenna polarization measurement method could be devised such that almost all of the information from the measurement derives from the amplitude measurement. Newell’s method, as it has come to be called, requires only that the sense (either positive or negative) of the 180° phase change associated with the null in the polarization pattern be known. This insensitivity to the measurement of phase is the primary advantage of Newell’s method, and is responsible for the improved accuracy of the method compared to those described in references 1-3. The method of polarization measurement described in this paper, like Newell’s method, depends on the fact that the form of the functional dependence of the measured quantities on the desired unknown polarization parameters is known. Instead of assuming phase information, as is done with Newell’s method, this new method uses the measured phase information but selectively removes measurement errors by digital signal processing, thereby improving the accuracy of the measurement.

The selective removal of errors is accomplished by a one- or two-dimensional discrete Fourier transform (DFT) which filters the measured data as a function of angle in the spatial frequency domain and leaves only the spatial frequency components that vary with angle as polarization patterns ideally should. A method of polarization measurement using a one-dimensional discrete Fourier transform was implemented in 1981 by workers at the Technical University of Denmark[5], but the method described herein based on a two-dimensional discrete Fourier transform has not been described before in the literature. The mathematical foundation for the method is described briefly in the next section.

II. Mathematical Basis

The three antenna method is carried out by measurement of the polarization patterns between successive pairs of antennas. The measurement configuration has one antenna transmitting an illuminating wavefront that is received by the second antenna. For a given pair the relative received phasor or normalized voltage response is given by the matrix product

$$\hat{V} = (W_r \; W_t) = W_r^{\dagger} \; W_t \tag{1}$$

where the superscript † denotes the complex conjugate of the transpose of a matrix and where the W_r and W_t are the column matrices

$$W_t = \begin{bmatrix} \cos \gamma_t \\ \sin \gamma_t \; e^{j\delta_{ct}} \end{bmatrix} \qquad W_r = \begin{bmatrix} \cos \gamma_r \\ \sin \gamma_r \; e^{j\delta_{cr}} \end{bmatrix} \tag{2}$$

W_t is the polarization column vector of the transmitting antenna--i.e. W_t is the polarization column vector of the wave which is incident on the receiver. W_r is the polarization column vector of the receiving antennaa--i.e. W_r is the polarization column vector of the wave to which the receiving antenna is perfectly polarization-matched.

The polarization matrices are related mathematically to the angles which specify the polarization of a wave by the location of its corresponding point on the Poincaré sphere[6]. See Figure I for an illustration of the Poincaré sphere showing the polar angle 2γ and the longitude δ_c. Each point of the Poincaré sphere is associated with a particular polarization state of a wave. There is a one-to-one mapping between points on the sphere and possible polarization states of the wave. The polar angle 2γ and the longitude δ_c are shown designating the polarization of the wave, with the poles of the sphere designating left-hand and right-hand circular polarization states.

If the antennas are physically rotated about the range axis, which is the line-of-sight joining them, the polarization matrices take on the following parametric dependence on the rotation angles ϕ and χ:

$$W_t(\phi) = \begin{bmatrix} \cos\gamma_t\, e^{+j\phi} \\ \sin\gamma_t\, e^{+j\delta_{ct}\,-j\phi} \end{bmatrix} \tag{3}$$

and

$$W_r(\chi) = \begin{bmatrix} \cos\gamma_r\, e^{-j\chi} \\ \sin\gamma_r\, e^{+j\delta_{cr}\,+j\chi} \end{bmatrix} \tag{4}$$

Then

$$\hat{V}(\phi,\chi) = \cos\gamma_t \cos\gamma_r\, e^{j(\phi+\chi)} + \sin\gamma_t \sin\gamma_r\, e^{-j(\phi+\chi+\delta_{cr}-\delta_{ct})} \tag{5}$$

where the time dependence e^{+jwt} has been assumed. The angles ϕ and χ are the angles through which the transmitting antenna and receiving antenna respectively are rotated in the clockwise direction as each is viewed from the other. Figure 2 illustrates the coordinate systems for the polarization measurements and defines the rotation angles.

Equation (5) will be recognized as a simple two-dimensional Fourier series in the angles ϕ and χ. Only two spatial frequency components in each variable are present--the positive and negative single cycle per cycle harmonics.

The coefficients of the Fourier series can be computed from the known parametric dependence of $\hat{V}$ on ϕ and χ by inversion:

$$\cos\gamma_t \cos\gamma_r = \left(\frac{1}{2\pi}\right)^2 \int_0^{2\pi}\int_0^{2\pi} \hat{V}(\phi,\chi)\, e^{-j(\phi+\chi)}\, d\phi\, d\chi \tag{6}$$

and

$$\sin\gamma_t \sin\gamma_r e^{-j(\delta_{cr}-\delta_{ct})} = \left(\frac{1}{2\pi}\right)^2 \int_0^{2\pi}\int_0^{2\pi} \hat{V}(\phi,\chi)\, e^{+j(\phi+\chi)}\, d\phi\, d\chi \qquad (7)$$

Taking the ratio of these two quantities we get that

$$M \equiv \left(\frac{\sin\gamma_t\, e^{+j\delta_{ct}}}{\cos\gamma_t}\right)\left(\frac{\sin\gamma_r\, e^{-j\delta_{cr}}}{\cos\gamma_r}\right) = \tan\gamma_t\, e^{+j\delta_{ct}} \tan\gamma_r\, e^{-j\delta_{cr}} \qquad (8)$$
$$= \hat{\rho}_{ct}\,\hat{\rho}_{cr}^*$$

where $\hat{\rho}_{ct}$ is the circular polarization ratio for the transmitting antenna and $\hat{\rho}_{cr}$ is the receiving circular polarization ratio for the receiving antenna.

Using the relation for a reciprocal antenna between the receiving circular polarization ratio and the transmitting circular polarization ratio that

$$\hat{\rho}_{cr}^* = \hat{\rho}_{ct} \qquad (9)$$

we see that this result (8) becomes simply

$$\hat{\rho}_c\,\hat{\rho}_c' = M \qquad (10)$$

where $\hat{\rho}_{ct}$ is written $\hat{\rho}_c$ and $\hat{\rho}_c'$ has replaced $\hat{\rho}_{cr}^*$. Now, both circular polarization ratios for the two antennas refer to transmission and the prime differentiates the two antennas.

Once we have the products of the complex circular polarization ratios for each of the three antenna pairs determined from successive polarization measurements, we can compute each of the circular polarization ratios from three equations in three complex unknowns derived as follows. For simplicity of notation we define the products of the complex circular polarization ratios as

$$\begin{aligned}\hat{\rho}_{cA}\,\hat{\rho}_{cB} &= M_{AB}\\ \hat{\rho}_{cA}\,\hat{\rho}_{cC} &= M_{AC}\\ \hat{\rho}_{cB}\,\hat{\rho}_{cC} &= M_{BC}\end{aligned} \qquad (11)$$

where the quantities M_{AB}, M_{AC}, and M_{BC} are complex numbers derived from the data reduction.

The solutions for the circular polarization ratios are given by

$$\hat{\rho}_{cA} = \sqrt{\frac{M_{AB}\, M_{AC}}{M_{BC}}} \qquad \hat{\rho}_{cB} = \sqrt{\frac{M_{AB}\, M_{BC}}{M_{AC}}} \qquad \hat{\rho}_{cC} = \sqrt{\frac{M_{AC}\, M_{BC}}{M_{AB}}} \tag{12}$$

Resolution of the sign ambiguity inherent in taking the square root follows from an a-priori knowledge of the approximate tilt angle of one of the three antennas.

Once the complex circular polarization ratio for an antenna is known, the axial ratio, tilt angle, and sense of polarization may be computed from it. The details of this computation are given in Hollis, Lyon, and Clayton.[6]

III. Experimental Procedure

The experimental procedure for any three-antenna polarization measurement method is to measure the relative received phasor voltage for the three antennas taken pairwise. The voltage as a function of rotation about the line of sight is recorded. This rotation is described by the angles ϕ and χ as shown in Figure 2. The measured data is reduced using expressions for the polarization parameters in terms of the relative received phasor voltage such as those given in Section II.

Initial alignment of the antenna coordinate system is important with any three-antenna polarization measurement method because of the need to maintain a constant orientation reference for measurement of the tilt angle. The coordinate systems of the antennas in the measurement must be defined by mechanical reference devices such as bubble levels mounted on the antennas. The measurement procedure described herein is used to measure directly the polarization of antennas in the coordinate systems so defined. No coordinate system redefinition is required. In practice, this means that the antennas are mounted with their mechanical reference devices in their reference orientation (e.g. with their bubble levels level). The angular offsets in ϕ and χ corresponding to this condition are noted and these offsets used to define the zero rotation angles for the coordinate systems.

The three-antenna polarization measurement method described herein is based on step-scan motion of the two polarization axes on which the antenna pairs are mounted. The complex voltage (phase and amplitude) at the port of a receiving antenna is measured by a phase/amplitude receiver and digitally recorded as one of the antennas is scanned. This scanning consists of rotating the source antenna through an angle $\phi = 360^\circ$ or the receiving antenna through an angle $\chi = 360^\circ$.

For single scan polarization measurements, which are reduced to polarization parameters by means of a one-dimensional DFT, this is the only measurement that is required for each antenna pair. The sample increment has only a secondary influence on the accuracy of the measurement. Our experimental results indicate that no further improvements in measurement accuracy result from use of sample increments less than 1°.

For multiple scan polarization measurements, the axes are rotated in a step-scan fashion. Following acquisition of single scan data as described above, the polarization axis which was not used for the scan (logically, this axis may be referred to as the step axis) is rotated through a discrete angular increment and the scan is repeated. The phase and amplitude data are recorded as functions of both the step and scan axis angles (ϕ and χ) and a two-dimensional DFT is used for reduction of the measured coupling data to polarization parameters. As in the single scan measurements neither the step nor the scan angle increment has a profound influence on the accuracy of the measurement. Our experimental results indicate that no further improvements in measurement accuracy result from use of step angle increments less than 15°.

IV. Experimental Results

A set of three single-ported antennas and one dual-ported, dual-polarized antenna was used to verify the measurement method. All possible combinations of the five antennas (the dual-ported antenna is considered to be two antennas and its two ports are referred to in the tables as antennas number 4 and number 5) were measured, with the results shown in Tables I and II.

For the single scan measurements shown in Table I the total variation in the measured axial ratio appears to be very large. However, because the axial ratio approaches infinity for nearly linear antennas, total variation in the axial ratio is a deceptive measure of accuracy. Note that the total variation in polar angle 2γ for all the antennas is 0.29° - 0.44°. This is a true indication of the size of the locus of uncertainty on the Poincaré sphere, while axial ratio variation is not. Note in particular that for antenna #2, with an axial ratio of 28.44 dB, a 0.78 dB total variation corresponds to a larger locus of uncertainty on the Poincaré sphere than does the 5.67 dB variation for antenna number 3 with an axial ratio of 49.31 dB, the 9.44 dB variation for antenna number 4 with an axial ratio of 57.50 dB or the 2.74 dB variation for antenna number 5, with an axial ratio of 38.92 dB.

The single scan measurements shown in Table I indicate the degree of accuracy achievable using a one-dimensional discrete Fourier transform for reduction of the polarization pattern data. Such a DFT tends to reject random phase and amplitude errors in the measurement such as stray reflections in the test volume. However, periodic errors, such as rotary joint phase and VSWR wow, are not rejected by the one-dimensional DFT since they have the same period as the polarization pattern (2π). But these periodic errors (in particular, rotary joint errors) are dependent only on the rotation angle of one polarization positioner (the one in which they are mounted), while the polarization pattern is dependent on the rotation angle of both polarization positioners. Therefore, the periodic errors in the measurement do not vary in the same way as the polarization pattern does as a function of both measurement angles, ϕ and χ, and they will tend to be rejected by the two-dimensional DFT in ϕ and χ which is used to reduce multiple scan polarization measurements.

Table 11 shows the results of multiple scan measurements on the same antennas. For multiple scan measurements, periodic as well as random errors tend to be filtered out, resulting in a 2 or 3 to 1 improvement in measurement accuracy over single scan measurements. Work on computer-simulated data indicates that multiple-scan data reduction (a two-dimensional instead of a one-dimensional DFT) reduces periodic errors by about 20 dB.

V. Conclusions

We have demonstrated an automated, highly accurate method of polarization measurement. The method is restricted to use under far-field conditions. The method is implemented as part of the spherical near-field subsystem of the Scientific-Atlanta 2022B antenna analyzer option 08A. The measurements required are simply phase/amplitude polarization patterns for multiple source orientations. No redefinition of coordinate systems (as in Newell's method) is required, and polarization parameters are computed automatically and displayed in the format shown in Figure 3. The automation makes this method easier to use than manual methods. This method has the further advantage that no longitudinal translation is needed to average out stray signals. Stray signals are rejected by the step-scan rotation.

VI. References

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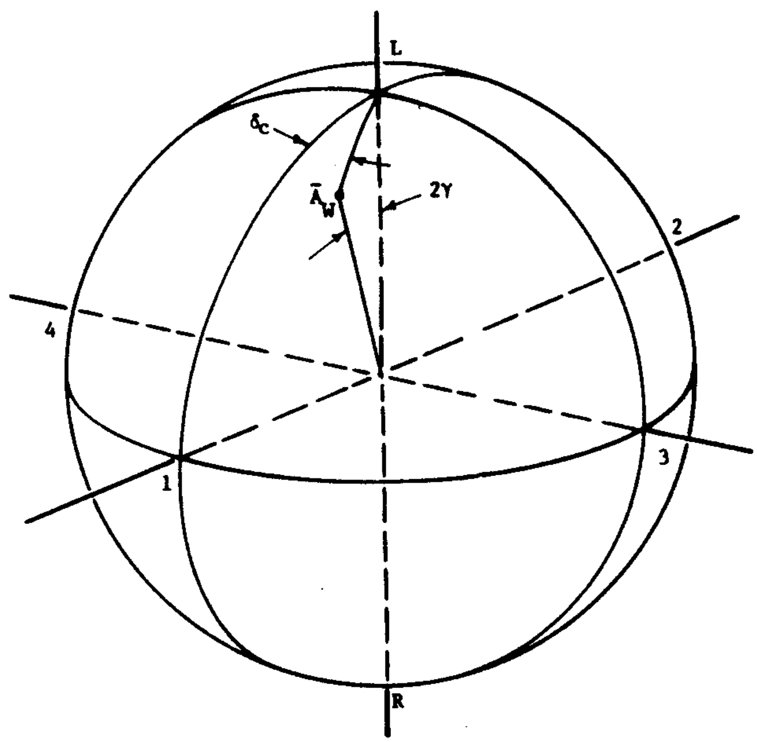

Figure 1. Poincare sphere with polar angle 2γ and longitude δ_c shown. Each point on the sphere represents a particular polarization state of a wave.

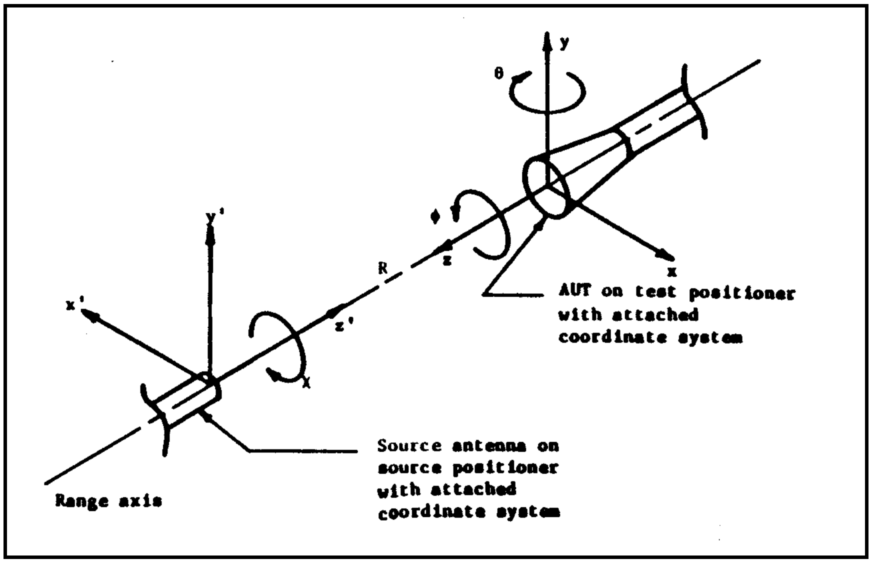

Figure 2. Coordinate system for polarization measurement.

POLARIZATION PARAMETERS FROM FILE DFIMS3

FREQUENCY (MHz)	AXIAL RATIO (DB)	SENSE	TILT ANGLE (DEGREES)
13000.0	53.22	RHE	88.98

ALTERNATIVE POLARIZATION REPRESENTATIONS

BASIS	COMPLEX POLARIZATION RATIOS		POINCARE SPHERE COORDINATES	
	AMPLITUDE	PHASE (DEGREES)	POLAR ANGLE (DEGREES)	LONGITUDE (DEGREES)
CIRCULAR	1.0044	177.9580	98.25	177.96
LINEAR	55.6969	-6.9888	177.94	-6.99
DIAGONAL	.9650	-.2504	87.96	-.25

Figure 3. Sample polarization parameters output list.

TABLE I
RESULTS OF THREE-ANTENNA SINGLE SCAN POLARIZATION MEASUREMENTS

Legend for table:

Antenna set number	Antenna numbers
1	1,2,3
2	1,2,4
3	1,2,5
4	1,3,5
5	1,4,5
6	2,3,5
7	2,4,5

Antenna Number	Polarization Parameter	THREE-ANTENNA SET 1	2	3	4	5	6	7	Nominal Value	Total Variation
1	Axial ratio (dB)	46.30	47.82	47.14	59.85	56.58	–	–	53.40	13.55
	Tilt angle (deg)	88.95	88.94	88.94	89.07	89.01	–	–	88.98	0.13
	Sense	RHE	RHE	RHE	RHE	RHE	–	–	RHE	–
	Polar angle 2γ (deg)	90.56	90.47	90.50	90.12	90.17	–	–	90.36	0.44
	Latitude δ_c (deg)	177.91	177.89	177.87	178.14	178.02	–	–	177.97	0.27
2	Axial ratio (dB)	28.28	28.10	28.17	–	–	28.88	28.73	28.44	0.78
	Tilt angle (deg)	90.44	90.45	90.46	–	–	90.53	90.51	90.48	0.09
	Sense	RHE	RHE	RHE	–	–	RHE	RHE	RHE	–
	Polar angle 2γ (deg)	94.42	94.51	94.47	–	–	94.12	94.19	94.34	0.39
	Latitude δ_c (deg)	-179.13	-179.10	-179.08	–	–	-178.94	-178.98	-179.05	0.19
3	Axial ratio (dB)	45.51	–	–	50.24	–	51.18	–	49.31	5.67
	Tilt angle (deg)	179.58	–	–	179.54	–	179.48	–	179.53	0.10
	Sense	LHE	–	–	LHE	–	LHE	–	LHE	–
	Polar angle 2γ (deg)	89.39	–	–	89.65	–	89.68	–	89.57	0.29
	Latitude δ_c (deg)	-0.84	–	–	-0.91	–	-1.04	–	-0.93	0.20
4	Axial ratio (dB)	–	62.12	–	–	54.69	–	52.68	57.50	9.44
	Tilt angle (deg)	–	89.63	–	–	89.58	–	89.56	89.59	0.07
	Sense	–	LHE	–	–	RHE	–	RHE	RHE	–
	Polar angle 2γ (deg)	–	89.91	–	–	90.21	–	90.27	90.13	0.36
	Latitude δ_c (deg)	–	179.26	–	–	179.15	–	179.11	179.17	0.15
5	Axial ratio (dB)	–	–	40.80	38.06	38.39	38.29	38.75	38.92	2.74
	Tilt angle (deg)	–	–	91.46	91.33	91.39	91.39	91.41	91.40	0.13
	Sense	–	–	RHE	RHE	RHE	RHE	RHE	RHE	–
	Polar angle 2γ (deg)	–	–	91.05	91.43	91.38	91.40	91.32	91.32	0.38
	Latitude δ_c (deg)	–	–	-177.08	-177.35	-177.23	-177.22	-177.19	-177.21	0.27

TABLE II
RESULTS OF THREE-ANTENNA MULTIPLE SCAN POLARIZATION MEASUREMENTS

Legend for table:

Antenna set number	Antenna numbers
1	1,2,3
2	1,2,4
3	1,2,5
4	1,3,5
5	1,4,5
6	2,3,5
7	2,4,5

		THREE-ANTENNA SET								
Antenna Number	Polarization Parameter	1	2	3	4	5	6	7	Nominal Value	Total Variation
1	Axial ratio (dB)	52.85	53.22	52.96	54.91	52.81	–	–	53.38	2.10
	Tilt angle (deg)	89.01	88.98	88.97	89.04	89.00	–	–	89.00	0.07
	Sense	RHC	RHC	RHC	RHC	RHC	–	–	RHC	–
	Polar angle 2γ (deg)	90.26	90.25	90.26	90.21	90.26	–	–	90.25	0.05
	Longitude δ_c (deg)	178.02	177.96	177.94	178.08	178.01	–	–	178.00	0.14
2	Axial ratio (dB)	28.67	28.66	28.80	–	–	28.91	28.78	28.76	0.25
	Tilt angle (deg)	90.43	90.47	90.49	–	–	90.51	90.52	90.48	0.09
	Sense	RHC	RHC	RHC	–	–	RHC	RHC	RHC	–
	Polar angle 2γ (deg)	94.22	94.22	94.16	–	–	94.11	94.17	94.18	0.11
	Longitude δ_c (deg)	-179.14	-179.06	-179.02	–	–	-178.98	-178.96	-179.03	0.18
3	Axial ratio (dB)	49.69	–	–	50.73	–	52.90	–	51.21	3.21
	Tilt angle (deg)	179.57	–	–	179.57	–	179.50	–	179.55	0.07
	Sense	LHC	–	–	LHC	–	LHC	–	LHC	–
	Polar angle 2γ (deg)	89.62	–	–	89.67	–	89.74	–	89.68	0.12
	Longitude δ_c (deg)	-0.86	–	–	-0.86	–	-1.00	–	-0.91	0.14
4	Axial ratio (dB)	–	55.05	–	–	55.70	–	52.69	54.57	3.01
	Tilt angle (deg)	–	89.62	–	–	89.59	–	89.56	89.59	0.06
	Sense	–	RHC	–	–	RHC	–	RHC	RHC	–
	Polar angle 2γ (deg)	–	90.20	–	–	90.19	–	90.27	90.22	0.08
	Longitude δ_c (deg)	–	179.24	–	–	179.18	–	179.12	179.18	0.12
5	Axial ratio (dB)	–	–	39.37	38.53	38.90	39.01	39.44	39.06	0.91
	Tilt angle (deg)	–	–	91.43	91.34	91.38	91.42	91.40	91.39	0.09
	Sense	–	–	RHC	RHC	RHC	RHC	RHC	RHC	–
	Polar angle 2γ (deg)	–	–	91.23	91.36	91.30	91.28	91.22	91.28	0.14
	Longitude δ_c (deg)	–	–	-177.13	-177.32	-177.24	-177.17	-177.19	177.21	0.15