MICROWAVE
ANTENNA
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CHAPTER 1
FOREWORD

This book was originally written for use as a text for a short course of the same title at San Fernando Valley State College, Northridge, California, July 14-18, 1969. The original was edited and revised for a second short course, this time at the Georgia Institute of Technology, in Atlanta, Georgia, July 20-24, 1970. This revision (1985) reflects current Instrumentation available for antenna testing including new trends in automation.

The editors recognize that further revisions and extensions are desirable and they welcome specific suggestions.

The contributions of a number of people in addition to the authors were necessary in the preparation of this text. The course was jointly conceived and outlined by Professor E. S. Gillespie of San Fernando Valley State College and by Scientific-Atlanta. Background material for the courses was presented in a monograph by Dr. Gillespie.* Thanks are due to the Printing Department of Scientific-Atlanta, to Mrs. Barbara B. Smith and Mrs. Jean A. Nichols, who typed the bulk of the manuscript, and to Mrs. Dianne Beaumont who typed the revised material.

Thanks are also due to Messrs. Ying-Tung Chou and Kai Hsu for preparation of the Index and for proof reading the original text and to Mr. Wayne K. Rivers of Georgia Tech for his criticisms and suggestions.

The rationalized MKS system is used throughout the text. Notation and terminology are defined at the point of introduction.

* See Reference 3, Chapter 2.
An antenna is a device for accomplishing a transition between a guided electromagnetic wave and a wave which propagates in free space. This book is concerned with measurements of the properties of antennas which operate in the microwave region of the spectrum.

Electromagnetic waves result from the acceleration of electric charges. While the electric field due to an un-accelerated charge (one at rest or in uniform motion in a straight line) is radially directed and decreases as the square of the distance from the charge, acceleration of the charge gives rise to a tangential component of electric field which decreases linearly with distance.\(^1\) This time varying electric field has associated with it a time varying magnetic field; together, they comprise an electromagnetic field. An electromagnetic field which decreases linearly with distance represents an outward radiation.

In practice, one is almost always concerned with macroscopic effects resulting from acceleration of gross numbers of charges. On the macroscopic scale, the interrelationship between electric and magnetic fields is described mathematically by Maxwell's equations.\(^2,3\) An additional set of equations called constitutive relationships\(^3,4\) specifies the characteristics of the medium in which the field exists.

The mathematics of electromagnetic fields and the associated media makes use of vector and tensor analysis; the analyses can become very involved in many problems, especially those involving propagation in non-isotropic, non-reciprocal, non-linear or non-homogeneous media. Although certain problems in antenna measurements can require application of more detailed mathematics, with few exceptions we will need consider only the relatively elementary aspects of vector analysis and electromagnetic theory in this book. The reader is assumed to be familiar with the basic vector operations and concepts of wave phenomena. Standard texts, such as the references cited in this and subsequent chapters, should be consulted as needed.
2.1 THE ANTENNA MEASUREMENT PROBLEM

The radiation characteristics of an antenna under a given set of conditions can be described by the functions \( G_1(\phi, \theta) \), \( G_2(\phi, \theta) \), \( \delta(\phi, \theta) \), \( \eta \), and \( Z \):

where \( G_1 \) and \( G_2 \) are the gain functions for two orthogonal polarizations, \( \delta \) is the phase angle between the output signals for the two polarizations, \( \eta \) is the antenna efficiency determined by its ohmic losses and \( Z \) is the impedance which the antenna presents at its input terminals. More generally, however, the radiation characteristics are of a family described by functions of variables such as \( (x_1, \phi, \theta) \).

\[
\begin{align*}
G_1(x_1, \phi, \theta) \\
G_2(x_1, \phi, \theta) \\
\delta(x_1, \phi, \theta) \\
\eta(x_1) \\
Z(x_1)
\end{align*}
\]

where the \( x_1 \) parameters represent such conditions as frequency, element phase, feed position, steady state temperature, differential temperature over the antenna, angular acceleration, pressure, wind distortion effects, etc.

†See Figure 2.1, page 2-4.

*\( G_1 \) and \( G_2 \) are usually responses to orthogonal linear polarizations or to right-hand and left-hand circular polarizations. In the latter case, \( \delta \) is usually designated \( \delta' \).
Evidently, measurement of the family of antenna characteristics which completely describe an antenna would be impossible. Representative measurements for increments of all parameters would present an insurmountable problem of information rate, data storage, and analysis, even if it were possible to realize the incremental values of the environmental parameters involved.

As in other fields where testing under all operational conditions cannot be realized, safety factors must be incorporated into design calculations to reduce the probability of significant variations of the antenna characteristics with parameters for which test data cannot be obtained. However, the performance of the antenna must almost always be measured over some region of solid angle and over some frequency range. These are most often defined by the requirements of a using system and represent areas where performance is critical and where it cannot be adequately or economically predicted by the design calculations. In addition, measurements may be required by the inter-system interference problem.

This text is primarily concerned with techniques for measuring the first four of the functions listed and their manifestations as they occur in such problems as boresight measurements. In addition, because of their close association with the antenna problem, chapters are included on radome measurement and on measurement of the scattering parameters of radar targets.

Before proceeding to the details of antenna measurement procedures, it is appropriate to review certain fundamental terms and relations which are basic to the antenna measurements field.

2.2 COORDINATE SYSTEMS AND ANTENNA POSITIONERS

Almost all antenna measurements involve determination in one way or another of signal levels as functions of position or direction in space, usually the latter. Because of the nature of radiation, the spherical coordinate system (Figure 2.1) is the system which is most often employed in antenna problems. Often the measurements of direction must be made to high degrees of precision because of the requirements of the operational system of which an antenna under test is a part.
Because of the relatively large distances that are often required between the antenna under test and the source antenna or the sampling antenna (depending on whether the antenna is tested on receiving or transmitting), it is often not practicable to explore the pattern of the test antenna in \((\phi, \theta)\) by movement of an antenna over a spherical surface. Instead, the line of sight between the test antenna and the associated antenna is typically held fixed in space while the antenna under test is changed in orientation to simulate movement of the line of sight over the sphere. This requirement has led to the development of special-purpose, precision antenna positioners. Antenna positioners and related coordinate systems are described in Chapter 5 and the mechanical aspects of positioners are described in Chapter 15.

2.3 **THE FRIIS TRANSMISSION FORMULA**

If it were necessary to determine the power transfer between two antennas by resorting to the basic processes which are defined by the field equations and diffraction theory, the calculations would be virtually impossible for all except the simplest of antennas. The Friis transmission formula permits the power transfer to be determined from knowledge of the measured directive...
properties and the dissipative attenuation of the antennas, independent of their detailed design. *

Let the total power accepted by an antenna from a source connected to its terminals be $P_0$. Let the efficiency of the antenna be $\eta$, so that the total radiated power is given by

$$P_t = \eta P_0 \quad \text{(watts)} \quad (2.1)$$

If the power $P_t$ were radiated isotropically, the radiation intensity $\Phi(\phi, \theta)$ in watts per steradian at a great distance from the antenna would be $P_t/4\pi$. Since in practice the power is never radiated isotropically, by definition the actual radiation intensity in the direction $(\phi, \theta)$ to the receiving antenna (Figure 2.2) will be given by

$$\Phi(\phi, \theta) = \frac{P_t}{4\pi} D(\phi, \theta) \quad \text{(watts/steradian)} \quad (2.2)$$

where $D(\phi, \theta)$ is the value of the directivity function ** in the direction $(\phi, \theta)$.

Since the steradian of solid angle *** subtends an area on the surface of a sphere equal to the square of the radius, the power density $S$ at the receiving antenna is given by

---

* The Friis transmission formula is based on the assumption that the antennas are polarization matched. See Section 2.4, and refer to Chapter 3 for discussion of power transfer between arbitrarily polarized antennas.

** The directivity as defined in (2.2) will be considered in this text to be a function $D(\phi, \theta)$, which describes the distribution in space of the energy radiated by an antenna. The IEEE Standard on Test Procedures for Antennas, January 1965, defines this property to be the directional gain and reserves the term directivity for the maximum value of the function. We will use the term directivity as the function and will call its maximum value the maximum directivity or simply the directivity if it is obvious that the maximum value is implied. This is in keeping with widespread usage in the field and is consistent with use of the gain $G(\phi, \theta)$ as the function and maximum gain or simply gain as the maximum value.

*** See Section 2.5.
FIGURE 2.2 Coordinates associated with transmitting and receiving antennas.

\[
S(R, \phi, \theta) = \frac{\Phi(\phi, \theta)}{K^2} = \frac{P_D(\phi, \theta)}{4\pi R^2} \quad \left(\text{watts meter}^{-2}\right)
\]  

(2.3)

If the receiving antenna is polarization matched to the incident field, by definition the received power \( P_r \) is given by

\[
P_r = S A_e(\phi', \theta') \quad \left(\text{watts}\right)
\]

(2.4)

where \( A_e(\phi', \theta') \) is the effective area of the receiving antenna in the direction \( (\phi', \theta') \) of the incident wave.

Equation (2.4) applies to any two polarization-matched antennas. If the receiving antenna is reciprocal, its effective area on receiving is related to its gain on transmitting by a universal constant \(^*\), \( \lambda^2 / 4\pi \), giving

\[
A_e(\phi', \theta') = G'(\phi', \theta') \frac{\lambda^2}{4\pi}
\]

(2.5)

\(^*\)See Appendix 2A.
Using (2.3) and (2.5) in (2.4) gives

\[ P_t = P_0 D(\phi, \theta) G'(\phi', \theta') \left( \frac{\lambda}{4\pi R} \right)^2. \]  

(2.6)

In terms of the input power to the transmitting antenna

\[ P_r = P_0 \eta D(\phi, \theta) G'(\phi', \theta') \left( \frac{\lambda}{4\pi R} \right)^2, \]  

(2.7)

or, since by definition

\[ \eta D(\phi, \theta) = G(\phi, \theta), \]  

(2.8)

we have

\[ P_r = P_0 G(\phi, \theta) G'(\phi', \theta') \left( \frac{\lambda}{4\pi R} \right)^2. \]  

(2.9)

Equations (2.4) and (2.9) can be modified for determining the power transfer between arbitrarily polarized antennas by introducing the polarization efficiency \( \Gamma \), giving

\[ P_r = S A_s(\phi', \theta') \Gamma \]  

(2.4a)

or

\[ P_r = P_0 G(\phi, \theta) G'(\phi', \theta') \left( \frac{\lambda}{4\pi R} \right)^2 \Gamma. \]  

(2.9a)

### 2.4 POLARIZATION

At large distances from a radiating antenna, the electric and magnetic vectors of the radiated field are at right angles to each other and to the direction of propagation. The two fields oscillate in time phase, and the ratio of their magnitudes (E/H) is a constant, \( \zeta \), the intrinsic impedance of free space, which has a value of approximately 120\( \pi \) ohms.
The polarization of an electromagnetic field is described in terms of the direction in space of the electric field. If the vector which describes the electric field at a point in space is always directed along a line, which is necessarily normal to the direction of propagation, the field is said to be linearly polarized. In general, however, the terminus of the electric vector describes an ellipse, and the field is said to be elliptically polarized.

Antennas exhibit polarization properties in relation to the fields they radiate or receive. If an antenna is operated on receiving, it will not in general be polarization matched to the incident field. If it is polarization matched, it will extract maximum power from the field, and its polarization efficiency is said to be unity. If its polarization is orthogonal to the field, it will extract zero power, and its polarization efficiency is consequently zero.

The polarization properties of fields and antennas are naturally of primary concern in any problems concerned with communication between antennas. Chapter 3 is devoted entirely to polarization analysis of electromagnetic fields and to calculation of polarization efficiency.

2.5 THE STERADIAN

Let a closed cone of rays of arbitrary shape (Figure 2.3) intersect a sphere of arbitrary radius centered at the apex of the cone. The ratio $\Omega$ of the surface area $A$ of the sphere subtended by the cone of rays to the square of the radius $R$ is defined as the magnitude of the solid angle defined by the cone of rays. It is evident that the magnitude of $\Omega$ is independent of the radius of the sphere since the surface area increases as the square of the radius.*

Since the total surface area of a sphere is given by

$$A_t = 4\pi R^2,$$  \hspace{1cm} (2.10)

*Compare the definition of the radian measure of plane angle where the radian is defined as the ratio of the subtended arc length to the radius, which is also independent of the radius.
the total solid angle contained within a sphere is

\[
\Omega = 4\pi \text{ steradians.} \tag{2.11}
\]

The steradian is evidently a dimensionless quantity.

\[\text{FIGURE 2.3 Geometry associated with steradian measure of solid angle.}\]

2.6 DECIBELS

The use of decibels in calculations and measurements of antenna characteristics is almost essential. The decibel, abbreviated dB, is a logarithmic unit which is used to measure the ratio between two amounts of power. By definition,

\[
\text{number of dB} = 10 \log \frac{P_1}{P_2} \tag{2.12}
\]

where \(P_1/P_2\) is a power ratio. When \(P_1/P_2\) is greater than unity, the number of dB representing the ratio \(P_1/P_2\) is positive; when \(P_1/P_2\) is less than unity, the number of dB representing \(P_1/P_2\) is negative. Where the power ratio is less than unity, the fraction is often inverted, and the ratio is expressed as a decibel loss.

Since power, voltage, and current are related by

\[
P = V^2/R = I^2 R \tag{2.13}
\]
number of dB = 10 \log \frac{(V_1^2/R_1)}{(V_2^2/R_2)} = 10 \log \frac{(I_1^2/R_1)}{(I_2^2/R_2)} \quad (2.14)

and, for the important case where R_1 = R_2,

number of dB = 20 \log \frac{V_1}{V_2} = 20 \log \frac{I_1}{I_2} \quad (2.15)

The value of the use of decibels in antenna work is largely based on two factors. First, if n_1 and n_2 are power ratios whose values in dB are N_1 and N_2, the product n_1n_2 is represented by N_1 + N_2 dB, and n_1/n_2 is represented by N_1 - N_2 dB. This permits the handling of products and quotients of large power ratios simply by the operations of addition and subtraction. Second, the decibel scale represents a compression of the power ratio scale, which is extreme for large ratios. This permits the display of tremendous power ratios on a single graph with equal resolution at all power levels.

To permit use of the decibel in specifying power levels, P_2 of equation (2.12) is sometimes set equal to a specific value of power. The dBm is such a measure which is almost universally used; it is defined with P_2 equal to one milliwatt: level of P_1 in dBm = 10 \log P_1/1 \text{ mw}.

Often the decibel is used informally to indicate a power level where the reference is understood. For example, it is customary to express side lobe levels as "-NdB" with no reference indicated. This is generally understood to mean "at a level of -NdB with reference to the maximum level of the main lobe." Sometimes side lobe levels are expressed without the minus sign, where an "NdB side lobe" is taken to mean a side lobe whose level is NdB below the maximum level of the main lobe. The specification of a side lobe gain per se as referenced to an isotropic standard, for example, is seldom used. In interpretation of specifications, one is cautioned to clarify any undefined terminology.

It is convenient to convert the Friis transmission formula (equation (2.9)) to decibels to simplify its application:

L_r = L_0 + g(\phi, \theta) + g'(\phi', \theta') - 20 \log (4\pi R/\lambda), \quad (2.16)
where

\[ L_r \] is the signal level at the output terminals of the receiving antenna in dBm,
\[ L_i \] is the signal level at the input terminals of the transmitting antenna in dBm,
\[ g(\phi, \theta) = 10 \log G(\phi, \theta), \]
\[ g'(\phi', \theta') = 10 \log G'(\phi', \theta'), \] and
\[ R \] is the transmitter-receiver separation.

In making pattern measurements, especially of high gain antennas, it is often desirable to measure over at least a 40 dB dynamic range to adequately investigate the character of the minor lobe structure of the antenna. Under this assumption we can replace \( g'(\phi', \theta') \) with \( g_{max} - 40 \) dB. In addition, \( g(\phi, \theta) \) will be replaced by \( g_{max} \) since the transmitting antenna will be directed with its beam maximum in the direction of the test antenna. Under these conditions,

\[ L_r = L_i + g_{max} + g'_{max} - 40 \text{ dB} - 20 \log \left( \frac{4 \pi R}{\lambda} \right). \quad (2.17) \]

This is the receiving level when the test antenna is oriented so that a -40 dB pattern level is directed toward the transmitting antenna.

A table for conversion of voltage and power ratios to decibels and a nomograph for evaluating the last term of (2.17) are given inside the back cover of the text. The quantity \(-20 \log \left( \frac{4 \pi R}{\lambda} \right)\), which is \(-10 \log \left( \frac{(4 \pi R)^2}{\lambda^2} \right)\), is often called space loss or space attenuation. We will call it the latter, although we prefer to think of it merely as a term in the equation.*

*This term should not be confused with the dissipative attenuation of a medium such as air which absorbs energy from the wave. The factor \( R^2 \) simply accounts for the fact that the power density in a spherical wave must decrease with distance as the energy in the wave spreads out over an ever increasing surface area as the wave progresses. The factor \( 4 \pi / \lambda^2 \), which relates the gain and effective area of the receiving antenna, is incorporated in the term for convenience. It does not imply that a higher frequency wave decreases in amplitude more rapidly than a lower frequency wave. It is simply a consequence
2.7 GAIN

The IEEE Test Procedure for Antennas of January 1965 defines the power gain* of an antenna in a specified direction as \( 4\pi \) times the ratio of the power radiated per unit solid angle in that direction to the net power accepted by the antenna from its generator. This is described mathematically by

\[
G(\phi, \theta) = \frac{4\pi \Phi(\phi, \theta)}{P_o},
\]  

(2.18)

where \( P_o \) is the power accepted by the antenna from its generator and \( \Phi(\phi, \theta) \) is the radiation intensity (power radiated per unit solid angle).

Rewriting (2.18) in the form

\[
G(\phi, \theta) = \frac{\Phi(\phi, \theta)}{P_o/4\pi}
\]

(2.19)

shows that the gain is the ratio of the power radiated per steradian in the specified direction \((\phi, \theta)\) to the power which would be radiated per steradian by a lossless isotropic antenna with the same input power \(P_o\) accepted at its terminals. This form gives a physical interpretation to the gain and also shows the relationship between gain and directivity from the definition of directivity

\[
D(\phi, \theta) = \frac{\Phi(\phi, \theta)}{P_t/4\pi}
\]

(2.2)

where \( P_t \) is the power radiated by the antenna.

Dividing (2.19) by (2.2) gives

\[
\frac{G(\phi, \theta)}{D(\phi, \theta)} = \frac{P_t}{P_o} = \eta,
\]

(2.20)

of the fact that for a given gain the effective area of a higher frequency antenna is smaller than that of a lower frequency antenna so that it intercepts a smaller amount of the power from the wave.

*The term gain used in this text is synonymous with the term power gain of the IEEE Test Procedures. See footnote on page 2-5.
the relationship indicated in (2.1). Since the efficiency can never be as great as unity, the gain must always be less than the directivity.

Techniques for measurement of gain are given in Chapter 8.

REFERENCES


3. E. S. Gillespie, Microwave Antenna Measurements - Background Material, a monograph prepared for the short course "Microwave Antenna Measurements," San Fernando Valley State College; July 1969.


APPENDIX 2A
RELATIONSHIP BETWEEN GAIN AND EFFECTIVE AREA

The purpose of this appendix is to derive the relationship between the gain $G(\phi, \theta)$ of an antenna and its effective area $A_e(\phi, \theta)$, which is given by

$$G(\phi, \theta) = \frac{4\pi}{A_e(\phi, \theta) \lambda}$$

(2A. 1)

Consider two arbitrary matched antennas 1 and 2 of Figure 2A. 1, which are located in free space and separated sufficiently that there is negligible interaction between them and that plane wave conditions exist. In (a) and (b) the

![Diagram](image)

FIGURE 2A. 1 Geometry showing reciprocal propagation between matched antennas.

*2A. 1 is a much used relationship and is virtually taken as a postulate in the field. It is usually derived from relationships involving dipoles. The derivation given here is from the viewpoint of aperture type antennas.*
direction of propagation is, respectively, from 1 to 2 and 2 to 1. In accordance with the reciprocity theorem, the power transfer which occurs is independent of the direction of power flow between the two. Thus from the Friis transmission formula (2.9) it can be seen that

$$\frac{G_1(\phi, \theta)}{A_{e1}(\phi, \theta)} = \frac{G_2(\phi', \theta')}{A_{e2}(\phi', \theta')} , \quad (2A.2)$$

where $G_1$ and $G_2$ are the antenna gains, and $A_{e1}$ and $A_{e2}$ are the effective areas of the antennas at a given frequency, which will be arbitrary but constant.

Since no restriction has been placed on antennas 1 or 2 or on their relative orientations it follows that for any antenna at a given frequency $G(\phi, \theta)$ and $A_e(\phi, \theta)$ are related by a constant, $K$; that is

$$\frac{G(\phi, \theta)}{A_e(\phi, \theta)} = K , \quad (2A.3)$$

and, if $K$ is evaluated for a particular case, it can be applied to every case. We must therefore show that $K$ is $4\pi/\lambda^2$ to prove (2A.1).

Now consider antenna 1 to be a lossless, directional antenna that is many, many wavelengths in diameter. In this limiting case the radiation from the antenna can be considered to be produced, in accordance with Huygens' principle, from the field over a planar aperture of area $A$ which is normal to the direction of propagation and immediately in front of the antenna, as in Figure 2A.2(a).

Let the antenna design be such that, when a power $P_0$ is fed into its terminals, the field $E$ is constant in phase, amplitude and direction over $A$. In this event,

$$P_0 = \frac{1}{2\varepsilon} \int_A E^2 \, ds = \frac{E^2 A}{2\varepsilon} . \quad (2A.4)$$

Again from the reciprocity theorem, if a plane wave of power density $S$ is incident, as in (b), on the same antenna from a direction such that the direction of power flow through the aperture is opposite to that on transmitting, then
the received power $P_r$ is given by

$$P_r = SA. \quad (2A. 5)$$

By definition the received power is the product of the incident power density times the effective area $A_e$; that is,

$$P_r = SA_e. \quad (2A. 6)$$

Thus from (2A. 5) and (2A. 6) the effective area of our hypothetical antenna is equal to its physical area.

Now consider the diffraction field of antenna 1. For the case of pure linear polarization, postulated here, the field magnitude $E_z$ at a point $P$ external to the antenna is approximated by the scalar diffraction integral $A^3$. 

FIGURE 2A. 2  Geometry showing reciprocal power flow through antenna aperture.
\[
E_p = \frac{1}{4\pi} \int_{\Sigma} E(o, y, z) e^{j2\pi/\lambda (y, z)} e^{-jkr} \left[ \frac{1}{r} \left( (jk + \frac{1}{r}) \mathbf{x} \cdot \mathbf{r} + jk \mathbf{x} \cdot \mathbf{p} \right) \right] ds
\]  

(2A. 7)

where \( k = 2\pi/\lambda \),

\( \mathbf{x}, \mathbf{r} \) and \( \mathbf{p} \) are unit vectors,
\( \mathbf{x} \) is normal to the aperture,
\( \mathbf{r} \) is in the direction to the field point \( P \),
\( \mathbf{p} \) is in the direction of local power flow through the aperture,
and \((o, y, z)\) denotes position in the aperture.

For our case the integral becomes equal to the magnitude of the radiated field with negligible error at a very large distance \( R \) from the aperture and for \( P \) on the \( x \) axis, giving

\[
E_p = \frac{E_A}{\lambda R} e^{-jkr}.
\]  

(2A. 8)

The power density \( S(R, \phi, \theta) \) at \( P \) is given by

\[
S(R, \phi, \theta) = \frac{E_p E_r^*}{2\pi} = \frac{E_A^2}{\lambda R} \frac{1}{2\pi}.
\]  

(2A. 9)

The gain of the antenna is defined by

\[
G(\phi, \theta) = \frac{\Phi(\phi, \theta)}{P_0/4\pi} = \frac{S(R, \phi, \theta)}{P_0/4\pi R^2}.
\]  

(2A. 10)

where \( \Phi(\phi, \theta) \) is the radiation intensity (the power radiated per steradian) in the direction \((\phi, \theta)\) and \( S(R, \phi, \theta) \) is the power density at \( P \).

Using (2A. 4) and (2A. 9) in (2A. 10) gives
but (2A. 5) and (2A. 6) show that for the antenna under consideration
\[ A_{\text{on axis}} = A, \text{ giving} \]
\[ \frac{G_{\text{on axis}}}{A} = \frac{4\pi}{\lambda^2} \quad (2A. 11) \]

The constant \( K \) of (2A. 3) has thus been evaluated for a particular case, proving (2A. 1).

Note: While the above was developed for linear polarization, it is also valid
for arbitrarily polarized antennas, where the total gain \( G \) is defined by

\[ G = G_a + G_b \quad (2A. 13) \]

and where \( G_a \) and \( G_b \) are the partial gains for orthogonal polarizations.

The development above applies to each polarization separately, giving

\[ \frac{G_a(\phi, \theta)}{A_{a\phi}(\phi, \theta)} = \frac{4\pi}{\lambda^2} \quad (2A. 14) \]

\[ \frac{G_b(\phi, \theta)}{A_{b\phi}(\phi, \theta)} = \frac{4\pi}{\lambda^2} \quad (2A. 15) \]

and

\[ \frac{G(\phi, \theta)}{A_{\phi}(\phi, \theta)} = \frac{4\pi}{\lambda^e} \quad (2A. 16) \]

where \( A_{\phi} = A_{a\phi} + A_{b\phi} \).
APPENDIX 2A
REFERENCES

A1. J. C. Slater, Microwave Transmission, McGraw-Hill, N.Y., 1942; Chapter VI.


3.1 INTRODUCTION

In this chapter the subjects of polarization of electromagnetic waves and the transfer of power between an arbitrarily polarized wave and a receiving antenna are discussed from a viewpoint that the authors have found helpful. Measurement of the polarization characteristics of antennas is covered in Chapter 10.

As an aid in understanding the polarization of light waves, the nineteenth century French mathematician, Poincaré, showed that it is advantageous in certain problems in optics to associate the polarization state of a wave with position on the surface of a sphere. Deschamps adapted the Poincaré sphere to radio waves and has written several papers on the subject.

In the treatment given here, the basic equations describing polarization phenomena are derived, and the mechanism for power transfer between a wave and an antenna is explained. The Poincaré sphere is developed to provide a graphical aid in visualizing and solving polarization problems.

* Note that the power transfer problem is specified to be between a wave and an antenna rather than between two antennas. This is because (1) the wave may be produced by either a passive scatterer or by an active antenna and because (2) the medium in which the wave travels can, in certain cases, alter the polarization of the wave as it appears at the aperture of the receiving antenna.

**A summary, which indicates the salient points of the chapter, is given on pages 3-46 through 3-48.
In addition to the basic development, certain extensions of the Poincare' sphere are made to show the relationship of the sphere to the multiple component method of polarization analysis and to aid in resolving waves into elliptical components. Some of the more detailed developments are given in appendices. Reference to directly related articles are presented at the end of the chapter preceding the appendices.

3.2 DERIVATION OF BASIC EQUATIONS

In Figure 3.1 let $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_p$ be mutually orthogonal unit vectors in free space, directed such that $\vec{u}_1 \times \vec{u}_2 = \vec{u}_p$. Assume a plane wave of arbitrary polarization traveling in the $\vec{u}_p$ direction. Phase fronts of the wave must be normal to $\vec{u}_p$ and therefore must be parallel with the 1,2 plane. The following development will be concerned with the field in the 1,2 plane.

![Figure 3.1 Geometry for plane electromagnetic wave traveling in free space in the $\vec{u}_p$ direction.](image)

**Linear Polarization Components** -- We will let $\vec{E}(t)$ be the vector representing the electric field in the 1,2 plane. It will be convenient to resolve $\vec{E}(t)$ into two orthogonal vector components, $E_1(t)\vec{u}_1$ and $E_2(t)\vec{u}_2$, which are aligned with the 1 and 2 directions respectively so that

---

*More specifically, the development will be concerned with the electric (E) field. Since the electric field and the magnetic (H) field are related by a constant, the E field is sufficient to our purposes here.

**See Appendix 3B for discussion of orthogonality.
If \( \vec{E}(t) \) is of a single frequency, \( f = \omega / 2\pi \), (3.1) can be written:

\[
\vec{E}(t) = E_{1t}\cos \omega t \hat{u}_1 + E_{2t}\cos (\omega t + \delta) \hat{u}_2 ,
\]

(3.2)

where we have chosen our time reference such that \( E_{1t}(t) \) is maximum when \( t \) is zero, as is shown in Figure 3.2. The terms on the right of (3.2) represent two orthogonal, linearly polarized fields, which differ in time phase by the angle \( \delta \). In general, the vector direction of \( \vec{E}(t) \) in the \( 1,2 \) plane will vary with time, and the tip of \( \vec{E}(t) \) will describe an elliptical locus, which is called the polarization ellipse. The field and the wave producing the field are then said to be elliptically polarized. If the vector direction of \( \vec{E}(t) \) does not deviate from a straight line, the wave is said to be linearly polarized. This occurs as \( E_1 \) or \( E_2 \) vanishes or as \( \delta \) approaches zero or \( n\pi \) (where \( n \) is an integer) causing the ellipse of Figure 3.2 to degenerate to a straight line.

![Figure 3.2. Electromagnetic field in the 1,2 plane resolved into orthogonal linear components, showing polarization ellipse. The electric vector \( \vec{E}(t) \) is shown for \( \omega t = 0 \).](image)

If \( \delta \) of (3.2) is in the third or fourth quadrant (i.e., negative and less than 180 degrees), \( \vec{E}(t) \) rotates from \( \hat{u}_1 \) toward \( \hat{u}_2 \).
In accordance with IEEE standards, the wave is said to have right-hand sense. Right-hand sense thus corresponds to right-hand screw rotation, with the direction of advance in the direction of propagation, \( \mathbf{\hat{u}}_p \). On the other hand, if \( \delta \) is in the first or second quadrant, the wave has left-hand sense.

The polarization properties of a wave can be defined by the shape and orientation of the polarization ellipse and by the sense of rotation of the electric vector in the \( 1,2 \) plane. We will define these properties in detail shortly, but it is convenient to consider first the special case of circular polarization and to explore certain other aspects of polarization which are necessary to our definitions.

**Exercise** -- Convince yourself that a wave has left-hand sense if \( \delta \) is in the first or second quadrant and right-hand sense if \( \delta \) is in the third or fourth quadrant.

**Circular Polarization Components** -- Let us return to (3.2) and consider a wave for which \( E_1 = E_2 \) and \( \delta = \pi/2 \). The polarization of this wave is left-hand circular, and (3.2) becomes

\[
\vec{E}_L(t) = E_L (\cos \omega t \mathbf{\hat{u}}_1 - \sin \omega t \mathbf{\hat{u}}_2) ,
\]

where \( E_L = E_{1z} = E_{2z} \), and where the total left circular field is designated \( \vec{E}_L(t) \).

On the other hand, with \( E_1 = E_2 \) and \( \delta = -\pi/2 \), the polarization is right-circular, and (3.2) becomes

\[
\vec{E}_R(t) = E_R (\cos \omega t \mathbf{\hat{u}}_1 + \sin \omega t \mathbf{\hat{u}}_2) .
\]

*The IEEE standards define the sense to be right-hand if the electric vector rotates clockwise for an observer looking in the direction of propagation. The definition given here is equivalent to that of the IEEE standard.

**We will usually say left-circular and right-circular for sake of brevity.*
Polarization ellipses for left-circular and right-circular fields are shown in Figure 3.3. The ellipses have, of course, degenerated to circles.

![Polarization ellipses](image)

**FIGURE 3.3** Polarization ellipses for left-circular and right-circular fields, showing sense of rotation. For right-circular polarization the rotation is from \( \vec{u}_1 \) toward \( \vec{u}_2 \). The fields are shown for \( \omega t = 0 \).

It can be shown that any field can be resolved into left-circular and right-circular components of appropriate magnitudes and relative phases. Equation (3.2) can then be written

\[
\vec{E}(t) = E_L [\cos (\omega t + \psi) \vec{u}_1 - \sin (\omega t + \psi) \vec{u}_2] +
E_r [\cos (\omega t + \psi + \delta') \vec{u}_1 + \sin (\omega t + \psi + \delta') \vec{u}_2]
\]  

(3.5)

The angle \( \psi \) is employed in all four terms of (3.5) to maintain the original time reference employed in (3.2) and the angle \( \delta' \) in the last two terms accounts for the difference in phase that may exist between the right-circular and left-circular components. *

---

*The angle \( \psi \) can be set equal to zero or omitted from (3.5) when it is not required to retain a specific time reference; it does not otherwise affect the characteristics of the polarization ellipse. We require \( \psi \) for later developments.
Equations for evaluation of $E_l$, $E_{\psi}$, $\psi$ and $\delta'$ from $E_{1x}$, $E_{2y}$ and $\delta$ are derived in Appendix 3A. Using values derived as indicated, the field of Figure 3.2 is shown in Figure 3.4, resolved into orthogonal circular polarization components in accordance with (3.5).

**FIGURE 3.4.** Elliptically polarized field resolved into orthogonal circular polarization components. In this example, $\psi$ is negative; (a), (b) and (c) occur in the time sequence (c), (a), (b).

(a) $\omega t = 0$

(b) $\omega t = -\psi$

(c) $\omega t = -(\psi + \delta'/2)$

In Figure 3.4a $\vec{E}(t)$ is shown for $\omega t = 0$. Note that $\vec{E}(0)$ is identical in Figure 3.2 and 3.4a. In Figure 3.4b, $\vec{E}(t)$ is shown for $\omega t = -\psi$; the field is given by

$$\vec{E}(-\psi/\omega) = E_l[\cos (0)\vec{u}_1 - \sin (0)\vec{u}_2] + E_r[\cos \delta'\vec{u}_1 + \sin \delta'\vec{u}_2].$$

(3.6)

$\vec{E}_l$ is oriented in the $\vec{u}_1$ direction and $\vec{E}_r$ is oriented at an angle $\delta'$ relative to $\vec{u}_2$.

In Figure 3.4c $\vec{E}(t)$ is shown for $\omega t = - (\psi + \delta'/2)$; $\vec{E}_l$ and $\vec{E}_r$ are co-aligned and are oriented at an angle,

$$\tau = \delta'/2$$

(3.7)
relative to \( \mathbf{u}_1 \). It is evident that the direction of alignment of \( \mathbf{E}_L \) and \( \mathbf{E}_R \) is the direction of the major axis of the polarization ellipse, and that the angle \( \tau \) defines the angle of the major axis relative to the \( \mathbf{u}_1 \) direction. The minor axis of the polarization ellipse is determined by the condition that \( \mathbf{E}_L \) and \( \mathbf{E}_R \) are oppositely directed. Observe that the sense of rotation is that of the larger circular component; that is, if \( \mathbf{E}_L \) is larger than \( \mathbf{E}_R \), the sense is left-hand.

Exercise - In (3.5), let \( \psi \) be zero and let \( \delta' = 30 \) degrees. (See note at the bottom of page 3-5.) What are the instantaneous directions of \( \mathbf{E}_R(t) \) and \( \mathbf{E}_L(t) \) when \( t = \pi/2\omega; \pi/\omega; 3\pi/2\omega \)? What is the tilt angle of the polarization ellipse? If \( E_R \) is 10 volts/meter and \( E_L \) is 5 volts/meter, what is the sense of rotation?

The Axial Ratio and the Circular Polarization Ratio -- Inspection of Figure 3.4c shows that the length of the major axis of the polarization ellipse is \( 2(E_L + E_R) \) and the length of the minor axis is \( 2|E_L - E_R| \). The axial ratio \( r \) is usually defined by

\[
r = \frac{E_L + E_R}{E_R - E_L}.
\]

The sign of the denominator thus determines the sense of polarization with \( r \) positive for right-hand sense and negative for left-hand sense. However, other definitions are sometimes employed; for example, Beckmann defines the axial ratio to be the negative of (3.8) while Deschamps employs the negative reciprocal. *We will use the definition of (3.8).

Study of Figure 3.4 shows that the polarization is left-hand circular when \( E_R/E_L \) is zero, approaches linear as \( E_R/E_L \) approaches unity, and approaches right circular as \( E_R/E_L \) approaches infinity. We will define this ratio by the symbol \( \rho \); that is,

\[
E_R/E_L = \rho.
\]

* Except that it has not been accepted as common practice, the definition employed by Deschamps would be the logical choice from the viewpoint of consistency with the Poincare sphere. See page 3-26.
We will designate it the circular polarization ratio (CPR).

We will usually use the quantity \( \rho \) rather than \( r \), in describing the polarization state of a wave. This is a departure from convention, although the circular polarization ratio has also been defined by Allen of the Naval Research Laboratory.\(^7\) As we proceed, it will be seen that the CPR is often more convenient than the axial ratio in describing polarization phenomena and that both simplification and symmetry result from its use. From (3.8) and (3.9) the CPR and axial ratio are related by

\[
\rho = \frac{r + 1}{r - 1} \quad (3.10)
\]

and

\[
r = \frac{\rho + 1}{\rho - 1} \quad (3.11)
\]

The axial ratio is often expressed in decibels, by the relation

\[
r(dB) = 20 \log \left| \frac{E_\theta + E_\phi}{E_\theta - E_\phi} \right|. \quad (3.8a)
\]

Note that the absolute value of \( r \) is required because one cannot take the logarithm of a negative number. Thus the sense of rotation must be indicated separately when \( r \) is expressed in decibels. The value of \( r(dB) \) varies from zero for circular polarization of either sense to infinity for linear polarization.

Returning to the circular polarization ratio for expressing the polarization state, if \( \rho \) is expressed in decibels by

\[
\rho(dB) = 20 \log \frac{E_r}{E_L} \quad , \quad (3.9a)
\]

the sense of rotation is included because \( \rho(dB) \) varies from minus infinity for LHC to zero for linear polarization to plus infinity for RHC.\(^*\)

\( \ast \) A table relating \( \rho, r, \rho(dB) \) as given in Appendix 3F. Note also that if \( \rho_1 = 1/\rho_2 \), \( r_1 = -r_2 \).
It will be seen in Chapter 10 that the method of measurement determines whether \( r(dB) \) or \( \rho(dB) \) is the more convenient form to use in visualizing the results of a measurement.

As a final note on the axial ratio, it is interesting that if a wave is resolved into linear components which are respectively aligned with and orthogonal to the major axis of the polarization ellipse, the two components will differ in phase by 90-degrees. See also (b) page 3-26.

**Diagonal Linear Components** -- We will now resolve the vector field \( \vec{E}(t) \) into orthogonal components \( E_3(t)\vec{u}_3 \) and \( E_4(t)\vec{u}_4 \), which are oriented at 45 degrees to the \( \vec{u}_1 \) and \( \vec{u}_2 \) components as shown in Figure 3.5.

![Figure 3.5](image)

**FIGURE 3.5** Vector field of Figure 3.2 resolved into orthogonal linear components along \( \vec{u}_3, \vec{u}_4 \) directions. The field is shown for \( \omega t = 0 \). Note that the field is identical in magnitude and direction in Figures 3.2, 3.4a and 3.5.

With the same time reference, the field of (3.2) can be written in terms of the \( \vec{u}_3, \vec{u}_4 \) components as

\[
\vec{E}(t) = E_{3n} \cos (\omega t + \phi) \vec{u}_3 + E_{4n} \cos (\omega t + \phi + \delta') \vec{u}_4 .
\]  

(3.12)
The angle $\phi$ is employed in both terms of (3.12) to retain the original time reference of (3.2) and (3.5). The angle $\delta''$ in the second term is employed to account for the difference in phase that may exist between the $\vec{u}_a$ and $\vec{u}_e$ field components.

Equation (3.12) can also be written

$$\vec{E}(t) = \frac{\sqrt{2}}{2} E_{a} \cos (\omega t + \phi) (\vec{u}_1 + \vec{u}_2)$$
$$+ \frac{\sqrt{2}}{2} E_{b} \cos (\omega t + \phi + \delta'') (\vec{u}_2 - \vec{u}_1), \quad (3.13)$$

since

$$\vec{u}_3 = \frac{\sqrt{2}}{2} (\vec{u}_1 + \vec{u}_2) \quad (3.14)$$

and

$$\vec{u}_4 = \frac{\sqrt{2}}{2} (\vec{u}_2 - \vec{u}_1). \quad (3.15)$$

Inter-relationships Among Polarization Components -- Equations (3.2), (3.5) and (3.13) are three different expressions describing the same elliptically polarized field $\vec{E}(t)$. These are repeated here for convenience:

$$\vec{E}(t) = E_{a1} \cos \omega t \vec{u}_1 + E_{b2} \cos (\omega t + \delta) \vec{u}_2, \quad (3.2)$$

$$\vec{E}(t) = E_{c} [\cos (\omega t + \psi) \vec{u}_1 - \sin (\omega t + \psi) \vec{u}_2] +$$
$$E_{r} [\cos (\omega t + \psi + \delta') \vec{u}_1 + \sin (\omega t + \psi + \delta') \vec{u}_2]. \quad (3.5)$$

$$\vec{E}(t) = \frac{\sqrt{2}}{2} E_{a} \cos (\omega t + \phi) (\vec{u}_1 + \vec{u}_2)$$
$$+ \frac{\sqrt{2}}{2} E_{b} \cos (\omega t + \phi + \delta'') (\vec{u}_2 - \vec{u}_1). \quad (3.13)$$

In Appendix 3A, these equations are employed to derive the inter-relationships among the several field components. These are shown in Table 3.1. In Section 3.5 we will see how (3.16) through (3.24) and (3.31) of Table 3.1 can be obtained by inspection from the Poincare' sphere. Equations (3.25) through (3.30) defining $\psi$ and $\phi$ are not necessary to the solution of polarization problems except those involving the instantaneous value and direction of $\vec{E}(t)$.

In Table 3.1 the inter-relationships are given in terms of peak values and effective values of the field components. The subject of effective values of elliptically polarized fields follows.
TABLE 3.1
INTER-RELATIONSHIPS AMONG POLARIZATION COMPONENTS

<table>
<thead>
<tr>
<th>Peak Values</th>
<th>Effective Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \delta = \frac{(E_L^2 - E_R^2)}{E_{1s}E_{2s}} ) (3.16)</td>
<td>( \sin \delta = \frac{(E_L^2 - E_R^2)}{2E_1E_2} ) (3.16a)</td>
</tr>
<tr>
<td>( \cos \delta = \frac{(E_{3s}^2 - E_{4s}^2)}{2E_{1s}E_{2s}} ) (3.17)</td>
<td>( \cos \delta = \frac{(E_{3}^2 - E_{4}^2)}{2E_1E_2} ) (3.17a)</td>
</tr>
<tr>
<td>( \tan \delta = \frac{2(E_{Ls}^2 - E_{Rs}^2)}{(E_{3s}^2 - E_{4s}^2)} ) (3.18)</td>
<td>( \tan \delta = \frac{(E_{L}^2 - E_{R}^2)}{(E_{3}^2 - E_{4}^2)} ) (3.18a)</td>
</tr>
<tr>
<td>( \sin \delta' = \frac{(E_{3s}^2 - E_{4s}^2)}{4E_{1s}E_L} ) (3.19)</td>
<td>( \sin \delta' = \frac{(E_{3}^2 - E_{4}^2)}{2E_1E_{L}} ) (3.19a)</td>
</tr>
<tr>
<td>( \cos \delta' = \frac{(E_{3s}^2 - E_{4s}^2)}{4E_{1s}E_L} ) (3.20)</td>
<td>( \cos \delta' = \frac{(E_{3}^2 - E_{4}^2)}{2E_1E_{L}} ) (3.20a)</td>
</tr>
<tr>
<td>( \tan \delta' = \frac{(E_{3s}^2 - E_{4s}^2)}{(E_{3s}^2 - E_{4s}^2)} ) (3.21)</td>
<td>( \tan \delta' = \frac{(E_{3}^2 - E_{4}^2)}{(E_{3}^2 - E_{4}^2)} ) (3.21a)</td>
</tr>
<tr>
<td>( \sin \delta'' = \frac{(E_{L}^2 - E_{R}^2)}{E_{3s}E_{4s}} ) (3.22)</td>
<td>( \sin \delta'' = \frac{(E_{L}^2 - E_{R}^2)}{2E_3E_4} ) (3.22a)</td>
</tr>
<tr>
<td>( \cos \delta'' = \frac{(E_{3s}^2 - E_{1s}^2)}{2E_{3s}E_{4s}} ) (3.23)</td>
<td>( \cos \delta'' = \frac{(E_{3}^2 - E_{1}^2)}{2E_3E_4} ) (3.23a)</td>
</tr>
<tr>
<td>( \tan \delta'' = \frac{2(E_{L}^2 - E_{R}^2)}{(E_{3s}^2 - E_{4s}^2)} ) (3.24)</td>
<td>( \tan \delta'' = \frac{(E_{L}^2 - E_{R}^2)}{(E_{3s}^2 - E_{4s}^2)} ) (3.24a)</td>
</tr>
<tr>
<td>( \sin \psi = \frac{-E_{2s} \cos \delta}{2E_{L}} ) (3.25)</td>
<td>( \sin \psi = \frac{-E_{2} \cos \delta}{2E_{L}} ) (3.25a)</td>
</tr>
<tr>
<td>( \cos \psi = \frac{(E_{1s} + E_{2s} \sin \delta)}{2E_{L}} ) (3.26)</td>
<td>( \cos \psi = \frac{(E_{1} + E_{2} \sin \delta)}{2E_{L}} ) (3.26a)</td>
</tr>
<tr>
<td>( \tan \psi = \frac{-E_{2s} \cos \delta}{(E_{1s} + E_{2s} \sin \delta)} ) (3.27)</td>
<td>( \tan \psi = \frac{-E_{2} \cos \delta}{(E_{1} + E_{2} \sin \delta)} ) (3.27a)</td>
</tr>
<tr>
<td>( \sin \psi = \frac{E_{2s} \sin \delta}{2E_{3s}} ) (3.28)</td>
<td>( \sin \psi = \frac{E_{2} \sin \delta}{2E_{3}} ) (3.28a)</td>
</tr>
<tr>
<td>( \cos \phi = \frac{(E_{1s} + E_{2s} \cos \delta)}{2E_{3s}} ) (3.29)</td>
<td>( \cos \phi = \frac{(E_{1} + E_{2} \cos \delta)}{2E_{3}} ) (3.29a)</td>
</tr>
<tr>
<td>( \tan \phi = \frac{E_{2s} \sin \delta}{(E_{1s} + E_{2s} \cos \delta)} ) (3.30)</td>
<td>( \tan \phi = \frac{E_{2} \sin \delta}{(E_{1} + E_{2} \cos \delta)} ) (3.30a)</td>
</tr>
<tr>
<td>( 2(E_{Rs}^2 + E_{Ls}^2) = E_{1s}^2 + E_{2s}^2 = E_{3s}^2 + E_{4s}^2 ) (3.31)</td>
<td>( E_{Rs}^2 + E_{Ls}^2 = E_{1}^2 + E_{2}^2 = E_{3}^2 + E_{4}^2 ) (3.31a)</td>
</tr>
</tbody>
</table>
Exercises -- (1) Convince yourself that (3.14) and (3.15) are true, so that (3.13) follows from (3.12). Convince yourself that if δ'' is in the second or third quadrant, the wave has right-hand sense of rotation.

(2) Verify the derivations of Appendix 3A.

Effective Values of Elliptically-Polarized Fields -- It is shown in Appendix 3B that the total power density $S$ (the time average of the Poynting vector) in any electromagnetic wave can be divided between any two orthogonal polarizations. This is expressed by the integral,

$$S = \frac{1}{\zeta T} \int_0^T \{E_1^2(t) + E_2^2(t)\} dt.$$  \hspace{1cm} (3.32)

where $E_1(t)$ and $E_2(t)$ are the coefficients of $\vec{u}_1$ and $\vec{u}_2$ in (3.5) and $\zeta$ is the impedance of space. Equation (3.5) is shown rearranged below with the $\vec{u}_1$ and $\vec{u}_2$ components collected:

$$\vec{E}(t) = [E_L \cos (\omega t + \psi) + E_R \cos (\omega t + \psi + \delta')] \vec{u}_1$$
$$+ [E_R \sin (\omega t + \psi + \delta') - E_L \sin (\omega t + \psi)] \vec{u}_2.$$  \hspace{1cm} (3.5a)

The indicated integration gives

$$S = (E_L^2 + E_R^2) / \zeta.$$  \hspace{1cm} (3.33)

If the effective value $E$ of the field is defined such that

$$S = E^2 / \zeta,$$  \hspace{1cm} (3.34)

then

$$E = (E_L^2 + E_R^2)^{\frac{1}{2}}.$$  \hspace{1cm} (3.35)
The maximum value of the total field is given by

\[ E_\pi = E_L + E_R . \]  

(3.36)

Therefore,

\[ \frac{E}{E_*} = \left( \frac{E^2_L + E^2_R}{E_L + E_R} \right)^{1/2}. \]  

(3.37)

In Figure 3.6, \( E/E_* \) is plotted as a function of \( \rho \) for \( \rho < 1 \) and as a function of \( 1/\rho \) for \( \rho > 1 \).

Note that for \( \rho = 1 \), that is for any linear polarization, \( E = (\sqrt{2}/2) E_* \), while for circular components \( E = E_r \). The development of the Poincare' sphere in Section 3.3 will be in terms of the effective values of the field components.

**FIGURE 3.6** The normalized effective values of elliptically polarized fields.

**Exercise** -- What is the effective value of an elliptically polarized field if the maximum value of \( \dot{E}(t) \) is 10 volts/meter and \( \rho = 2 \) from equation 3.37? If \( \zeta \) is 377 ohms, what is the power density in watts/equal meter? If \( \rho = 0.5 \), what is the effective value? Do these results agree with Figure 3.6?
Elliptical Polarization Components -- Thus far we have resolved $\mathbf{E}(t)$ into orthogonal linear and circular field components. However, the total power in an electromagnetic wave is the sum of the powers in any two orthogonal components. Thus $\mathbf{E}(t)$ of (3.2) can also be resolved into any two orthogonal elliptical polarization components $M$ and $N$ where $E_M$ and $E_N$ are the effective values of the two elliptical components. We will designate the ratio $E_N/E_M$ by $\rho_E$ and will call it the elliptical polarization ratio; $\rho_E$ is not unique for a given wave because it depends on the individual circular polarization ratios, $\rho_M$ and $\rho_N$, of the two orthogonal elliptical polarizations into which the wave is resolved. The ratios $\rho_M$, $\rho_N$ and $\rho_E$ are given by

$$
\begin{align*}
\rho_M &= \frac{E_{RM}}{E_{LM}}, \\
\rho_N &= \frac{E_{RN}}{E_{LN}}, \\
\rho_E &= \frac{E_N}{E_M}.
\end{align*}
$$

(3.38)

where we have designated the right and left circular components of the two elliptical components $E_{RM}$, $E_{LM}$, $E_{RN}$ and $E_{LN}$.

It is shown in Appendix B that for two orthogonal elliptical polarizations

$$
\rho_N = \frac{1}{\rho_M}
$$

(3.39)

and

$$
\tau_N = \tau_M \pm 90^\circ.
$$

In Figure 3.7, the field of Figure 3.2 is resolved into elliptical components,
FIGURE 3.7a Vector field of Figure 3.2 resolved into orthogonal elliptical polarization components.

where we have defined the relative phase angle $\delta''$ by the phase of $\vec{E}_{R}$ relative to that of $\vec{E}_{R_{m}}$ the direction of $\vec{E}_{L_{m}}$ and $\vec{E}_{R_{m}}$ defines $\tau_{n}$. For $\tau_{n} = 0$, $\delta''$ approaches $\delta'$ as $\vec{E}_{R_{m}}$ and $\vec{E}_{L_{m}}$ approach zero.

The vectors of Figure 3.7a are rearranged in Figure 3.7b, grouping the left-circular and right-circular components. The CPR of the total field is seen to be

$$\rho = \frac{E_{a}}{E_{c}} = \frac{\left[\frac{E_{R_{m}}^{2} + E_{R_{N}}^{2} + 2E_{R_{m}}E_{R_{N}}\cos\delta''}{E_{L_{m}}^{2} + E_{L_{N}}^{2} - 2E_{L_{m}}E_{L_{N}}\cos\delta''}\right]^{\frac{1}{2}}}{\delta}. \quad (3.40)$$
The tilt angle $\tau$, defined in (3.7) and Figure 3.4, is given by

$$\tau = \frac{\delta'}{2} = \tau_n + \frac{1}{2} (\eta + \nu), \quad (3.41)$$

since $\vec{E}_L$ must rotate through the angle $(\eta + \tau_n)$ to reach the $\vec{u}_1$ direction, while $\vec{E}_N$ rotates through the same angle in the opposite direction. The angles $\eta$ and $\nu$ are determined by the law of sines in their respective triangles.

A method for using the Poincare' sphere to determine the inter-relationships among $(\rho, \delta')$, $(\rho_n, \delta_n')$ and $(\rho, \delta''_1)$ is developed in Appendix 3C.

Exercise -- Consider an elliptically polarized wave which is resolved into two orthogonal elliptical components M and N. Let $E_m = 10$ volts/meter, $\rho = 1/3$, $E_{mN}/E_{mL} = 0.53$, $\tau_n = -10^\circ$ and $\delta''_1 = 107.5^\circ$. Determine $\rho$, $\tau$, $\delta'$ and $\tau$ for the wave. What is the sense of rotation? Determine $E_1$, $E_2$, $\delta$, $E_3$, $E_4$, $\delta''$. \[ \text{In Figure 3.7b } \tau_2 \text{ is negative and larger than } \eta. \text{ Therefore } \vec{E}_L \text{ has already passed the } \vec{u}_1 \text{ axis.} \]
3. 3 THE POINCARE' SPHERE

Development of the Sphere - - It has been established previously (Appendix B) that two elliptically-polarized fields are orthogonal if, and only if, their circular polarization ratios are reciprocal and if their tilt angles differ by 90 degrees. If two fields are orthogonal, the sum of the powers contained in the two polarizations is equal to the total power in the field.* Thus, given two orthogonal plane waves with polarizations M and N, the total power density, $S_w$, is given by

$$S_w = S_M + S_N.$$  \hspace{1cm} (3.42)

In accordance with (3.34)

$$S_w = \frac{E_M^2}{\zeta}, \quad S_M = \frac{E_M^2}{\zeta} \text{ and } S_N = \frac{E_N^2}{\zeta},$$ \hspace{1cm} (3.43)

where $E_w, E_M$ and $E_N$ are the effective values of their electric field vectors. Therefore, from (3.42) and (3.43)

$$E_M^2 + E_N^2 = E_w^2.$$ \hspace{1cm} (3.44)

Equation (3.44) forms the basis for the Poincare' sphere, however, it will be convenient to normalize $E_M$ and $E_N$ to $E_w$ because this will result in a sphere with a dimensionless radius of unity. To normalize we will write

$$F_M = \frac{E_M}{E_w} \hspace{1cm} (3.45)$$

and

$$F_N = \frac{E_N}{E_w} \hspace{1cm} (3.46)$$

so that

$$F_M^2 + F_N^2 = 1.$$ \hspace{1cm} (3.47)

*See Appendix 3B.
As a result of (3.47), we can construct triangle MBO of Figure 3.8 and erect semi-circle MWN of unit radius about O. It is evident from Figure 3.8 that the location of the point W on the semi-circle determines the division of power between polarizations M and N since it controls the ratio $\frac{E_M}{E_N}$.

It is further apparent that as $\theta$ approaches zero so that W approaches M, $E_N$ approaches zero and $E_M$ approaches unity. The converse is true as $\theta$ approaches 180 degrees. It is convenient to think of M, N and W as polarizations M, N, and W even though we do not yet have enough information to completely specify their polarizations.

*In the development and use of the Poincare' sphere, the subscribed dot, indicating division by $E_N$, will be omitted in expressions involving ratios, since the factor $E_N$ cancels in taking the ratio.*

FIGURE 3.8 Geometry leading to development of the Poincare' sphere
Thus far, we have let $M$ and $N$ represent arbitrary, elliptical polarizations. Now let $M$ and $N$ represent left circular (L) and right circular (R) polarizations respectively. The semi-circle of Figure 3.8 then becomes semi-circle LWR of Figure 3.9a. We will call angle LOW, $2\gamma$.

![Diagram](image)

**FIGURE 3.9** (a) Semi-circle of Figure 3.8 with $M$ and $N$ representing orthogonal circular polarizations L and R respectively. (b) Semi-circle of (a) with $2\gamma=90^\circ$ and with $W$ representing specifically linear polarization in the $\bar{u}_l$ direction. $W$ is replaced by the designator 1.

$E_m$ and $E_n$ become $E_L$ and $E_R$ and

$$\tan \gamma = \frac{E_R}{E_L} = \frac{E_R}{E_L} = \rho$$

in accordance with (3.9).
In Figure 3.9b, $2\gamma$ has been set equal to 90 degrees so that it represents some linear polarization ($\rho = 1$). Further, $W$ will represent the specific linear polarization for which $\delta' = 0$. $W$ is replaced by the designator 1, which indicates that the polarization is linear and that the major axis of the degenerate polarization ellipse lies along the $\vec{u}_1$ axis.

Let us now rotate semi-circle LWR about diameter LR from L1R through the angle $\delta'$. As LWR rotates through 360 degrees, the sphere of Figure 3.10a is generated.

![Diagram](image)

**Figure 3.10** (a) Generation of Poincare' sphere from circular polarization components, (b) Polarization state of wave represented by point $W$ on sphere.

The equator of the sphere represents the family of linear polarizations because it is the locus for which $\rho = 1$. In accordance with (3.7), the points on the equator for which $\delta'$ is successively zero, 90 degrees, 180 degrees and 270 degrees must represent respectively linear polarizations 1, 3, 2 and 4.* Thus, the axes $(1,2)$, $(3,4)$ and $(L,R)$ are mutually orthogonal. As $2\gamma$ varies from zero to 180 degrees and $\delta'$ varies from zero through 360 degrees (Figure 3.10b), each point

---

*1 for $\vec{u}_1$, 2 for $\vec{u}_2$, 3 for $\vec{u}_3$, and 4 for $\vec{u}_4$
W on the surface of the sphere uniquely represents a polarization \( W(2\gamma, \delta') \), the significance of \( 2\gamma \) is indicated by (3.48).

We can now let \( M \) and \( N \) of Figure 3.8 represent, successively, linear polarizations \((1,2)\) and \((3,4)\) and generate the spheres of Figures 3.11 and 3.12 by rotation through the phase angles \( \delta \) and \( \delta' \) about \((1,2)\) and \((3,4)\) as polar axes, respectively.

\[
\alpha = \tan^{-1}\left(\frac{E_2}{E_1}\right), \quad (3.49)
\]

and

\[
\beta = \tan^{-1}\left(\frac{E_4}{E_3}\right). \quad (3.50)
\]
Study of Figures 3.10, 3.11 and 3.12 indicates the mutual consistency of the three spheres at the orthogonal axes. To determine mutual consistency over the entire surface of the three spheres, we return to the basic semi-circle of Figure 3.8, which is redrawn in Figure 3.13 to show normal WD.

![Figure 3.13 Semi-circle of Figure 3.8 redrawn to show normal WD.](image)

Simple trigonometric identities\(^*\) show that

\[
OD = E_M^2 - E_N^2
\]

and

\[
WD = 2E_M E_N .
\]

Letting M and N represent their corresponding polarizations in the three spheres, gives

\[
E_1^2 - E_2^2 = \cos (2\alpha)
\]
\[
2E_1 E_2 = \sin (2\alpha)
\]

\[
E_3^2 - E_4^2 = \cos (2\beta)
\]
\[
2E_3 E_4 = \sin (2\beta)
\]

\(^*\)cos 2\theta = \cos^2 \theta - \sin^2 \theta; \sin 2\theta = 2 \sin \theta \cos \theta.
\[ E_{l}^2 - E_{r}^2 = \cos (2\gamma) \]

and

\[ 2E_{l}E_{r} = \sin (2\gamma) \]

(3.55)

In Figure 3.14, W is located at the same point on the three spheres; thus it is described by \((2\alpha, \delta), (2\beta, \delta'),\) or \((2\gamma, \delta')\). The right parallelepiped, whose major diagonal is OW, is inscribed in the sphere. Its sides, diagonals and angles are related by (3.53), (3.54) and (3.55). Further, the inter-relationships among the field components derived from (3.2), (3.5) and (3.9) and given in Table 3.1 are consistent with the geometry of Figure 3.14. Thus, the point W on the composite sphere represents the polarization of a given wave in terms of \((2\alpha, \delta)\) \((2\beta, \delta'\)) or \((2\gamma, \delta')\).*

Further, Figure 3.14, which can be simply committed to memory, serves as a valuable graphical aid in determining the inter-relationships among the various polarization equations. We will call this figure the polarization box.**

*It is also shown in Appendix 3C that poles M and N can be defined at any diametrically opposite points on the sphere and that the polarization of W is consistent in terms of \(2\theta, \delta''\), \(2\gamma_M\) and \(\delta'_M\) (see inset). The angles \(2\gamma\) and \(\delta'\) can be determined from \(2\theta, \delta''\), \(2\gamma_M\) and \(\delta'_M\) by standard Euler-angle transformations (see Chapter 5).

This extension of the Poincare' sphere, using arbitrary elliptical poles, provides a tool for analyzing errors in polarization measurements and for solution of complex polarization problems which may not yield to direct solution using the basic Poincare' sphere. The development of the sphere given here is somewhat different from that employed by Poincare' in several respects. We have employed three polar angles \(2\alpha, 2\beta\) and \(2\gamma\) with the corresponding phase angles \(\delta, \delta''\) and \(\delta'\). Poincare' and subsequent writers have used a polar angles for the \((1,2)\) components but have used

** See Appendix 3G for examples of the use of the polarization box in solving problems.
FIGURE 3.14 The polarization box inscribed in the Poincare sphere. This figure serves as a graphical aid in constructing the equations given in Table 3.1, describing the inter-relationships among the three basic sets of polarization components, (E₁, E₂), (E₃, E₄) and (Eₓ, Eᵧ). See also Appendix 3G.

a latitude angle for the (L, R) components. This is our angle $2\lambda (=2\gamma-90^\circ)$. Poincare' did not define the (3, 4) coordinate system. One further difference in the development here from that of Poincare' is that Poincare' developed the sphere by projection from a tangent plane onto the sphere rather than from an internal construction. The development here and the definition of the three orthogonal axes leads to the polarization box and is consistent with the multiple component method of polarization analysis, which is described in Section 3.4(c) and in Chapter 10.
3.4 SUMMARY OF METHODS OF DESCRIBING POLARIZATION

It has been shown in the foregoing sections that the polarization state of an arbitrary, elliptically-polarized electromagnetic wave can be described by any of the three sets of parameters \((E_1, E_2, \delta), (E_1^*, E_2^*, \delta'), \) and \((E_1, E_4, \delta'').\) Through the years these parameters have been used in a number of ways in polarization description and analysis.

The Poincare' sphere and the polarization box of Figure 3.14 provide convenient means for summarizing the various methods of describing polarization parameters. These are indicated in the following paragraphs.

(a) The Polarization Ratios - The circular polarization ratio has already been defined by

\[
\rho = \frac{E_2}{E_1} = \tan \gamma .
\]

The following polarization ratios are evident from symmetry:

**Linear Polarization Ratio:**

\[
\rho_L = \frac{E_2}{E_1} = \tan \alpha .
\]  \hspace{1cm} (3.56)

**Diagonal Polarization Ratio:**

\[
\rho_D = \frac{E_4}{E_3} = \tan \beta .
\]  \hspace{1cm} (3.57)

Finally the

**Elliptical Polarization Ratio:**

\[
\rho_T = \frac{E_m}{E_n} = \tan \theta
\]

follows from Figure 3.8.

*Because of the frequent usage of the circular polarization components in polarization analysis, the symbol \(\rho,\) designating the circular polarization ratio is written without a subscript for the convenience of the user. The less frequently used polarization ratios are designated by subscripted symbols.*
(b) The relationship of the axial ratio to the Poincare' sphere is evident from the construction of Figure 3.15. Axis A B is a polar axis in the equator of the sphere, which is rotated an angle $\delta' = 2\pi$ in the positive sense from the 1,2 axis. This orientation of the polar axis results in a 90-degree phase lead of $E_\theta$ relative to $E_A$. The polarization $W$ then lies in the ALBR plane, and $E_\theta/E_A$ defines the ratio of the length of the minor axis of the polarization ellipse to the major axis. It can be seen that

$$E_\theta/E_A = \frac{E_\theta - E_A}{E_\theta + E_A} = \tan \lambda.$$  \hspace{1cm} (3.59)

where $2\lambda$ is the latitude of $W$.

This logical definition of axial ratio was proposed by Deschamps. The definition which has come into common usage is given by

$$E_A/E_\theta = \frac{E_\theta + E_L}{E_\theta - E_L} = -\cot \lambda.$$  \hspace{1cm} (3.59a)

*The polar axis AB is in the equator of the sphere of Figure 3.10, which has $L$ and $R$ as poles. A and B must therefore represent linear polarizations. See Figure 3.15, inset. See also inset in footnote of page 3-23.*
(c) Multiple Amplitude Components. From Table 3.1 we have

\[ \tan \delta' = \frac{E_2^g - E_4^g}{E_1^g - E_2^g} \]  

(3.21)

This is shown in Figure 3.16, where the Poincare' sphere and polarization box are shown in plan view, looking down from the L pole.

It is thus seen that \( E_L, E_R \) and \( \delta \) (and thus the polarization) are uniquely determined by the three orthogonal pairs of amplitude components \((E_1, E_2), (E_3, E_4)\) and \((E_L, E_R)\). The polarization \( W \) can be thought of as given by the direction angles \( 2\alpha, 2\beta \) and \( 2\gamma \) of Figure 3.14, or by the three polarization ratios \( \rho, \rho_L \) and \( \rho_R \).

The multiple amplitude components provide a means of determining the polarization from specification of the amplitudes of field components without the necessity for specifying a phase angle or the tilt angle of the polarization ellipse. The multiple component method is described in more detail in Chapter 11.

(d) Hatkin defined \( E_R \) and \( E_L \) of Figure 3.16, where

\[ E_R = \sin \gamma = \frac{E_R}{(E_L^2 + E_R^2)^{1/2}} \quad \text{and} \quad E_L = \cos \gamma = \frac{E_L}{(E_L^2 + E_R^2)^{1/2}} \]  

(3.61)
and showed that the equation for the efficiency of power transfer between a wave

and an antenna is quite simple in terms of these ratios. The circular polariza-

tion ratio, which is the ratio of Hatkin's ratios, that is,

\[ \rho = \tan \gamma = \frac{\sin \gamma}{\cos \gamma} = \frac{E_R}{E_L} \quad , \quad (3.62) \]

is easier to specify from measured data and also results in a simple equation

for specifying the efficiency of power transfer. Therefore, we will generally

use \( \rho \) rather than Hatkin's ratios directly. Hatkin's ratios are also the magnitudes

of the circular polarization components of the polarization matrix; see (h), page

3-30.

(e) The Degree of Polarization - Beckmann defines the degree of polarization

\( P \) as a quantity which reduces, in our notation, to

\[ \rho = 2E_R E_L = 2 \sin \gamma \cos \gamma \quad . \quad (3.63) \]

This ratio is unity for linear polarization and vanishes for circular polarization

of either sense. \( P \) is the diagonal of the polarization box to the L-R axis. It

is plotted in Figure 3.18 in terms of \( \rho \) and \( 1/\rho \).
(f) Degree of Circular Polarization - The term degree of polarization defined in (e) is a carry-over from optics, where a polarized wave implies a linearly polarized wave. The degree of circular polarization, which we will define by

$$CP = E_L^2 - E_R^2 = \cos 2\gamma$$  \hspace{1cm} (3.64)

would seem to be of greater value since it indicates sense as well as degree. The degree of circular polarization is the height of the Polarization Box. It is plotted in terms of $\rho$ and $1/\rho$ in Figure 3.19.
(g) **The Stokes Parameters** - In investigations of partially polarized light, Stokes introduced parameters which turn out to be the major diagonal and the particular sides of the polarization box which are shown in heavy lines in Figure 3.20.

![FIGURE 3.20 Polarization box showing Stokes parameters.](image)

(h) **The Polarization Matrices** - For some polarization theory applications it is advantageous to express the polarization of a wave in matrix form. Any n-dimensional vector can be expressed in matrix form by

\[
\mathbf{x} = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\vdots \\
x_n
\end{bmatrix}
\]

(3.65)

where the \(x_i\) may be complex and are the \(i^{th}\) components of the vector. In polarization theory, the state of a vector in a plane is described, and only a two dimensional vector is required. When a wave is described in matrix form, all the

*See Chapter 13, Section 1.*
well-defined rules for matrix manipulations can be applied.*

The general form for the matrix notation of a normalized vector that describes the polarization of a wave is

\[
\begin{bmatrix}
E_M \\
E_N e^{j\delta''}
\end{bmatrix} = \begin{bmatrix}
\cos \theta \\
\sin \theta e^{j\delta''}
\end{bmatrix}
\] (3.66)

where \(E_M\) and \(E_N e^{j\delta''}\) are the components of basis vectors that are orthogonal in average power. **

\(M, N, \theta,\) and \(\delta''\) can assume the specific designations

\((1, 2, \alpha, \delta)\)

\((3, 4, \beta, \delta'')\)

and

\((L, R, \gamma, \delta')\)

for the respective basis vectors in addition to those for the general elliptical form. †

(i) **The Polarization Pattern** - The polarization pattern, as the name implies, is a measured quantity. †† It should not be confused with the polarization ellipse. The polarization pattern is the amplitude response of a linearly polarized antenna to an elliptically polarized wave as the linearly polarized antenna rotates through 360 degrees about the \(\bar{u}_x\) axis (Figure 3.1).

*Typical matrix manipulations that will be found useful are multiplication, inner product, transposition and conjugation.

**See Appendix 3B for the definition of orthogonality in average power.

†See pages 3-14 and 3-15 for explanation of elliptical components.

†† See Chapter 10.
Mathematically, the polarization pattern is the linearly polarized component of the total field in the \( \chi \) direction (Figure 3.21) as \( \chi \) varies from zero through 360 degrees.

![Figure 3.21 Polarization pattern, showing relation to polarization ellipse and showing values which are proportional to \( E_1, E_2, E_3 \) and \( E_4 \).]

The polarization pattern is tangent to the polarization ellipse at the major and minor axes. Thus, it indicates the tilt angle \( \tau \) and the magnitude of the axial ratio. Further, the ratios

\[
\frac{E_4}{E_1} = \rho_0 \tag{3.67}
\]

and

\[
\frac{E_4}{E_3} = \rho_0 \tag{3.68}
\]

are available from the polarization pattern. It is important to note, however, that the polarization pattern does not indicate the sense of polarization.

Derivation of the polarization pattern from the Poincare' sphere will be accomplished in Section 3.6.
3.5 POWER TRANSFER BETWEEN A WAVE AND AN ANTENNA

One of the principal motivations for analyzing antenna polarization is the requirement for calculating the efficiency of power transfer between terminal antennas of communication links. Of particular significance are problems associated with spacecraft and missiles. In the present discussion the problem is treated from the viewpoint of power transfer between an incident wave and a receiving antenna. The role of the transmitting antenna in producing the incident wave is described in the development of the Friis transmission formula in Chapter 2.

In the previous sections, we have developed means for describing the polarization of an electromagnetic wave and have related the polarization state of the wave to the Poincare sphere. We will now investigate the polarization efficiency, which determines the power transfer between an incident wave and a receiving antenna.

In the development, an approach similar to that taken in describing wave polarization will be followed; the polarization efficiency will be derived from a physical viewpoint with detailed derivations relegated to the Appendices. Then it will be related to the Poincare sphere, which will serve as an aid in visualizing and solving power transfer problems.

Antenna Polarization -- In keeping with accepted standards, the polarization of an antenna is defined by the polarization of the wave it radiates, where the polarization of the wave is defined by one of the means described in Section 3.4.

In order to make the polarization meaningful, it is necessary to refer it to some system of reference, usually the coordinate system of the antenna. With reference to Figure 3.22, the polarization of an antenna is defined for any direction.
(ϕ, θ) in the far zone of the antenna in a plane, which we will as usual designate the $1, 2$ plane. In keeping with IRIG standards, the $u_1$ direction has been chosen in the direction of positive $θ$, with $u_p$ in the direction of propagation.

**FIGURE 3.22** IRIG standard for defining polarization of an antenna by the polarization of the wave it radiates. The $u_1$ direction is the direction of positive $θ$, and $τ$ is measured from $u_1$ toward $u_2$, where $u_2 \times u_3 = u_p$.

The polarization of an antenna indicates the polarization of the wave it radiates in free space. It does not take into account polarization shift which may be introduced by the medium of propagation.

**Polarization Efficiency** -- Let an elliptically polarized wave $W$, of power density $S$, approach the antenna of Figure 3.23 from a distant source in the direction $ϕ, θ$. 

3-34
If the antenna and wave are polarization matched, from the definition of the effective area $A_e$ of the antenna, the power $P_r$ received by a matched load at the antenna terminals will be given by

$$P_r = S A_e \omega,$$  \hspace{1cm} (3.69)

where we will assume $A_e$ to define $A_e(\phi, \theta)$.

If the wave and the receiving antenna are not polarization-matched, it is necessary to introduce the polarization efficiency $\Gamma$ to account for the polarization mismatch. For general cases, (3.69) becomes

$$P_r = S A_e \Gamma \omega,$$ \hspace{1cm} (3.70)

If $W$ and $A_r$ are polarization-matched, $\Gamma$ is unity and (3.70) reduces to (3.69). For all other cases $\Gamma$ is less than unity, vanishing when the wave and antenna are orthogonally polarized. Derivation of an expression for $\Gamma$ is given in Appendix 3D.

Receiving Polarization -- In the following paragraphs we will use the Poincare sphere to determine $\Gamma$ of (3.70). However, we will first define the receiving polarization. As was indicated above, the defined polarization of an antenna

---

*See Chapter 2.
describes the polarization of the wave which is launched into space by the antenna. It is a common practice to say that the polarization of a reciprocal antenna is the same on receiving as on transmitting. It is true, as is shown in Appendix 3D, that a reciprocal antenna is polarization matched to a wave which has the same CPR and whose polarization ellipse is oriented in space identically with that of the wave which the antenna transmits in the direction of arrival of the incident wave under consideration. However, the polarization of this wave is not the same as the polarization of the wave which was employed to define the polarization of the antenna on transmission. This results because the two waves are traveling in opposite directions and are defined in different coordinate systems.

This is illustrated in Figure 3.24 where, in order to orient \( \mathbf{u}_{pr} \) in the direction of propagation of the incident wave, it is necessary to set

\[
\mathbf{u}_{pr} = -\mathbf{u}_{pt} \tag{3.71}
\]

It is consequently necessary to reverse the direction of either \( \mathbf{u}_{1r} \) or \( \mathbf{u}_{2r} \) in order to have

\[
\mathbf{u}_{1r} \times \mathbf{u}_{2r} = \mathbf{u}_{pr} . \tag{3.72}
\]

Since the tilt angle is measured from \( \mathbf{u}_{1r} \) toward \( \mathbf{u}_{2r} \), \( \tau_r \) is given by *

\[
\tau_r = 180^\circ - \tau \quad \text{or} \quad \tau_r = -\tau . \tag{3.73}
\]

Since from Appendix 3D,

\[
\rho_r = \rho , \tag{3.74}
\]

\( \rho_r \) and \( \tau_r \) can be used to define the receiving polarization, where \( \rho \) and \( \tau \) describe the antenna polarization. **

---

* \( \tau_r \) is given by \( 180^\circ - \tau \) or \( -\tau \) because the tilt angle is measured to either end of the major axis of the polarization ellipse. See Figure 3.24.

** For the definition of (3.73) to hold, it is necessary for the direction of either \( \mathbf{u}_{1r} \) or \( \mathbf{u}_{2r} \) to be the same as that of \( \mathbf{u}_{1t} \) or \( \mathbf{u}_{2t} \), respectively. In Figure 3.24 we have chosen to hold \( \mathbf{u}_{1r} = \mathbf{u}_{1t} \); the results would have been the same if we had maintained \( \mathbf{u}_{2r} = \mathbf{u}_{2t} \).
The above is basic to the solution of problems concerning power transfer between a wave and an antenna because the efficiency of power transfer from a wave to an antenna is related to the angular separation of $A_r$ and $W$ on the Poincare' sphere, as in Figure 3.25, where $A_r$ is the receiving polarization and $W$ is the polarization of an incident wave. The two polarizations must be put into the same coordinate system, as was done above, for their separation to determine the polarization efficiency.
FIGURE 3.25 Incident wave $W$ and receiving polarization $A_r$ plotted on Poincare' sphere.

As an alternative procedure to that outlined, the polarization of both the wave and the antenna can be transferred to the coordinate system of the wave which would be transmitted by the receiving antenna when employed as a transmitting antenna. This is shown in Figure 3.26.

On the right, $W$ is the polarization of the incident wave as defined in the normal manner, with $u_1$ in the direction of propagation, and $A_r$ is the receiving polarization of the antenna as defined in (3.67) and (3.68).

FIGURE 3.26 Alternate conventions for plotting incident wave and polarization of an antenna on Poincare' sphere.
On the left, the polarization $A$ of the antenna is the antenna polarization and the polarization of the incident wave is transformed to $W_t$, defined by

$$\rho_{wt} = \rho_w$$  \hspace{1cm} (3.75)

and

$$\tau_{wt} = 180^\circ - \tau_w \text{ or } \tau_{wt} = -\tau_w$$  \hspace{1cm} (3.76)

The resulting plots are symmetrical about the $L1R$ plane and the distances $WA_r$ and $W_tA$ are identical. In general we will use the configuration on the right of Figure 3.26 because it involves the Poincare' sphere in its defined manner, that is to graph the polarization of waves in a right-hand coordinate system with $u_p$ in the direction of propagation of the wave, for both $W$ and $A_r$. However, in certain situations the alternate configuration has merit and either method can be used. The power transfer is identical.

3.6 EVALUATION OF THE POLARIZATION EFFICIENCY BY MEANS OF THE POINCARÉ SPHERE.

In Figure 3.27 let $W$ represent the polarization of a wave which is approaching an antenna whose receiving polarization is $A_r$. The unit Poincare' sphere is shown with poles $M$ and $N$ representing any two orthogonal elliptical polarizations (See Appendix 3C). The angular separations between $M$ and $W$, $M$ and $A_r$, and $W$ and $A_r$ are $2\theta_w$, $2\theta_r$, and $2\phi$, respectively, and the polar angle of separation between $W$ and $A_r$ is $\Delta_r$

![Figure 3.27 Geometry for determination of polarization efficiency from Poincare' sphere.](image)
As shown in Figure 3.8, the power density of the incident wave may be divided between the receiving polarization and the orthogonal polarization, $A_r$ and $A_r^\perp$, respectively. The fraction of the power density of polarization $W$ which is matched to $A_r$ is $\cos^2 \phi$; thus

$$\Gamma = \cos^2 \phi \quad (3.77)$$

Evaluation of $\cos^2 \phi$ in terms of $2\theta_w$, $2\phi_r$ and $\Delta_e$ can be accomplished by reduction from the spherical triangle $MWA_r$, where

$$\cos 2\phi = \cos 2\theta_w \cos 2\phi_r + \sin 2\theta_w \sin 2\phi_r \cos \Delta_e \quad (3.78)$$

Using the identities

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \quad (3.79)$$
$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \quad (3.80)$$

and

$$\cos 2\phi = 2 \cos^2 \phi - 1 \quad (3.81)$$

gives

$$\cos^2 \phi = \frac{1 + \tan^2 \theta_w \tan^2 \phi_r + 2 \tan \theta_w \tan \phi_r \cos \Delta_e}{(1 + \tan^2 \theta_w)(1 + \tan^2 \phi_r)} \quad (3.82)$$

Using (3.58) and (3.77) in (3.82) gives

$$\Gamma = \frac{1 + \rho_w^2 \rho_r^2 + 2 \rho_w \rho_r \cos \Delta_e}{(1 + \rho_w^2)(1 + \rho_r^2)} \quad (3.83)$$

Note that equation (3.83) is identical with (3D.13). The elliptical polarization ratio $\rho_e$ in (3.83) can assume any of the forms, $\rho$, $\rho_L$, and $\rho_R$, when the poles $M$ and $N$ represent poles $(L, R)$, $(1, 2)$ and $(3, 4)$, respectively. In particular, for the case of the $(L, R)$ poles, the polarization efficiency may be expressed as

$$\Gamma = \frac{1 + \rho_w^2 \rho_r^2 + 2 \rho_w \rho_r \cos \Delta_e}{(1 + \rho_w^2)(1 + \rho_r^2)} \quad (3.84)$$

*See note on page 3-48 for physical interpretation of the angle $\Delta$. 

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It is interesting to note that (3.84) is invariant to substitution of the inverse of both $\rho_w$ and $\rho_r$ for $\rho_w$ and $\rho_r$.

We will apply (3.84) to certain special cases. In Figure 3.28 the receiving antenna is assumed to be linearly polarized, so that $\rho_r$ is unity. For this case equation (3.84) reduces to

$$\Gamma = \frac{1 + \rho_w^2 + 2\rho_w \cos \Delta}{2(1 + \rho_w^2)}$$

(3.85)

![Figure 3.28 Relationship of polarization pattern to Poincaré sphere.](image)

The polarization pattern of the polarization $W$ is given by

$$\Gamma_w^2(\tau_r) = \cos \phi$$

(3.86)

Note that $\tau_r$ is $\chi$ of Figure 3.21. $A_r$ and $W$ can be interchanged in Figure 3.28 so that the polarization pattern is that of the receiving antenna rather than that of the incident wave, and is given by

$$\Gamma_r^2(\tau_w) = \cos \phi$$

(3.87)

If $\rho_w$ and $\rho_r$ are both unity (linear polarization) in (3.84), then
\[ \Gamma = \frac{1 + \cos \Delta}{2} \]  \hspace{1cm} (3.88)

This results in the polarization pattern of Figure 3.29 where \( \Gamma \) is unity when \( W \) and \( A_r \) are coincident on the sphere (\( \Delta = 0 \)) and zero when \( W \) and \( A_r \) are diametrically opposite (\( \Delta = 180 \) degrees).

**FIGURE 3.29** Polarization pattern for linearly polarized field (\( \rho = 1 \)). The axial ratio of the polarization ellipse is infinite.

Figure 3.30 is for \( \sigma_r = 0 \) (LHC). In this event \( \Gamma \) is independent of \( \Delta \) and is given by

\[ \Gamma = \frac{1}{1 + \sigma_r^2} \]  \hspace{1cm} (3.89)

If \( \sigma_r \) is infinite so that \( A_r \) is at the right-circular pole, (3.84) is an indeterminate form, however division by \( \rho_x^2 \rho_r^2 \) results in the form

\[ \Gamma = \frac{\rho_x^2}{1 + \rho_x^2} \]  \hspace{1cm} (3.90)
As indicated previously if the position of $A_r$ and $W$ are interchanged on the sphere, (3.89 and 3.90) become* 

\[ \Gamma = \frac{1}{1 + \rho_r^2} \quad (3.89a) \]

and

\[ \Gamma = \frac{\rho_r^2}{1 + \rho_r^2} \quad (3.90a) \]

The expression for the polarization efficiency in terms of the axial ratios may be obtained from (3.84) by substitution of (3.10) and reduction, which yields

\[ \Gamma = \frac{(1+r_w^2)(1+r_r^2)+4r_w r_r + (1-r_w^2)(1-r_r^2) \cos \Delta}{2(1+r_w^2)(1+r_r^2)} \quad , \quad (3.91) \]

where $r_w$ and $r_r$ are the axial ratios of the polarizations $W$ and $A_r$, respectively. This expression becomes an indeterminate form for linear

*It is important to note that (3.89), (3.90), (3.89a), and (3.90a) are not invariant to substitution of the inverse of $\rho_w$ or $\rho_r$. 

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polarization of either wave or antenna \((r_w \to \infty\), or \(r_r \to \infty\)) however after division by \(r_w^2 r_r^2\), it can be written as

\[
\Gamma = \frac{1 + r^2 - (1 - r^2) \cos \Delta}{2(1 + r^2)}
\]

where \(r\) is the finite axial ratio.

3.7 POLARIZATION EFFICIENCY IN MATRIX FORM

If the polarizations of an antenna and a wave are given in matrix form (Section 3.4(i)), the polarization efficiency can be determined from the inner product,

\[
\overline{V} = (\overline{A_r}, \overline{W})
\]

where \(\overline{A_r}\) is the matrix form of the receiving polarization, \(\overline{W}\) is that of the incident wave and where the superscribed bar denotes a phasor quantity. The inner product, defined for two-dimensional column vectors, \(A\) and \(B\), is given by

\[
(A, B) = A^\dagger B = \bar{a_1} b_1 + \bar{a_2} b_2
\]

where \(T\) denotes the transpose of the matrix, the asterisk denotes the complex conjugate, and where \((\bar{a_1}, \bar{a_2})\) and \((\bar{b_1}, \bar{b_2})\) are the components of matrices \(A\) and \(B\), respectively.

The receiving polarization \(\overline{A_r}\) is obtained from the transmitting polarization by the rules in Appendix 3D.

In terms of general elliptical components, \(\overline{A_r}\) and \(\overline{W}\) are given by

\[
\overline{A_r} = \begin{bmatrix}
\cos \theta_r \\
\sin \theta_r e^{j\phi_r}
\end{bmatrix}
\]

(3.95)
and

\[ \mathbf{W} = \begin{bmatrix} \cos \theta_w & \sin \theta_w e^{j\gamma_w} \\ \sin \theta_w e^{j\gamma_w} & \cos \theta_w \end{bmatrix} \]  

(3.96)

so that

\[ \mathbf{V} = \cos \theta_t \cos \theta_r + \sin \theta_t \sin \theta_r e^{j\Delta_t}, \]  

(3.97)

where

\[ \Delta_t = \delta_w - \delta_r. \]

This process accomplishes the matrix operation so that the receiving polarization and the polarization of the incident wave are described in the same coordinate system, Figure 3.30(b) and (c).

In terms of circular polarization components,

\[ \mathbf{V} = \cos \gamma_t \cos \gamma_r + \sin \gamma_t \sin \gamma_r e^{j\Delta}, \]  

(3.98)

where

\[ \Gamma = \mathbf{V} \mathbf{V}^* = |\mathbf{V}|^2 = \cos^2 \gamma_w \cos^2 \gamma_r + \sin^2 \gamma_w \sin^2 \gamma_r + 2 \cos \gamma_w \cos \gamma_r \sin \gamma_w \sin \gamma_r \cos \Delta. \]  

(3.99)
This can be written

\[
\Gamma = \frac{E_r^2}{P_{rr}^2} + \frac{E_L^2}{P_{LL}^2} + 2 \frac{E_{RL} E_{LR}}{P_{RL} P_{LR}} \cos \Delta .
\]  \hspace{1cm} (3.100)

This equation was given by Hatkin.* Division of (3.100) by \( \frac{E_{LL}^2}{E_{LR}^2} \), and use of (3.9) gives \( \Gamma \) in the form of (3.84), or

\[
\Gamma = \frac{1 + \rho_r^2 \rho_L^2 + 2 \rho_{rl} \rho_{lr} \cos \Delta}{(1 + \rho_r^2)(1 + \rho_L^2)} .
\]  \hspace{1cm} (3.84)

3.8 SUMMARY

Polarization

The polarization state of an electromagnetic wave is defined by the shape and orientation of the polarization ellipse (page 3-3) and by the sense of rotation of the electric field vector at a designated point in space. The IEEE standard defining sense is given on page 3-4.

The coordinate system which is used to describe the polarization state is given in Figure 3.1. In addition to the orthogonal axes designated \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \), a second orthogonal pair is important. These are designated \( \mathbf{u}_3 \) and \( \mathbf{u}_4 \) in Figure 3.5. They are oriented at 45 degrees relative to the \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) axes, respectively.

The polarization of a wave can be described in terms of the relative amplitude and phase of any two orthogonal polarization components of the field. Usually the two are orthogonal linear or circular components.

The amplitude of the right-circular component (\( E_R \)) relative to that of the left-circular component (\( E_L \)) is the circular polarization ratio. The amplitude of the linear component in the \( u_2 \) direction (\( E_2 \)) relative to that in the \( u_1 \) direction (\( E_1 \)) is the linear polarization ratio. The relative amplitudes of the two linear components which differ in phase by 90 degrees is the axial ratio of the polarization ellipse. The axial ratio is defined such that it is equal to or greater than unity.
The phase angle $\delta'$ of $E_\phi$ relative to $E_L$ is numerically equal to twice the tilt angle of the polarization ellipse, measured from the $\overline{u}_1$ direction toward the $\overline{u}_2$ direction. The phase $\delta$ of $E_\phi$ relative to $E_1$ is positive and less than 180 degrees for left-elliptical polarizations.

**The Poincare Sphere.** Poincare showed that the polarization state of a wave can be represented by a unique position on a unit sphere. The sphere can serve as a useful graphical aid in visualizing and solving polarization problems, much like the Smith chart does in solving reflection coefficient problems.

Three mutually orthogonal axes on the sphere are chosen to represent the $(L, R)$, $(1, 2)$ and $(3, 4)$ polarizations respectively (See pages 3-19 through 3-21). Position on the sphere can be defined by any of three interrelated spherical coordinate systems, Figures 3.10, 3.11 and 3.12. The colatitude or polar angle in each coordinate system is determined by the corresponding polarization ratio, (page 3-24). The longitude in each coordinate system, measured in the positive sense, is the phase angle by which the second component of each set in parenthesis leads the first.

The basic generator of the Poincare' sphere is the semicircle of Figure 3.8, which results from the fact that the total power density in a wave is the sum of the power densities in any two orthogonal polarization components. The spheres of Figures 3.10 through 3.12, with poles along the $(L, R)$, $(1, 2)$ and $(3, 4)$ axes, respectively, are special cases of a general sphere with poles representing any orthogonal elliptical polarizations (See Appendix 3C).

**The Polarization Box.** The polarization box serves to prove the mutual consistency of the three spherical coordinate systems of the Poincare' sphere (See Figure 3.14, page 3-24, and Table 3.1, page 3-11). It also aids in the specific solution of polarization problems (Appendix 3G).

**The Polarization Matrices** represent an alternative method of defining polarization (See pages 3-30 and 3-44). Note that the two magnitudes of the elements of the matrix are the cosines and sines of $\alpha$, $\beta$, or $\gamma$, rather than the tangent as in the polarization ratio representation (See Figures 3.8 and 3.9).
The Polarization Efficiency. The following points are basic to the calculation of polarization efficiency.

(1) The polarization of an antenna is defined by the polarization of the wave it radiates.

(2) A reciprocal antenna is polarization matched to an incident wave whose polarization ellipse is identical to and oriented in space identically with that of the wave which would be transmitted by the same antenna.

(3) The polarization of the receiving antenna and the polarization of the incident wave must be described in the same coordinate system to permit calculation of the polarization efficiency.

(4) We accomplish (3) herein by defining the receiving polarization of the antenna as the polarization of the incident wave which is polarization matched to the antenna.

As an alternate, the polarization of the incident wave can be transformed to the coordinate system of the receiving antenna; the results are identical (See Figure 3. 26).

(5) The polarization of the incident wave and the receiving polarization of the antenna on which the wave is incident can now be (a) plotted on the Poincare' sphere (b) used in any of the equations for polarization efficiency (e.g. 3. 84 or 3. 91) or (c) used in the matrix formulation (3. 98 and 3. 99).

The angular separation between W and A, on the sphere determines the polarization efficiency. The polar angle Δ between W and A, on the Poincare' sphere (See Figure 3. 28) and in all of the equations involving circular polarization components is twice the physical angle in space between the major axes of the polarization ellipses defining the polarizations of the incident wave and the receiving antenna.

(6) Development of the polarization efficiency by means of the Poincare' sphere is given in Section 3. 6, page 3-39. Proof of statement (2) is given in Appendix 3D. Representation of the polarization efficiency in matrix form is given in Section 3. 7, page 3-44.
REFERENCES

CHAPTER 3


For a more complete list of related works the reader is referred to the excellent bibliography given by Pike in reference 13.
The following appendix presents the derivations of the equations in Table 3.1.

The arbitrary polarization of a plane wave can be expressed in terms of the $\vec{u}_1$ and $\vec{u}_2$ components as:

$$\vec{E}(t) = E_{1a} \cos\omega t \; \vec{u}_1 + E_{2a} \cos(\omega t + \delta) \; \vec{u}_2 = \vec{E}_{1a}(t) + \vec{E}_{2a}(t)$$  \hspace{1cm} (3A.1)

It can also be expressed in terms of orthogonal circular polarization components as:

$$\vec{E}(t) = \vec{E}_q(t) + \vec{E}_g(t),$$

where

$$\vec{E}_q(t) = E_q \left[ \cos(\omega t + \psi) \; \vec{u}_1 - \sin(\omega t + \psi) \; \vec{u}_2 \right],$$  \hspace{1cm} (3A.2)

and

$$\vec{E}_g(t) = E_g \left[ \cos(\omega t + \psi + \delta') \; \vec{u}_1 + \sin(\omega t + \psi + \delta') \; \vec{u}_2 \right].$$  \hspace{1cm} (3A.3)

Therefore

$$E_{1a} \cos\omega t \; \vec{u}_1 + E_{2a} \cos(\omega t + \delta) \; \vec{u}_2 = E_q \left[ \cos(\omega t + \psi) \; \vec{u}_1 - \sin(\omega t + \psi) \; \vec{u}_2 \right]$$

$$+ E_g \left[ \cos(\omega t + \psi + \delta') \; \vec{u}_1 + \sin(\omega t + \psi + \delta') \; \vec{u}_2 \right].$$  \hspace{1cm} (3A.4)

The $\vec{u}_1$ components and the $\vec{u}_2$ components are orthogonal and yield two independent equations. Each of these equations when expanded can be separated into two independent equations corresponding to the $\sin\omega t$ and $\cos\omega t$ terms because of the orthogonality of sines and cosines. The resulting four
independent equations are:

\[ E_L \cos \psi + E_R \cos (\psi + \delta') = E_{1z}, \tag{3A. 5} \]
\[ E_L \sin \psi + E_R \sin (\psi + \delta') = 0, \tag{3A. 6} \]
\[ -E_L \sin \psi + E_R \sin (\psi + \delta') = E_{2z} \cos \delta, \tag{3A. 7} \]
\[ -E_L \cos \psi + E_R \cos (\psi + \delta') = -E_{2z} \sin \delta. \tag{3A. 8} \]

Combining these equations simultaneously gives the following equations:

\[ \sin \psi = -\frac{E_{2z} \cos \delta}{2E_L}, \tag{3. 25} \tag{3A. 9} \]
\[ \cos \psi = \frac{E_{1z} + E_{2z} \sin \delta}{2E_L}, \tag{3. 26} \tag{3A. 10} \]
\[ \sin \delta' = \frac{E_{1z} E_{2z} \cos \delta}{2E_R E_L}, \tag{3A. 11} \]
\[ \cos \delta' = \frac{E_{1z}^2 - E_{2z}^2}{4E_R E_L} \tag{3. 20} \tag{3A. 12} \]
\[ \sin \delta = \frac{E_{1z}^2 - E_{2z}^2}{E_{1z} E_{2z}}, \tag{3. 16} \tag{3A. 13} \]
\[ 2(E_L^2 + E_R^2) = E_{1z}^2 + E_{2z}^2. \tag{3. 31} \tag{3A. 14} \]

Dividing (3A. 9) by (3A. 10), we get

\[ \tan \psi = \frac{-E_{2z} \cos \delta}{E_{1z} + E_{2z} \sin \delta}. \tag{3. 27} \tag{3A. 15} \]

The arbitrary polarization \( \vec{E}(t) \) can also be expressed in terms of the \( \bar{u}_3 \) and \( \bar{u}_4 \) components as

\[ \vec{E}(t) = \vec{E}_{3z}(t) + \vec{E}_{4z}(t), \tag{3A. 16} \]

where

\[ \vec{E}_{3z}(t) = KE_{3z} \cos(\omega t + \phi)(\bar{u}_1 + \bar{u}_2) \tag{3A. 17} \]
\[ \vec{E}_{4z}(t) = KE_{4z} \cos(\omega t + \phi + \delta')(\bar{u}_1 + \bar{u}_2) \tag{3A. 18} \]
The following four independent simultaneous equations result from similar reasoning to that employed previously:

\[ KE_{3z} \cos \phi - KE_{4z} \cos(\phi + \delta'') = E_{1z} \]  \hspace{1cm} (3A.19)  
\[ -KE_{3z} \sin \phi + KE_{4z} \sin(\phi + \delta'') = 0 \]  \hspace{1cm} (3A.20)  
\[ KE_{3z} \cos \phi + KE_{4z} \cos(\phi + \delta'') = E_{2z} \cos \delta \]  \hspace{1cm} and \hspace{1cm} (3A.21)  
\[ -KE_{3z} \sin \phi - KE_{4z} \sin(\phi + \delta'') = -E_{3z} \sin \delta. \]  \hspace{1cm} (3A.22)  

These equations are now combined simultaneously as in the previous case, resulting in the following equations:

\[ E_{1z}^2 + E_{2z}^2 = E_{3z}^2 + E_{4z}^2 \] \hspace{1cm} (3.31) \hspace{1cm} (3A.23) 
\[ \cos \delta = \frac{E_{3z} - E_{4z}}{2E_{1z}E_{2z}} \] \hspace{1cm} (3.17) \hspace{1cm} (3A.24) 
\[ \cos \delta'' = \frac{E_{2z} - E_{1z}}{2E_{3z}E_{4z}} \] \hspace{1cm} (3.23) \hspace{1cm} (3A.25) 
\[ \sin \delta'' = \frac{E_{1z} - E_{2z}}{E_{3z}E_{4z}} \] \hspace{1cm} (3.22) \hspace{1cm} (3A.26) 
\[ \sin \phi = \frac{E_{2z} \sin \delta}{\sqrt{2} E_{3z}} \] \hspace{1cm} (3.28) \hspace{1cm} (3A.27) 
\[ \cos \phi = \frac{E_{1z} + E_{2z} \cos \delta}{\sqrt{2} E_{3z}} \] \hspace{1cm} (3.29) \hspace{1cm} (3A.28) 

Dividing (3A.13) by (3A.24), we get:

\[ \tan \delta = \frac{2(E_{1z}^2 - E_{2z}^2)}{E_{3z}^2 - E_{4z}^2} \] \hspace{1cm} (3.18) \hspace{1cm} (3A.29) 

Substituting (3A.24) into (3A.11), we get:

\[ \sin \delta' = \frac{E_{3z}^2 - E_{4z}^2}{4E_{3z}E_{4z}} \] \hspace{1cm} (3.19) \hspace{1cm} (3A.30) 

(3A.30) is now divided by (3A.12) to give:

\[ \tan \delta' = \frac{E_{3z}^2 - E_{4z}^2}{E_{1z}^2 - E_{2z}^2} \] \hspace{1cm} (3.21) \hspace{1cm} (3A.31)
Finally (3A.27) can be divided by (3A.28) to give:

\[ \tan \phi = \frac{E_{2a} \sin \delta}{E_{1a} + E_{2a} \cos \delta} \]  

(3.22)  (3A.32)

*Note: All equations in this appendix are in terms of peak values. See page 3-10 through 3-13 for discussion of effective value of elliptical polarization components.
ORTHOGONALITY OF PLANE ELECTROMAGNETIC WAVES

Let a plane electromagnetic wave of a single frequency, \( \omega/2\pi \), be resolved into two component plane waves. The two component waves are defined to be orthogonal if the sum of the average power densities in the two waves is equal to the total average power density in the wave for any ratio of amplitudes and for any arbitrary, relative phase of the component waves.

We will show here that two component waves, M and N, are orthogonal if their circular polarization ratios \( \rho_M \) and \( \rho_N \) are reciprocal and if their tilt angles, \( \tau_M \) and \( \tau_N \), differ by 90 degrees; that is, if

\[
\rho_M = 1/\rho_N, \tag{3B.1}
\]

and if

\[
\tau_M = \tau_N \pm 90^\circ. \tag{3B.2}
\]

We must show that for the conditions of B3.1 and B3.2 and for arbitrary relative amplitudes and phases of the components M and N, that

\[
\vec{E} \times \vec{H}^* = \vec{E}_M \times \vec{H}_M^* + \vec{E}_N \times \vec{H}_N^*, \tag{3B.3}
\]

where \( \vec{E} \) and \( \vec{H} \) are complex vectors representing the total field in some plane which is normal to the direction of propagation (see Figure 3.1, page 3-2), and where \( \vec{E}_M, \vec{H}_M, \vec{E}_N \) and \( \vec{H}_N \) represent the component fields. We have

\[
\vec{E} \times \vec{H}^* - (\vec{E}_M + \vec{E}_N) \times (\vec{H}_M^* + \vec{H}_N^*) = \vec{E}_M \times \vec{H}_M^* + \vec{E}_N \times \vec{H}_N^* + (\vec{E}_M \times \vec{H}_M^* + \vec{E}_N \times \vec{H}_N^*), \tag{3B.4}
\]

*Orthogonality of polarization components implies orthogonality of average power. See note on spatial orthogonality of electric vectors at the end of this appendix.*
It is necessary to show that the term in parenthesis on the right-hand side of B3.4 is zero for the defined conditions.

Let
\[ \overrightarrow{E}_m = A e^{j\omega t} \overrightarrow{u}_1 + B e^{j(\omega t + \delta_n)} \overrightarrow{u}_2 \]  
(3B.5)

and
\[ \overrightarrow{E}_n = C e^{j(\omega t + \phi)} \overrightarrow{u}_1 + D e^{j(\omega t + \phi + \delta_n)} \overrightarrow{u}_2. \]
(3B.6)

Then
\[ \overrightarrow{H}_m = K A e^{j\omega t} \overrightarrow{u}_2 - K B e^{j(\omega t + \delta_n)} \overrightarrow{u}_1 \]
(3B.7)

and
\[ \overrightarrow{H}_n = K C e^{j(\omega t + \phi)} \overrightarrow{u}_2 - K D e^{j(\omega t + \phi + \delta_n)} \overrightarrow{u}_1, \]
(3B.8)

where \( K \) is a constant of proportionality and \( \overrightarrow{u}_1 \) and \( \overrightarrow{u}_2 \) are unit vectors which define directions of the field vectors. Since we require
\[ (C e^{j\phi} \overrightarrow{u}_1 + D e^{j(\phi + \delta_n)} \overrightarrow{u}_2) \times (K A \overrightarrow{u}_2 - K B e^{-j\delta_n} \overrightarrow{u}_1) = 0, \]
(3B.10)

Expanding (3B.10) we obtain
\[ K u_p [A e^{-j\phi} + B e^{j(\delta_n - \delta_n - \phi)} + A e^{j\phi} + B e^{-j(\delta_n - \delta_n - \phi)}] = 0, \]
(3B.11)

where \( \overrightarrow{u}_1 \times \overrightarrow{u}_n = \overrightarrow{u}_p. \)

The quantity in parenthesis must be made to vanish, since \( K \) and \( \overrightarrow{u}_p \) are non-zero.
The imaginary component is identically zero, and the real component vanishes if

$$AC \cos \phi + BD \cos (\delta_H - \delta_N - \phi) = 0. \quad (3B.12)$$

Thus

$$\frac{A}{B} \cos \phi = -\frac{D}{C} \cos (\delta_H - \delta_N - \phi). \quad (3B.13)$$

Expansion of (3B.13) gives

$$\frac{A}{B} \cos \phi = -\frac{D}{C} \left[ \cos (\delta_H - \delta_H) \cos \phi + \sin (\delta_H - \delta_H) \sin \phi \right] = 0. \quad (3B.14)$$

Inspection of (3B.14) shows that, for A, B, C and D to be positive reals, it is necessary for

$$\delta_H - \delta_H = \pi. \quad (3B.15)$$

This causes the second term within brackets to vanish and gives

$$\frac{A}{B} = \frac{D}{C}. \quad (3B.16)$$

In terms of the nomenclature employed in (3.2) on page 3.1, 3B.16 becomes

$$\frac{E_{2M}}{E_{1M}} = \frac{E_{2N}}{E_{1N}}. \quad (3B.17)$$

Note that (3B.15) and (3B.17) do not impose restrictions on the relative amplitudes or phases of the two elliptical polarization components.

The equations in Table 3.1 on page 3.11, which define the inter-relationships among the polarization parameters, can be employed to prove 3B.1 and 3B.2 from 3B.15 and 3B.17. However, this procedure is cumbersome, and we will use the Poincare' sphere for proof.
Letting $E_{\text{in}} = E_{\text{2m}}$, $E_{\text{1n}} = E_{\text{2n}}$, $\delta_m = -\pi/2$ and $\delta_n = \pi/2$, so that the polarizations of the resulting M and N components are right-circular and left-circular respectively proves that circularly polarized components of opposite sense are orthogonal by substitution in 3B.15 and 3B.17.

In the development of the sphere, it was shown that diametrically opposite points on the sphere represent orthogonal polarizations on the assumption that the three orthogonal axes of the sphere represent orthogonal polarizations. We can postulate that the (1, 2) and (3, 4) linear polarizations are orthogonal, and we have proven here that the (L, R) polarizations are likewise orthogonal.

We can now define two polarizations to be orthogonal if, and only if, they are represented by diametrically opposite points on the Poincare sphere. Thus if two polarizations M and N are orthogonal,

\[
\rho_M = 1/\rho_N \text{ and } \delta'_M = \delta'_N \pm 180^\circ \quad (\tau_M = \tau_N \pm 90^\circ)
\]
\[
\rho_{LM} = 1/\rho_{LN} \text{ and } \delta_M = \delta_N + 180^\circ
\]
\[
\rho_{BM} = 1/\rho_{BN} \text{ and } \delta''_M = \delta''_N \pm 180^\circ
\]
and
\[
\rho_{EM} = 1/\rho_{EN} \text{ and } \delta'''_M = \delta'''_N \pm 180^\circ
\]

where $\rho$, $\rho_L$, $\rho_D$ and $\rho_E$ are defined in Section 3.4 and where $\delta'$, $\delta$, $\delta''$ and $\delta'''$ are the corresponding relative phase angles.

A Note on Spatial Orthogonality of Electric Field Vectors* -- We will define two vector fields to be spatially orthogonal if they are always mutually perpendicular

*The term electric field vectors is used here to emphasize that we are not considering the orthogonality of the electric and magnetic vectors of a single polarization component.
in space. Spatial orthogonality of electric fields implies orthogonality in instantaneous power, because at every instant

\[ \mathbf{E} \times \mathbf{H} = \mathbf{E}_m \times \mathbf{H}_m + \mathbf{E}_n \times \mathbf{H}_n \]  

(B3.22)

where \( \mathbf{E}_m \) and \( \mathbf{E}_n \) are electric vectors which are spatially orthogonal.

The following conditions are necessary and sufficient for spatial orthogonality of two electric vectors:

1. They must define polarization components which have identical (not reciprocal) circular polarization ratios.

2. The tilt axes of the defined polarization ellipses must be orthogonal.

3. At some instant of time, the vectors must be mutually orthogonal.

The sum of two spatially orthogonal fields is a field which has the same circular polarization ratio as the component fields. Examples of spatially orthogonal electric vectors are shown in Figure 3B.1. In (a) both components are linear, in (b) both are left-elliptical with circular polarization ratios of 0.5, and in (c) they are both left-circular.

The linear polarization components are both spatially orthogonal and orthogonal in average power, while the elliptical and circular components are spatially orthogonal but are not orthogonal in average power.

---

**FIGURE 3B.1** Examples of Spatially Orthogonal Electromagnetic Fields

---

3B.5
It is interesting to note that the sum of the power densities in two spatially orthogonal fields is equal to the total power density in the field even though the fields are not orthogonal in average power. However, orthogonality in average power requires that the sum of the component power densities be equal to the total power density for any relative phase of the two components. This condition is met if, and only if, the two polarizations are diametrically opposite on the Poincare' sphere. The two linear polarization components of Figure 3B.1 are diametrically opposite on the sphere, Figure 3B.2; the elliptical and circular components are not diametrically opposite.

The following is the important practical difference between spatially orthogonal fields and fields which are orthogonal in average power. If two fields which are orthogonal in average power are incident on an antenna that is polarization matched to one field, it will receive zero power from the orthogonal field. This is not true for spatially orthogonal fields except for the special case of linear polarization.

Acknowledgement

The authors gratefully acknowledge the contribution of Dr. Donald G. Bodner of the Georgia Institute of Technology to this Appendix.
APPENDIX 3C
RESOLUTION OF ELLIPTICALLY POLARIZED WAVES INTO ORTHOGONAL ELLIPTICAL COMPONENTS

The composite Poincare' sphere developed in Section 3.3 has polar axes which define the polarizations (L, R), (1, 2) and (3, 4). It will be shown here that poles can be defined for arbitrary, orthogonal polarizations (M, N) and that the polarization \( W \) of an arbitrary elliptically polarized wave is represented by the spherical coordinates \((\theta, \phi')\) of Figure 3C.1 where the polar angle \( \theta \) is defined by the equation

\[
\theta = \tan^{-1}\left(\frac{E_M}{E_N}\right),
\]

and the phase angle \( \phi' \) is defined as in Figure 3.7 and the accompanying discussion. *

![Figure 3C.1 Extended Poincare' sphere, with poles M and N representing orthogonal elliptical polarization components.](image)

From (3.39) we have that \( \rho_N = 1/\rho_M \) for orthogonal polarizations. Using (3.38) in (3.39) and rearranging gives

\[
\frac{E_{RN}}{E_{LM}} = \frac{E_{LN}}{E_{RN}}.
\]

*See also Appendix 3.E.
From Figure 3.7b,

\[ E_r^2 + E_l^2 = (E_{RM}^2 + E_{RN}^2 + 2E_{RM}E_{RN}\cos \delta'') + (E_{LM}^2 + E_{LN}^2 - 2E_{LM}E_{LN}\cos \delta''), \quad (3C.3) \]

but, if the total power in the wave is to be equal to the sum of the powers in the two orthogonal components M and N,

\[ E_r^2 + E_l^2 = E_{LM}^2 + E_{LN}^2 + E_{RM}^2 + E_{RN}^2, \quad (3C.4) \]

Therefore, it must hold that

\[ E_{RM}E_{RN}\cos \delta'' - E_{LM}E_{LN}\cos \delta'' = 0. \quad (3C.5) \]

Since \( \delta'' \) can have any value, (3C.5) reduces to (3C.2), indicating the consistency in the definition of \( \delta'' \) given in Figures 3.7a and 3.7b.

From Figures 3.7a and 3.6 and equation (3.39), if (3C.2) holds, then

\[ \frac{E_{RN}}{E_{LM}} = \frac{E_{LN}}{E_{RM}} = \frac{E_{LN} + E_{RN}}{E_{LM} + E_{RM}} = \frac{E_{RN}}{E_{RN}} = \frac{E_{N}}{E_{N}}, \quad (3C.2a) \]

Now from Figure 3C.1,

\[ \cos 2\gamma_m = \cos 2\gamma_m \cos 2\theta - \sin 2\gamma_m \sin 2\theta \cos \delta''. \quad (3C.6) \]

From (3.51) and (3.52),

\[ \cos 2\theta = \frac{E_{RM}^2 - E_{RN}^2}{E_{N}}, \quad (3.51) \quad (3C.7) \]

and

\[ \sin 2\theta = 2E_{RM}E_{RN}, \quad (3.52) \quad (3C.8) \]

where the subscribed dots indicate normalization to the effective value \( E \) of the total field as before.

From (3.55)

\[ \cos 2\gamma_m = \frac{E_{LM}^2 - E_{RM}^2}{E_{N}^2}, \quad (3C.9) \]
and
\[ \sin 2\gamma_M = \frac{2E_{LM}E_{RM}}{E_M^2}. \]  
(3C. 10)

since \( E_M \) is the effective value of the field of the M polarization component.

Dividing numerators and denominators of (3C. 9) and (3C. 10) by \( E \) gives
\[ \cos 2\gamma_M = \frac{E_{LM}^2 - E_{RM}^2}{E_M^2} \]  
(3C. 11)

and
\[ \sin 2\gamma_M = \frac{2E_{LM}E_{RM}}{E_M^2}. \]  
(3C. 12)

Substituting in (3C. 6) and reduction gives
\[ \cos 2\gamma_M - \frac{E_{LM}^2}{E_M^2} \left( \frac{E_{RM}^2}{E_M^2} \right) + \frac{E_{LM}^2}{E_M^2} \left( \frac{E_{RM}^2}{E_M^2} \right) - 4E_{LM}E_{LN} \cos \delta'''. \]  
(3C. 13)

Substituting from (3C. 2) gives, after simple reduction,
\[ \cos 2\gamma_M = \frac{E_{LM}^2}{E_M^2} - \frac{E_{RM}^2}{E_M^2} + \frac{E_{LN}^2}{E_M^2} - 4E_{LM}E_{LN} \cos \delta''', \]  
(3C. 14)

and since from (3C. 2)
\[ E_{LM}E_{LN} = E_{RM}E_{RN}, \]  
(3C. 15)

\[ \cos 2\gamma_M = \left( \frac{E_{LM}^2}{E_M^2} + \frac{E_{LN}^2}{E_M^2} - 2E_{LM}E_{LN} \cos \delta''' \right) - \left( \frac{E_{RM}^2}{E_M^2} + \frac{E_{RN}^2}{E_M^2} + 2E_{RM}E_{RN} \cos \delta''' \right). \]  
(3C. 16)

But since
\[ \cos 2\gamma_M = \frac{E_{LM}^2}{E_M^2} - \frac{E_{RM}^2}{E_M^2}, \]  
(3. 55)

the terms in parenthesis are \( E_{RM}^2 \) and \( E_{RM}^2 \) in accordance with (3C. 3) and Figure 3. 7b.

3C. 3
Since the validity of (3C. 1) is evident from inspection of Figure 3. 8, any point \( W \) on the Poincare' sphere can be expressed by the coordinates \( \theta \) and \( \delta''' \) where \( \delta''' \) is defined in accordance with Figure 3. 7 and the accompanying discussion. The required interrelationships between the circular and elliptical components are given by

\[
\begin{align*}
\cos 2\theta &= \cos 2\gamma_w \cos 2\gamma_w + \sin 2\gamma_w \sin 2\gamma_w \cos 2(\tau_w - \tau_W), \\
\cos 2\gamma_w &= \cos 2\gamma_w \cos 2\theta - \sin 2\gamma_w \sin 2\theta \cos \delta''', \\
\cos 2\gamma_w &= \cos 2\gamma_w \cos 2\theta + \sin 2\gamma_w \sin 2\theta \cos [\delta'''' - 2(\tau_w - \tau_W)], \\
\sin \delta'''' &= \frac{\sin 2(\tau_w - \tau_W)}{\sin 2\theta} = \frac{\sin [\delta'''' - 2(\tau_w - \tau_W)]}{\sin 2\gamma_w}.
\end{align*}
\]

Acknowledgement

The authors gratefully acknowledge the contribution of Mr. Raymon A. Heaton of the staff of Scientific-Atlanta to this Appendix.
APPENDIX 3D
DERIVATION OF RECEIVING POLARIZATION
AND POLARIZATION EFFICIENCY

Let the effective area $A_e$ of the receiving antenna of Figure 3.26 be divided between two arbitrary, orthogonal, elliptical* components $A_M$ and $A_N$, so that

$$A_e = A_M + A_N \quad \text{(3D. 1)}$$

Further, let $S$ be similarly divided such that

$$S = S_M + S_N \quad \text{(3D. 2)}$$

where $S_M$ and $S_N$ are polarization-matched, respectively, to $A_M$ and $A_N$.

If $S_M$ or $S_N$ should exist alone, we would have from (3.69) that

$$P_{rM} = S_M A_M \quad \text{or} \quad P_{rN} = S_N A_N \quad \text{(3D. 3)}$$

However, $S_M$ and $S_N$ usually exist simultaneously, and in general

$$P_r \neq P_{rM} + P_{rN} \quad \text{(3D. 4)}$$

because $P_{rM}$ and $P_{rN}$ do not exist in orthogonal fields at the antenna terminals.

*In the development here we let the subscripts $M$ and $N$ represent arbitrary, orthogonal, elliptical components as in Section 3.2, page 3-14. In practice, $M$ and $N$ will usually be replaced by the orthogonal components (L, R), (1, 2) or (3, 4), the components which define the orthogonal axes of the Poincaré sphere. More often than not, they will be L and R. Here it does not add complication and, we think it is instructive to develop the polarization efficiency in terms of general elliptical components and then to specialize, rather than vice versa.
We can, however, consider the terminal fields,

\[ E_r = KP_r^3, \quad E_{rM} = KP_{rM}^3, \quad \text{and} \quad E_{rN} = KP_{rN}^3, \quad (3D.5) \]

where \( K \) is a constant which is determined by the characteristics of the matched transmission line terminating the antenna. If \( A_R \) is a linear device, as is usually the case, the field, \( E_r \), is proportional to the incident field, and the principle of superposition holds. To determine \( E_r \), we can add the fields \( E_{rM} \) and \( E_{rN} \), (Figure 3D.1), if we know their relative phase \( \Delta_e \). *

![Diagram showing phasor summation of received fields due to orthogonal elliptical polarization components processed by polarization matched orthogonal components of the effective area of a receiving antenna.](image)

From Figure 3D.1, using phasor notation,

\[ \overline{E}_r = \overline{E}_{rM} + \overline{E}_{rN} e^{j\Delta_e} \quad (3D.6) \]

which gives, using the law of cosines,

\[ E_r^2 = E_{rM}^2 + E_{rN}^2 + 2E_{rM}E_{rN}\cos\Delta_e. \quad (3D.7) \]

But substituting from (3D.5) into (3D.7) gives

\[ P_r = P_{rM} + P_{rN} + 2(P_{rM}P_{rN})^{\frac{1}{2}}\cos\Delta_e. \quad (3D.8) \]

*At this point, we do not know \( \Delta_e \), but we will proceed and leave \( \Delta_e \) for later evaluation.
Using (3D. 3) in (3D. 8) gives

\[ P_r = S_N A_M + S_N A_N + 2 (S_N A_M S_N A_N)^{\frac{1}{2}} \cos \Delta \]  

(3D. 9)

We will now define

\[ \rho_{tr} = \left( \frac{A_N}{A_M} \right)^{\frac{1}{2}} \text{ and } \rho_{tw} = \left( \frac{S_N}{S_M} \right)^{\frac{1}{2}} \]  

(3D. 10)

This is in keeping with the definition of \( \rho_e \) in (3. 58) where \( \rho_e \) represents ratios of the amplitudes of elliptically polarized field components, and where the second subscripts denote wave (w) and receiving antenna (r). From (3D. 1) and (3D. 2) with (3D. 10),

\[ A_M = \frac{A_e}{1 + \rho_{tw}^2} \text{ and } S_M = \frac{S}{1 + \rho_{tw}^2} \]  

(3D. 11)

and from (3D. 1) and (3D. 2) with (3D. 11),

\[ A_N = \frac{\rho_{tw}^2}{1 + \rho_{tw}^2} A_e \text{ and } S_N = \frac{\rho_{tw}^2}{1 + \rho_{tw}^2} S \]  

(3D. 12)

Finally, substitution of (3D. 11) and (3D. 12) into (3D. 9) gives

\[ P_r = S A_e \frac{1 + \rho_{tw}^2 \rho_{tr}^2 + 2 \rho_{tw} \rho_{tr} \cos \Delta}{(1 + \rho_{tw}^2)(1 + \rho_{tw}^2)} \]  

(3D. 13)

Comparison of (3D. 13) with (3. 76) shows that the quantity in brackets is , the polarization efficiency.

Equation (3D. 13) has been developed in terms of the elliptical polarization ratios \( \rho_{tr} \) and \( \rho_{tw} \). We can now substitute for \( \rho_e \) any of the three ratios \( \rho \), \( \rho_0 \), or \( \rho_\perp \), defined in (3. 9), (3. 56), and (3. 57). When this is done, \( \Delta_e \) will take on specific values \( \Delta_e \), \( \Delta_\perp \), or \( \Delta_0 \), which are given on page 3D. 8. Let us look at two special cases of (3D. 13). Consider (3. 86) for \( \rho_{tr} = \rho_{tw} \) and
\( \Delta_\varepsilon = 0 \). It is seen that \( \Gamma \) is unity. Thus \( A_r \) is polarization matched to \( W \) if the antenna and field have the same values of \( \rho_{E_r} \), and \( \Delta_\varepsilon \) is zero. Next consider (3. D13) for \( \rho_{E_r} = 1/\rho_{E_w} \) and \( \Delta_\varepsilon = 180 \) degrees. In this event, \( \Gamma \) goes to zero, and \( A_r \) is orthogonal to \( W \).

Evaluation of \( \Delta_r \) -- We have already defined the receiving polarization to be the polarization of the incident wave to which the antenna under consideration is polarization matched, and based on prior knowledge, have indicated that it is given by (3. 73) and (3. 74). We must now justify these equations.

Consider (3D. 13) with \( S \) and \( A_e \) divided into orthogonal linear components along the \( \vec{u}_1 \) and \( \vec{u}_2 \) directions of Figure 3D. 2. In this event, \( \rho_{E_w}, \sigma_{E_r}, \) and \( \Delta_\varepsilon \) become \( \sigma_{L_w}, \sigma_{L_r} \) and \( \Delta_\ell \) and

\[
P_r = SA_e \frac{1 + \rho_{L_w}^2 \rho_{L_r}^2 + 2 \rho_{L_w} \rho_{L_r} \cos \Delta_\ell}{(1 + \rho_{L_w}^2)(1 + \rho_{L_r}^2)}.
\]

(3D.14)

![Figure 3D.2](image)

**FIGURE 3D.2** Antenna being employed as (a) transmitting antenna and (b) receiving antenna for use in deriving receiving polarization.

In Figure 3D. 2, let the antenna we have been considering as a receiving antenna be employed on transmission, and let the polarization for a direction
(\phi, \theta) be defined in the 1, 2 plane by the point \text{A} in the first octant* of the Poincare' sphere, Figure 3D.3. The polarization is left-hand elliptical, and the polarization box is similar to that shown in Figure 3.14. We must now determine the polarization A_r of the incident wave which will make \Gamma equal to unity.

We have already seen that the required conditions are

\[ \rho_{LW} = \rho_{LR} \quad , \]

and

\[ \Delta_L = 0 \quad . \] (3D.15) (3D.16)

Therefore, we will postulate (3D.15) and determine the conditions which will satisfy (3D.16). With reference to Figure 3D.2a with the antenna on transmitting, consider the relative phase delay of the \( \tilde{u}_1 \) and \( \tilde{u}_2 \) components between some reference plane R across the terminals of the antenna and the 1, 2 plane.

With the antenna receiving, as in Figure 3D.2b, we can invoke the principle of reciprocity to determine the relative phase delay of the \( \tilde{u}_1 \) and \( \tilde{u}_2 \) components from the 1, 2 plane to the R plane is the same as that in the opposite direction with the antenna transmitting. We note, however, that, on transmitting, the relative phase in the 1, 2 plane is measured between the field components in the positive \( \tilde{u}_{1t} \) and \( \tilde{u}_{2t} \) directions. We must therefore use these same directions for our references in specifying equal relative

---

*Octant definition for Poincaré Sphere:

---

[Diagrams of octants]
phase delay between the 1, 2 plane and the R plane on receiving. Thus, the specified phase delay will be that of the \( \mathbf{E}_{2r} \) component relative to the \( \mathbf{E}_{1r} \) component of the wave.

Figure 3D. 3a is a phasor diagram of the fields in the 1, 2 plane on transmission where specific values of \( \rho_l \) and \( \delta \) have been assumed.

\[
\begin{align*}
\delta_r &= 180^\circ - \delta \quad (3D.17) \\
\rho_{lr} &= \rho_l \quad (3D.18)
\end{align*}
\]

specifies the receiving polarization \( A_r \), that is, the polarization of the wave to which the antenna is polarization matched. This amounts to construction of the polarization box for \( A_r \) as a mirror image in the L1R plane of the polarization box for \( A \). This is shown in Figure 3D. 4. Also see Figures 3.24 and 3.25 and discussion of receiving polarization.
Inspection of Figures 3D.4 and 3.14 shows that the following definitions apply in relating $A_r$ to $A$:

1, 2 components

\[ 2\alpha_r = 2\alpha \quad \rho_{\alpha r} = \rho \quad \delta_r = 180^\circ - \delta \]

(3D.19)

3, 4 components

\[ 2\beta_r = 180^\circ - 2\beta \quad \rho_{\beta r} = 1/\rho_0 \]

(3D.20)

\[ \delta''_r = \delta'' \]

(3D.21)

L, R components

\[ 2\gamma_r = 2\gamma \quad \rho_r = \rho \]

(3D.22)

\[ \delta''_r = -\delta' \quad \tau_r = \delta''_r/2 = 180^\circ - \tau = -\tau \]

(3D.23)
The above conditions apply for the polarization-matched wave which defines $A_r$. For any other wave

\[ \Delta = \delta'_r - \delta'_w \]  \hspace{1cm} \text{(Circular components)} \hspace{1cm} (3D. 25)

\[ \Delta_L = \delta_r - \delta_w \]  \hspace{1cm} \text{(Linear components)} \hspace{1cm} (3D. 26)

\[ \Delta'' = \delta''_r - \delta''_w \]  \hspace{1cm} \text{(Diagonal linear components)} \hspace{1cm} (3D. 27)
APPENDIX 3E
DISCUSSION OF AN INTERESTING INCONSISTENCY
IN DEFINITION OF $\delta$ AND $\delta'$

From Figure 3C.1, it is evident that the general elliptical polar axis MN can degenerate to the (L, R) and the (1, 2) directions. For these two cases $\delta''$ should degenerate to $\delta'$ and $\delta$ respectively.

It can be seen that $\delta''$ degenerates to $\delta'$ in the former case, but degenerates to $\delta + 90^\circ$ in the latter. This points out an interesting inconsistency in the definitions of $\delta$ and $\delta'$. Consider Figure 3E.1, where orthogonal circular and linear polarizations are represented as degenerate cases of the orthogonal elliptical polarizations shown in the center of each set. The vectors on each ellipse indicate conditions of zero phase difference. In (a) the phase reference for all three cases is that employed in defining $\delta$; that is, when the component fields are both maximum and in the $\vec{u}_1$ and $\vec{u}_2$ directions respectively. In (b) the phase reference is that employed in defining $\delta'$; that is, when the component fields are both in the $\vec{u}_1$ direction.

The standard employed in this text is indicated in (c). Note that the phase references for the linear and circular components are not from the same set. We have chosen the phase reference for $\delta''$ to agree with that for $\delta'$ because it makes the angle transformations given in Appendix 3C agree with the standard Euler angle transformations given in Appendix 5A. Neither of the consistent sets of definitions indicated in (a) or (b) would be particularly satisfying for defining both $\delta$ and $\delta'$. It is certainly desirable for $\delta'$ to be defined such that the tilt angle $\tau$ is numerically equal to $\delta'/2$ rather than $(\delta'/2 - \pi/4)$, which would result from the definition of set (a). In addition it would be rather confusing to use set (b) for defining $\delta$ since this would result in writing $\vec{E}(t)$ in the form

3E-1
\[ \vec{E}(t) = E_{1a} \cos \omega t \vec{u}_1 + E_{2a} \sin (\omega t + \delta) \vec{u}_2 \]  

This inconsistency in definition of \( \delta \) and \( \delta' \) is probably of more interest than importance. We point it out here to explain the inconsistency between \( \delta'' \) and \( \delta \), which occurs when \( \rho_m \) is unity and \( \tau_m \) is zero.

**FIGURE 3E.1** Illustration of three sets of standards for defining \( \delta, \delta'', \) and \( \delta' \). All vectors are shown at \( t = 0 \) and for \( \delta, \delta'', \) or \( \delta' \) equal to zero. The vertical line in the left-hand set with the dot at zero represents the \( \vec{u}_2 \) component of (3E.1), which is zero at \( t = 0 \).
APPENDIX 3 F
POLARIZATION CONVERSION TABLES

The tables given on the following pages list values for use in the equations defining (1) the interrelationships among the polarization components on page 3-11 and in the polarization box of page 3-24, and (2) the relationships between the axial ratio and the circular polarization ratio given in equations (3.8a), (3.9a), (3.10), and (3.11).

Although the relationships are written in terms of circular polarization components, they also apply to equations (3.56) through (3.58) by making the appropriate substitutions. In terms of these latter components, the axial ratio is not defined; therefore the parameters r and r(dB) do not have counterparts.

The tables are direct computer printouts, with minor modifications, using the BASIC language. The symbol E-X indicates $10^{-X}$ (where X represents an integer).
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APPENDIX 3G
USE OF THE POLARIZATION BOX

The polarization box, which was given in Figure 3.14 and derived on pages 3-22 through 3-24, can serve as a valuable graphical aid in solution of polarization problems. To this end let us define

\[ X_E = 2 E_M E_N = \sin 2\theta = 2 \sin \theta \cos \theta \quad (3G. 1) \]

and

\[ Y_E = E_M^2 - E_N^2 = \cos 2\theta = \cos^2 \theta - \sin^2 \theta. \quad (3G. 2) \]

Now since

\[ E_M = \frac{E_M}{(E_M + E_N)^2} = \cos \theta, \quad (3G. 3) \]

\[ E_N = \frac{E_N}{(E_M + E_N)^2} = \sin \theta, \quad (3G. 4) \]

and

\[ \rho_E = \frac{E_N}{E_M} = \tan \theta, \quad (3.58) (3G. 5) \]

we can write (3G. 1) and (3G. 2):

\[ X_E = \frac{2\rho_E}{1 + \rho_E^2} \quad (3G. 6) \]

and

\[ Y_E = \frac{1 - \rho_E^2}{1 + \rho_E^2}. \quad (3G. 7) \]
When $\rho_t$ takes on the identities $\rho, \rho_L$ and $\rho_0$, (See (3. 9), (3. 56) and (3. 57), then M and N take on the designations (L, R), (1, 2) and (3, 4) and ($X_\ell, Y_\ell$) will take on the designations (X, Y), ($X_L, Y_L$) and ($X_0, Y_0$), respectively. Using this terminology, the polarization box is as presented in Figure 3G.1.

From Figure 3G.1, (3. 16a) through (3. 24a) on page 3.11 can be written.

\[
\begin{align*}
\sin \delta &= \frac{Y}{X_L} & (3.16b) \\
\cos \delta &= \frac{Y_\ell}{X_L} & (3.17b) \\
\tan \delta &= \frac{Y}{Y_0} & (3.18b) \\
\sin \delta' &= \frac{Y_0}{X} & (3.19b) \\
\cos \delta' &= \frac{Y_L}{X} & (3.20b) \\
\tan \delta' &= \frac{Y_0}{Y_L} & (3.21b)
\end{align*}
\]
\[
\sin \delta'' = \frac{Y}{X_0} \quad (3.22b)
\]
\[
\cos \delta'' = -\frac{Y_L}{X_0} \quad (3.23b)
\]
and
\[
\tan \delta'' = -\frac{Y}{Y_L} \quad . \quad (3.24b)
\]

The amplitudes of the elements of the polarization matrix are given by

\[
\cos \theta = \left( \frac{1 + \cos 2\phi}{2} \right)^{\frac{1}{2}} = \left( \frac{1 + Y_L}{2} \right)^{\frac{1}{2}} \quad (3G. 8)
\]

and
\[
\sin \theta = \left( \frac{1 - \cos 2\phi}{2} \right)^{\frac{1}{2}} = \left( \frac{1 - Y_L}{2} \right)^{\frac{1}{2}} \quad . \quad (3G. 9)
\]

The elliptical polarization ratio \( \rho_e \) is defined by

\[
\rho_e = \tan \theta = \left( \frac{1 - Y_L}{1 + Y_L} \right)^{\frac{1}{2}} \quad . \quad (3G. 10)
\]

Letting \( \theta \) take on the identities \( \alpha, \beta, \) and \( \gamma \) as \( Y_L \) goes to \( Y_L, Y_0 \) and \( Y \) in (3G. 8) through (3G. 10) results in the following specific relations for the amplitudes of the elements of the polarization matrixes and for the polarization ratios:

\[
\begin{bmatrix}
\cos \alpha = \left( \frac{1 + Y_0}{2} \right)^{\frac{1}{2}} \\
\sin \alpha = \left( \frac{1 - Y_0}{2} \right)^{\frac{1}{2}}
\end{bmatrix} \quad (3G. 11)
\]

\[
\begin{bmatrix}
\cos \beta = \left( \frac{1 + Y_L}{2} \right)^{\frac{1}{2}} \\
\sin \beta = \left( \frac{1 - Y_L}{2} \right)^{\frac{1}{2}}
\end{bmatrix} \quad (3G. 12)
\]

*See page 3-30.
In these equations $Y_L$, $Y$, and $Y$ can be determined by inspection of Figure 3G.1 to be given by,

$$Y_L = X \cos \delta' = -X_0 \cos \delta''$$
(3.20b)(3.23b)

$$Y = X_0 \cos \delta = X \sin \delta'$$
(3.17b)(3.19b)

$$Y = X_0 \sin \delta = X_0 \sin \delta''$$
(3.16b)(3.22b)

The following examples serve to illustrate the application of the above equations to the solution of typical polarization problems.

**Example 1** - It is determined that $\rho_L$ of a radiated wave is 2.5 and that $\delta$ is 35 degrees. Find $\rho$, $\rho$(dB), $r$(dB), $\delta'$, $\tau$. What is the sense of rotation?

**Solution:**
Consideration of $\rho_L$ and $\delta$ along with the Poincare' sphere shows from inspection that the wave is left-elliptical ($\delta < 180^\circ$) and that $\tau$ is between 45$^\circ$ and 90$^\circ$ ($\delta < 90^\circ$, $\rho_L > 1$); however, the following quantitative solution is not
dependent on these conclusions.

Determine $\rho$ from (3G. 16) and (3. 16b)

$$\rho = \left(\frac{1 - X_L \sin \delta}{1 + X_L \sin \delta}\right)^{\frac{3}{2}} = \left(\frac{1 - 0.69(0.574)}{1 + 0.69(0.574)}\right)^{\frac{3}{2}} = 0.66$$

(3G. 16)(3. 16b)

where

$$X_L = \frac{2 \rho_L}{1 + \rho_L^2} = \frac{5}{7.25} = 0.69$$

(3G. 6)

Since $\rho$ is less than unity, the wave is left-elliptical.

To determine $\delta'$, write

$$x = \frac{2 \rho}{1 + \rho^2} = \frac{2(0.66)}{1 + 0.436} = \frac{1.32}{1.436} = 0.919$$

(3G. 6)

$$y_L = \frac{1 - \rho_L^2}{1 + \rho_L^2} = \frac{1 - 6.25}{1 + 6.25} = \frac{-5.25}{7.25} = -0.724$$

(3G. 7)

and

$$\cos \delta' = \frac{y_L}{x} = \frac{-0.724}{0.919} = -0.788$$

(3. 20b)

The ambiguity in defining $\delta'$ from its cosine can be resolved by use of (3. 17b) and (3. 19b).

$$y_\delta = X_L \cos \delta = (0.69)(0.819) = 0.565$$

(3. 17b)

and

$$\sin \delta' = \frac{y_\delta}{x} = \frac{0.565}{0.919} = 0.615$$

(3. 19b)

Since the cosine is negative and the sine positive, $\delta'$ is in the second quadrant. Thus $\delta' = 142.08$ and $\tau = \delta'/2 = 71.04$. 

3G-5
Finally \( r \) is given by

\[
\begin{align*}
  r &= \frac{\rho + 1}{\rho - 1} = \frac{1.66}{-0.34} = -4.88, \\
  r &\equiv \rho - 1 = -3.88, \quad (3.11)
\end{align*}
\]

and

\[
\begin{align*}
  r(dB) &= 20 \log |r| = 13.78. \quad (3.8)(3.8a)
\end{align*}
\]

Note that \( r(dB) \) does not indicate the sense of polarization. On the other hand,

\[
\begin{align*}
  \rho(dB) &= 20 \log \rho = -3.6 \quad (3.9)(3.9a)
\end{align*}
\]

indicates that the sense is left-elliptical, since \( \rho < 1 \).

Example 2 - Let the polarization of a wave be described by the matrix

\[
\begin{bmatrix}
  \cos \alpha \\
  \sin \alpha e^{i\delta}
\end{bmatrix} = \begin{bmatrix}
  0.5 \\
  0.866/30^\circ
\end{bmatrix}
\]

what is the polarization matrix in terms of circular polarization components, diagonal components, tilt angle? What are \( \rho_L, \rho_0, \rho, r \) and \( \tau \)?

Solution:

Use

\[
\begin{align*}
  \cos \gamma &= \left( \frac{1 + Y}{2} \right)^{\frac{1}{2}}, \\
  \sin \gamma &= \left( \frac{1 - Y}{2} \right)^{\frac{1}{2}}.
\end{align*}
\] (3G.13)

where

\[
Y = X_L \sin \delta \quad (3.16b)
\]

and where

\[
X_L = 2 \sin \alpha \cos \alpha \quad (3G.1)
\]
Proceed in a similar manner to obtain \( \cos \beta \) and \( \sin \beta \).

Determine \( \delta' \) and \( \delta'' \) as in Example (1) to give

\[
\begin{bmatrix}
\cos \gamma \\
\sin \gamma \angle \delta'
\end{bmatrix} = \begin{bmatrix}
.846 \\
.532 \angle 155.2^\circ
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cos \beta \\
\sin \beta \angle \delta''
\end{bmatrix} = \begin{bmatrix}
.935 \\
.353 \angle 41^\circ
\end{bmatrix}
\]

Determine \( \rho_L, \rho_o \) and \( \rho \) from \( \tan \alpha, \tan \beta, \tan \gamma \), respectively, to be

\[
\tan \alpha = \rho_L = .629
\]
\[
\tan \beta = \rho_o = .378
\]
\[
\tan \gamma = \rho = 0.628
\]

Example 3 - Suppose we have an antenna characterized by a 20 dB axial ratio and a tilt angle of 45°. The sense of this polarization is right-hand. In order to make a statistical assessment of information retrieval from several antennas of various polarizations, the above polarization must be converted to the linear matrix format to be compatible with the information available on the other antennas for an existing program.

Determining \( \rho_L \) from \( Y_L \), we find

\[
\alpha = 45^\circ,
\]
\[
\sin \alpha = \sqrt{2}/2, \text{ and}
\]
\[
\cos \alpha = \sqrt{2}/2
\]

Finding \( Y, Y_o, \) and \( X_L \) from \( \rho, X, \) and \( \rho_L \), respectively, we find \( \delta = 348 \ 2/3^\circ \). Therefore

\[
\begin{bmatrix}
\cos \alpha \\
\sin \alpha \angle \delta
\end{bmatrix} - \begin{bmatrix}
\sqrt{2}/2 \\
\sqrt{2}/2 \angle 348 \ 2/3^\circ
\end{bmatrix}
\]

3G-7
CHAPTER 4
SIGNAL DETECTION, NOISE, AND DYNAMIC RESPONSE
R. E. Pidgeon, Jr.

4.1 INTRODUCTION

In antenna and radar reflectivity measurements discussed in this book it is necessary to detect and measure the presence of microwave signals. This chapter is concerned with certain fundamental limitations in the accuracy of various techniques commonly employed to make these measurements. Three topics which affect the measurement accuracy are discussed: sensitivity, response to time-varying signals, and dynamic range.

The sensitivity of the measurement system determines the minimum level at which signals can be detected and measured. The transient response characteristics of the system further limit the measurement accuracy since antenna patterns and radar returns are generally measured while the antenna is being rotated to generate the desired cut. Errors caused by system noise and transient response characteristics are related to the system bandwidth. A trade-off in bandwidth is usually necessary since a reduction in bandwidth decreases the error due to noise but increases the transient response error. The dynamic range is limited by the sensitivity of the measuring system to weak signals and by saturation or non-linearity errors for strong signals.

In section 4.2 a discussion of some of the basic principles of noise is given as a background for the discussions that follow on the sensitivity of microwave detectors and receiving systems. Additional reading material on the subject is included in the bibliography. In section 4.5 the measurement error for a time-varying signal is discussed.
4.2 NOISE

Physical Sources of Noise - - - The term noise refers to the unwanted electromagnetic energy which varies randomly with time and interferes with the ability of the system to measure weak signals. Noise may originate in the measurement system or it may enter by the antenna terminals along with the desired signal. Externally generated noises may be man-made or due to natural environmental causes. The man-made type of disturbance may be due to electromagnetic pick-up of another signal near the same frequency as the desired signal or at a spurious frequency to which the system will respond. Other transmitting systems can be a serious source of interference. Interference may come from an adjacent antenna range testing a similar antenna or microwave system. Other erratic disturbances may be caused by pick-up of radiation from dc motors, power line transients, ignition systems, etc. Noise from these sources is usually insignificant at microwave frequencies. Problems from this type of noise getting into lower frequency circuits of the measuring system can be minimized or eliminated by proper design, shielding, and filtering.

Environmental noise due to natural causes includes cosmic noise, atmospheric noise, solar noise, etc. Noise from these sources is relatively low at microwave frequencies and is not a significant factor in limiting the sensitivity of measuring antenna patterns.

Noise is inherent in electrical circuits because of the random motion of electrons or charges which make up the electric current. The types of noise most often encountered are thermal and shot noise. Thermal noise is caused by the random motion of electrons in a conductor. Shot noise in a vacuum tube is caused by the random variation in the rate of emission of electrons from the cathode and the non-uniform rate at which they strike the plate. This type of noise occurs not only in vacuum tubes but also in semiconductor diodes and transistors.

Thermal Noise - - - Thermal noise (so called because of its dependence on temperature) was studied experimentally by J. B. Johnson and theoretically by H. Nyquist and reported in a classical paper by these authors in 1928.¹

¹Thermal noise is also referred to as Johnson noise.
Johnson demonstrated that a metallic resistor could be considered as the source of spontaneous voltage fluctuations with mean-squared value

$$\bar{e}^2 = 4kTR\Delta f,$$  \hspace{1cm} (4.1)

where
- \( k = \) Boltzmann's constant \( = 1.38 \times 10^{-23} \) joule/degree
- \( T = \) temperature of the resistor in degrees Kelvin
- \( R = \) resistance in ohms
- \( \Delta f = \) frequency increment of interest.

Thermal noise produces a noise spectrum which is theoretically constant throughout the frequency range of interest here. Because of the flat spectrum of noise power, thermal noise is called white noise (since white light contains all frequencies).

A resistor (or lossy passive device) may be considered as the source of thermal noise given by (4.1). A voltage-model representation is given in Figure 4.1, where the resistor \( R \) is assumed to be noiseless, and the source of noise is a noise generator whose mean-squared value is given by (4.1). An equivalent current-model representation is also given in Figure 4.1. These models may be used for all resistive elements in a passive network to calculate the thermal noise voltage or current at any point.

![Diagram](image_url)

**FIGURE 4.1** Thermal noise circuit models.
It is often more useful to work with noise power rather than noise voltage. Maximum power is transferred from a generator to a load when the load impedance is the conjugate of the generator impedance. The maximum noise power from a noise source of resistance $R$ will be transferred when the resistance of the load is also $R$. The power transferred is $\frac{e^2}{4R}$, and the maximum available noise power $N$ is

$$N = kT\Delta f.$$  \hspace{1cm} (4.2)

Note that the available noise power is independent of impedance, and that the same noise power is predicted by both the voltage model and the current model.

**Noise Bandwidth** - - - The power available from a thermal noise source is uniformly distributed in frequency. The noise is filtered, however, by the networks and amplifiers which the noise source feeds. If $H(f)$ represents the frequency-dependent voltage gain or transfer function of the filter, then the output noise power is distributed as $|H(f)|^2$. The noise bandwidth $B$ is defined as

$$B = \int_{\Delta f} H(f) |H(f)|^2 df,$$  \hspace{1cm} (4.3)

where $H(f_c)$ is the maximum voltage gain (usually at the center frequency of the filter). The noise bandwidth may be thought of as the bandwidth of a rectangularly shaped filter of height $H(f_c)$ and width $B$ and whose noise power output is the same as that for the actual filter. This concept is illustrated in Figure 4.2.

![Figure 4.2 Illustration of noise bandwidth.](image)

*See footnote on page 4-39.*
If the noise bandwidth is $B$, the available thermal noise power is

$$N = kTB, \quad (4.4)$$

The noise power may be expressed in decibels relative to a milliwatt (dBm) by

$$N(dBm) = -114 + 10 \log_{10} B + 10 \log_{10} \frac{T}{290^\circ K} \quad (4.5)$$

where $B$ is the noise bandwidth in megahertz.

**Signal-to-Noise Ratio** - - - Noise tends to mask or add an error of uncertainty to the signal being measured. A measure of the relative amount of noise present is given by the signal-to-noise ratio. Since the noise fluctuates randomly, it is the noise power averaged over a period of time ($\sigma^2$) that is of interest.

The signal-to-noise ratio is a power ratio, and is the ratio of signal and noise powers at a particular point in the system. The signal-to-noise ratio may be different at different points in the system because of attenuation of the signal, addition of noise from other sources, or reduction of the noise by filtering.

**Noise Figure** - - - An ideal amplifier would generate no noise of its own. All practical amplifiers must generate some noise, however. A measure of the noise produced by a practical amplifier compared with the noise of an ideal amplifier is given by the noise figure $F$ of the amplifier. The noise figure of a linear system may be defined as

$$F = \frac{(S/N)_i}{(S/N)_o}, \quad (4.6)$$

where $(S/N)_i$ is the signal-to-noise ratio at the input (source), and $(S/N)_o$ is the signal-to-noise ratio at the output. From this definition, it is evident that the noise figure of an amplifier may be defined as the degradation in the signal-to-noise ratio produced by that amplifier.

An alternate definition of noise figure is
\[ F = \frac{N_0}{N_s G}, \quad (4.7) \]

or

\[ F = \frac{N_0}{kTBG}, \quad (4.8) \]

where \( N_0 \) is the available output noise power, \( N_s \) is the available input (source) noise power, and \( G \) is the available amplifier signal gain.

From this definition, noise figure may be interpreted as the ratio of the actual available output noise power to the noise power which would be available at the output if the amplifier or network amplified just the thermal noise of the source.

It is often convenient to refer the amplifier noise to its input terminal. The noise referred to the input is the actual available output noise divided by the available gain of the amplifier, or \( N_0 / G \). From (4.8) the noise figure is seen as the ratio of the output noise referred to the input of the amplifier to the thermal noise produced by the source alone. Note that noise figure is a dimensionless quantity, and is expressed in terms of power ratios. Noise figure is often expressed in decibels, where

\[ F(dB) = 10 \log_{10} F. \quad (4.9) \]

The noise of an amplifier referred to its input is usually expressed in dBm. If the noise figure is known, the input noise \( N_i \) can be calculated from

\[ N_i = \frac{N_s}{G} = FkTB. \quad (4.10) \]

Expressed in dBm, the input noise power is

\[ N_i(dBm) = -114 + F(dB) + 10 \log_{10} B, \quad (4.11) \]

where \( B \) is the noise bandwidth in megahertz. This expression is used to calculate the sensitivity of an amplifier or receiver (for a signal-to-noise ratio
of unity). As an example, if the noise figure of a receiver is 4.5 dB and its noise bandwidth is 0.5 MHz, then its sensitivity is

\[ N_1 (\text{dBm}) = -114 + 4.5 + 10 \log_{10}(0.5) = -112.5 \text{ dBm}. \]

**Noise in a Square-Law Detection System** - - The discussion to this point has been concerned with linear systems. Noise calculations are not as straightforward in non-linear systems involving devices such as mixers and detectors. In this section we will discuss the response of a square-law detector to a signal in the presence of noise.

Antenna measurement receivers convert the RF signal to an intermediate frequency, amplify the signal, and feed a square-law detector. * The IF bandwidth is typically 0.1 to 0.5 MHz, and the signal is modulated, usually at a 1 KHz rate. If the RF input is a CW signal, modulation is accomplished by sweeping the frequency of the local oscillator causing the intermediate frequency to vary across the passband of the IF filter. A simplified block diagram of the portion of the receiver of interest here is given in Figure 4.3.

**FIGURE 4.3** Block diagram of square-law detection system with predetection and postdetection bandpass filters.

*Phase-locked receivers are not in this category. See Chapter 15 for the principles of operation of phase-locked systems.*
A bolometer is used as the square-law detector. The bolometer may be an ordinary 1/200A fuse. A bolometer has the characteristic that its change in resistance is proportional to the input power. A direct current is passed through the bolometer to obtain an output voltage $e_2$ proportional to input power. Thus,

$$e_2 = Ke_1^2$$  \hspace{1cm} (4.12)

where $e_1$ is the input voltage and $K$ has the dimension volt$^{-1}$. The bolometer maintains this square-law characteristic over a wide dynamic range.

Consider first the case in which the system is noiseless. Assume the signal applied to the receiver is sinusoidally 100 percent amplitude modulated. For this case, the input to the detector is

$$e_1 = E_0 [1 + \cos \omega_s t] \cos \omega_c t,$$ \hspace{1cm} (4.13)

where $2E_0$ is the peak value of the IF signal, and $\omega_s$ and $\omega_c$ are the modulating frequency and the intermediate frequency. The output of the detector is

$$e_2 = Ke_1^2,$$ \hspace{1cm} (4.12)

or

$$e_2 = KE_0^2 \left[ \frac{3}{4} + \cos \omega_s t + \frac{1}{4} \cos 2\omega_s t + \cos 2\omega_c t \left( \frac{3}{4} + \cos \omega_s t + \frac{1}{4} \cos 2\omega_s t \right) \right].$$ \hspace{1cm} (4.14)

Equation (4.14) contains components at twice the carrier frequency, $2\omega_c$, and sidebands at $\omega_s$ and $2\omega_s$. These components are filtered out by the thermal time constant of the detector. The output of the filter tuned to the modulation frequency is

$$e_2 = KE_0^2 \cos \omega_s t.$$ \hspace{1cm} (4.15)

Consider now the case in which there is noise but no signal present at the input to the detector. Assume, for simplicity, that the bandpass characteristic
of the IF amplifier is rectangular as illustrated in Figure 4.4. The noise power spectrum will also be rectangular for a white noise source. If the power spectral density is \( w \) watts/Hz, the total mean noise power at the input to the detector is

\[
\overline{e_n^2} = wB_1,
\]  

(4.16)

**I. F. AMPLIFIER BANDPASS CHARACTERISTIC**

**NOISE POWER-DENSITY SPECTRUM**

**DISCRETE APPROXIMATION OF NOISE POWER SPECTRUM**

\[
e_l \cos[(\omega_s + \omega_1)t + \varphi_1]
\]  

(4.17)

**FIGURE 4.4** Illustration of the approximation of noise power spectrum by discrete sinusoidal components.

The continuous power-density spectrum can be approximated by a discrete spectrum of \( N \) sinusoids as illustrated in Figure 4.4. Each component is of the form...
where \( \phi_i \) is a random variable equally distributed between 0 and \( 2\pi \) radians. The power in each term is \( e_i^2/2 \). The total power \( N e_i^2/2 \) must equal the power given by the continuous spectrum (4.16); therefore,

\[
N \frac{e_i^2}{2} = wB_1,
\]

(4.18)

or

\[
e_i^2 = 2w \frac{B_1}{N}.
\]

(4.19)

The noise at the output of the detector is given by \( e_2 = K e_i^2 \). Squaring (4.17) and summing the result leads to the output expressed as a summation of discrete sinusoids from which we get the continuous distribution. The output voltage is given by

\[
e_2 = K \left\{ \sum_{N/2}^{N/2} e_i \cos \left[ (\omega_c + \omega_i)t + \phi_1 \right] \right\}^2.
\]

(4.20)

Expanding (4.20) results in the terms of the form

\[
e_i^2 \cos^2 \left[ (\omega_c + \omega_i)t + \phi_1 \right]
\]

(4.21)

and cross-product terms of the form

\[
e_i e_j \cos \left[ (\omega_c + \omega_i)t + \phi_1 \right] \cos \left[ (\omega_c + \omega_j) + \phi_j \right].
\]

(4.22)

The squared terms (4.21) do not contain components in the low frequency pass band of interest. Therefore, these terms can be ignored. The cross-product terms produce low frequency components of the form

\[
\frac{e_i e_j}{2} \cos \left[ (\omega_i - \omega_j)t + \phi_1 - \phi_j \right]
\]

(4.23)
plus high frequency components (sidebands about \(2\omega\)) which are filtered out. The low frequency terms of the same frequency add, but these terms must be added statistically, not directly, since, the phase of each sinusoid varies randomly and independently. The power in each component is added to obtain the power at each discrete frequency. There are \(2N\) cross-product terms for the lowest frequency \(\omega_1\) and one cross-product term for the highest frequency \(\omega = 2\pi D_1\). The summation of the low frequency components produces the low frequency discrete power spectrum shown in Figure 4.5.²

The continuous noise power spectrum is also shown. Note that the amplitude of the power spectrum is proportional to the predetection bandwidth \(B_1\).

The output of the detector is fed to a narrow band filter with bandwidth \(B_a\) centered about the modulation frequency. The total noise power at the output of the postdetection filter is

\[
N_o = 2K^2 \omega B_1 B_a
\]

(4.24)

for \(B_a << B_1\), and \(f_a << B_1\).

The noise bandwidth \(B_a\) for a square-law detector may be defined as the bandwidth of an input filter that would produce the actual output noise (4.24) if the output noise were \(K^2\) times the square of the noise input to the detector. By this definition

\[
K^2 (wB_a)^2 = 2K^2 \omega B_1 B_a,
\]

(4.25)

and

\[
B_a = \sqrt{2B_1 B_a}.
\]

(4.26)

Thus the noise bandwidth is \(\sqrt{2}\) times the geometric mean of the predetection and postdetection bandwidths.

The preceding derivation illustrates what happens physically in a square-law detector. Noise components beat together to produce a triangular-shaped output noise spectrum with maximum output at \(f = 0\) and zero output at \(f = B_1\). As a result, the noise output is a function of the predetection and postdetection
FIGURE 4.5 Low frequency noise power spectrum at output of square-law detector (dc component not shown): (a) discrete approximation, (b) continuous spectral distribution, (c) noise power at output of postdetection filter.
bandwidths. However, the information bandwidth is approximately equal to one-half the postdetection bandwidth only.

Consider now the case in which both a modulated carrier and noise are present at the input of the detector. The input voltage $e_1$ is now the sum of the signal and noise voltages which were considered separately in the previous derivations. As before, $e_1$ is squared to obtain the output voltage, and only the low frequency components (in the range $0 < f < B_1$) are retained.

The input $e_1$ is given by

$$e_1 = E_0 (1 + \cos \omega_0 t) \cos \omega_0 t + \sum_{i=1}^{N-1} e_i \cos [(\omega_0 + \omega_i) t + \phi_i].$$  \hfill (4.21)

Squaring $e_1$ shows that noise is produced at the output of the detector from two causes:

(1) Noise components mixing with each other, and
(2) Noise components mixing with the carrier and its signal sidebands.

The output noise due to components of the input noise mixing with each other is the same as that when there is no carrier present. The noise power spectrum due to this cause was shown in Figure 4.5 and is shown again in Figure 4.6.

The noise spectrum due to components of the input noise mixing with the carrier and its sidebands is shown also in Figure 4.6. Figure 4.6A shows the noise spectrum for an unmodulated carrier plus noise at the input of the detector. Adding signal sidebands to the carrier produces the output spectrum shown in Figure 4.6B. The noise density due to noise mixing with the carrier is proportional to the carrier power $S_c$. The noise density due to noise mixing with the modulated carrier is proportional to the total input power (carrier plus sideband power) $S_1$ for $f < B_1/2 - f_s$. Note that the output noise increases when a carrier is applied. This is the effect we observe when we hear the noise in a radio increase when it is tuned to a weak signal.
FIGURE 4.6 Noise power spectrum at output of square-law detector (dc component not shown): (a) unmodulated carrier, amplitude $E$, plus noise; (b) carrier modulated 100% at frequency $f_0$, plus noise; (c) output of post-detection filter with bandwidth $B_2$.

NOTE: (1) Output noise due to input noise mixing with itself.
(2) Output noise due to input noise mixing with carrier.
(3) Output noise due to input noise mixing with carrier and its sidebands.
The noise spectrum at the output of the postdetection filter is shown in Figure 4.6C. Here the bandpass characteristic is assumed to be rectangular with bandwidth $B_2$. The output noise power $N_o$ is equal to the area under the curve in 4.6C, which is

$$N_o = K^2 B_2 (2w^2 B_1 + 4ws_1). \quad (4.28)$$

Here it is assumed that $f_s$ and $B_2$ are both small compared with $B_1$. This assumption is valid for the typical antenna measurement receiver as stated at the beginning of this section.

The noise bandwidth $B_n$ was given previously (4.26) in the absence of a signal as $(2B_1 B_2)^{\frac{1}{2}}$. The noise bandwidth can be expressed in a similar manner when in the presence of a signal by relating the total noise output with a signal (4.28) to the noise into the detector for a detector power gain of $K^2$. If the noise bandwidth in the presence of a signal is $B_{ns}$, then

$$N_o = K^2 (wB_{ns})^2, \quad (4.29)$$

and

$$P_{ns} = P_n \sqrt{1 + 2 \frac{S_t}{N_t}}. \quad (4.30)$$

Now we are in a position to derive the desired result: the output signal-to-noise ratio $S_o/N_o$ as a function of input signal-to-noise ratio $S_I/N_I$ and bandwidth ratio $B_o/B_1$. The signal output power $S_o$ can be related to the total signal input power $S_I$ by

$$S_o = CK^2 S_I^2. \quad (4.31)$$

since $e_o = Ke_I$. The constant $C$ will depend on the method of modulation employed (sine wave or square wave, for example). For sinusoidal modulation $C = 8/9$.

Combining (4.31) and the expressions for the input noise (4.16) and output noise (4.28) gives
For $S_1/N_1 >> 1$

\[
\frac{S_o/N_o}{S_1/N_1} = \frac{B_1}{B_2} \cdot \frac{C}{4\left(1 + \frac{1}{2S_1/N_1}\right)}.
\]  

(4.32)

For $S_1/N_1 << 1$

\[
\frac{S_o/N_o}{S_1/N_1} \approx \frac{CB_1}{4B_2} (S_1/N_1).
\]

(4.33)

Thus, $S_o/N_o$ varies linearly with $S_1/N_1$ for large input signal-to-noise ratios. This is the case for a mixer in which the local oscillator signal is strong. For small input signal-to-noise ratios $S_o/N_o$ varies as $(S_1/N_1)^2$.

From (4.32) the sensitivity of a receiver feeding a square-law detector can be calculated for a given ratio of predetection and postdetection bandwidths and for a specific output signal-to-noise ratio. For $S_o/N_o = 1$, and $B_2 << B_1$, $S_1/N_1$ is given by (4.34). The sensitivity can be related to the noise figure of the amplifier as discussed previously. Thus,

\[
S_1 = FkT \sqrt{\frac{2B_1B_2}{C}}
\]

(4.35)

for $S_o/N_o = 1$. Again it is seen that the noise bandwidth is proportional to the geometric mean of the predetection and postdetection bandwidths. In contrast, for a phase detector or mixer in which a strong local oscillator signal is present the noise bandwidth is determined only by the bandwidth of the circuits following the converter. This is because the noise resulting from noise mixing with the carrier predominates when a strong carrier is present. The frequency spectrum containing the noise and input signal is translated by an amount equal to the local
oscillator frequency and is then filtered by the output circuits.

4.3 DIRECT MICROWAVE DETECTORS

Microwave detectors are used to convert radio frequency signals to dc and to detect the modulation of the RF signal. Microwave diode detectors are often referred to as video detectors since they are designed to detect RF signals modulated by short pulses or wide-band signals in the video frequency range. However, in most applications for measuring antenna patterns or testing RF components in the laboratory, modulating frequencies are relatively low and bandwidths are kept as narrow as practical in order to maximize dynamic range and sensitivity. The RF signal is often square-wave modulated at 1 KHz and fed to the microwave detector. The output of the detector is amplified by a narrow-band low noise 1 KHz amplifier, called a crystal bolometer amplifier.

The two types of detectors commonly used in making antenna measurements are the diode detector and the bolometer. In the following sections three types of diode detectors are discussed briefly and compared: The point-contact diode, the hot-carrier (Schottky barrier) diode, and the backward tunnel diode (back diode). These diodes and the bolometer are compared as to their threshold sensitivity, frequency response, and dynamic range of square-law operation. The discussion that follows is not intended to be a comprehensive treatment on the subject but to be a review of the important characteristics of certain microwave detectors. The detector characteristics depend on the specific detector used and on the design of the detector mount. Other types of solid-state or thermal detectors are used in the laboratory to a lesser extent and are not discussed here.

Detector Transfer Characteristics — — — A detector characteristic that is of particular interest is the relationship of its output signal to the input power, or the sensitivity of the detector. The term sensitivity has a different meaning in this case than when used to measure the ability of the detector to distinguish signals from noise (as in threshold sensitivity). For a diode detector, the current sensitivity $\beta$ is the change in short-circuit output current due to a change of a microwave power $\Delta P$ as given by

$$\beta = \frac{\Delta i}{\Delta P}. \quad (4.36)$$

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Current sensitivity may be expressed in units of $\mu a/\mu w$ (or $\text{volt}^{-1}$). The sensitivity of a diode detector may also be measured by the change in open-circuit output voltage to a change in RF input power, and may be expressed in $\text{mv}/\mu w$. For a bolometer, sensitivity may be expressed as the change in bolometer resistance to a change in microwave power, or

$$\beta = \frac{\Delta R}{\Delta P}.$$  \hfill (4.37)

The response characteristic of a rectifier can be determined analytically by expressing the non-linear response in the form of a Taylor Series expansion about the operating point and evaluating the lower-order terms. For the diode detector illustrated in Figure 4.7 the current can be expressed as a function of the RF input voltage $v$ by the equation,

$$i = f(v) = I_s \left[ e^{\frac{qv}{nkT}} - 1 \right].$$  \hfill (4.38)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4.7.png}
\caption{Illustration of rectification by a diode detector.}
\end{figure}
The RF input voltage can be expressed as

\[ v = V \cos \omega t, \]  

(4.39)

and (4.38) can then be expanded in a Taylor Series. The series will contain a linear term, a second-power term, and higher-order terms. The linear term does not contribute to rectification and can be dropped. The higher-order terms are negligible in comparison to the second-power term if the input signal is small. These terms contain the input frequency and harmonics of the input and are filtered out by the detector circuit. The result is that for low-level signals the rectified current must be proportional to the square of the input voltage. For this reason, low-level detectors are called square-law detectors. However, as the input signal increases, a point is reached where the higher-order terms are no longer negligible. The upper limit of the useful square-law range can be determined by evaluating the contribution of the higher-order terms in the series.

For antenna measurement applications microwave detectors are usually operated in the square-law range. This results in a 2:1 ratio in the number of decibels measured at the output and input of the detector for a change in signal input. That is, a 1-dB change in signal power produces a 2-dB change in output signal. This becomes more dramatic when one considers that a 40-dB change in input signal produces a 80-dB change in the output signal. (That is to say a 100/1 input voltage ratio results in a 10000/1 output voltage ratio.) This range of signal levels must be measured accurately in a typical antenna instrumentation system.

**Threshold Sensitivity** - - - The threshold sensitivity of a system is the minimum input signal which will give a specified output signal-to-noise ratio. The sensitivity of microwave detectors is commonly specified by manufacturers as the tangential sensitivity (TSS). The tangential sensitivity can be estimated by observing the output noise of the device on an oscilloscope and applying a calibrated pulsed signal to its input. This procedure is illustrated in Figure 4.8. The signal is adjusted until the lower edge of the trace representing signal plus noise is tangent with the upper edge of the trace representing noise alone.
Although it may be useful in certain cases to measure and interpret tangential sensitivity in this manner, a tangential signal may be defined more rigorously as a signal 8 dB above the RMS noise level measured at the output of the detector. Note that a signal-to-noise ratio of 8 dB at the output of a square-law detector corresponds to a signal-to-noise ratio of 4 dB (or a ratio of 2.5) referred to the input of the detector since the number of dB of signal variation observed in the output circuit is twice the number observed at the input.

Tangential sensitivity is a function of the noise bandwidth of the output circuit. The output noise is directly proportional to the amplifier bandwidth $B$, assuming a constant noise power spectrum or a passband that is narrow relative to its center frequency. In this case, the sensitivity varies as $B^{1/2}$. However, because of the large amount of low frequency flicker noise in some detectors (see Figure 4.10), care must be exercised in calculating the tangential sensitivity for bandwidths other than that stated by the manufacturer in specifying tangential sensitivity. Unless stated otherwise herein, tangential sensitivity is normalized to a bandwidth of 1 Hz. For other bandwidths, the tangential signal is increased.
by $B^\frac{1}{2}$.

**Hot-Carrier and Point-Contact Diode Detectors** - Hot-carrier (Schottky-barrier) diodes and point-contact crystal rectifiers are metal-semiconductor devices which provide rectification by the nature of the metal-semiconductor contact. Point-contact diodes are fabricated by pressing a fine metal point (whisker) into the surface of a semiconductor. Hot-carrier diodes fabricated by depositing a metal film on the surface of a semiconductor are variously called planar diodes, hot-carrier diodes, and simply Schottky diodes. The operation of both types of diodes is explained by the Schottky theory of rectification, although the name Schottky diode is often used intentionally to distinguish planar hot-carrier diodes from point-contact diodes. Conduction of current is by the flow of majority carriers, whereas p-n junction diodes depend on minority carrier operation. The characteristic differences between planar Schottky-barrier and point-contact diodes are due to the difference in construction and geometry. Hot-carrier diodes can be produced with better control of the geometry, better repeatability, and higher resistance to mechanical shock. Hot-carrier diodes also exhibit much less low frequency flicker noise. However, point-contact diodes generally have lower capacitance and are usually superior for operation at millimeter-wave frequencies.

Hot-carrier and point-contact diodes can be represented electrically by the RF equivalent circuit shown in Figure 4.9. The components $R_s$ and $C_s$ are functions of the voltage across the diode junction. $R_s$ is the parasitic resistance and the contact resistance. These terms can be determined from low frequency measurements or from the theory of the rectification process.

\[
R_s = \text{PARASITIC SERIES RESISTANCE} \\
R_b = \text{BARRIER RESISTANCE} \\
C_b = \text{BARRIER CAPACITANCE}
\]

**RF EQUIVALENT CIRCUIT**

\[
\left(\frac{4kTt}{R_v}\right)^\frac{1}{2} \\
i_s = \beta P_{sr}
\]

**OUTPUT EQUIVALENT CIRCUIT**

**FIGURE 4.9** Equivalent circuit of solid-state diode detector.
Figure 4.9 also shows the equivalent circuit of the diode as seen from the output terminals. The signal current $i_s$ is the short-circuit current produced by application of RF power and $\beta$ is the short-circuit current sensitivity of the diode. The noise current $i_n$ is composed of the thermal noise associated with a passive resistance (see Figure 4.1) plus the excess flicker noise which causes an increase in the effective noise temperature of the resistive element. Tangential sensitivity is determined by relating the minimum signal current to the noise current (for signal-to-noise ratio of 8 dB as discussed previously). These characteristics are discussed further in the following paragraphs.

RF power must be absorbed by the non-linear resistance $R_B$ in order for rectification to take place. From Figure 4.9, the ratio of the RF power absorbed can be calculated. The junction capacitance causes the power absorbed by $R_B$ (and therefore, the current sensitivity) to vary as a function of frequency. From the equivalent circuit of the diode, it can be seen that the current sensitivity varies as $1 + (f/f_c)^2$ where $f_c$ is the RF cut-off frequency determined by $C_R$, $R_B$, and $R_S$. Typical values for $f_c$ are given in Table 4.1 at the end of this section.

The dc or video resistance is equal to the sum of the barrier resistance $R_B$ and series resistance $R_S$. The video resistance of point-contact diodes is typically several thousand ohms when the RF signal is within the square-law operating range. The resistance is also a function of the dc operating point. Therefore, detector diodes are often biased with a direct current to improve the match to the RF and output circuits. The video resistance as a function of bias current is given in Figure 4.11 for a hot-carrier diode.

In order to calculate the threshold sensitivity, the noise characteristics of the diode must be known. Hot-carrier and point-contact diodes produce thermal noise which is proportional to the video resistance $R_V$ and a constant called the noise temperature ratio $t_R$. The noise temperature ratio is the ratio of the actual power produced to the noise power of a resistor of $R_V$ ohms. Both diode types generate low frequency flicker noise in addition to the thermal noise.

---

*The discussion in this section assumes that all of the RF power is absorbed by the diode. Parasitic inductance and capacitance in the diode holder further reduce the sensitivity.*

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FIGURE 4.10 Noise characteristics of typical hot-carrier and point-contact diodes. (From Cowley and Sorensen, IEEE Trans.)

FIGURE 4.11 Barrier resistance and capacitance vs bias for typical hot-carrier diode. (From Cowley and Sorensen, IEEE Trans.)
The flicker noise varies as $1/f$; therefore, the noise temperature ratio varies as $1/f$ for low frequencies where flicker noise predominates. The complete expression for the noise temperature ratio is

$$t_n = t_w (1 + f_n / f),$$

(4.40)

where $t_w$ is the white noise temperature and $f_n$ is the noise corner. A plot of the noise temperature ratio for a point-contact diode and a hot-carrier diode is given in Figure 4.10.

From Figure 4.10 it is seen that the hot-carrier diode exhibits much lower flicker noise than the point-contact diode. Indeed, the noise corner of biased point-contact diodes is orders of magnitude higher than that for hot-carrier diodes. For this reason hot-carrier diodes are generally superior to point-contact diodes (except for operation at millimeter frequencies where the shunt capacity lowers the sensitivity) when operating at low modulating frequencies.

Characteristics of typical hot-carrier and point-contact diodes are summarized in Table 4.1. Data are also given for tunnel and back diodes, which are discussed briefly in the next section. The data in this table were extracted from "Quantitative Comparison of Solid State Microwave Devices" by A. M. Cowley and H. O. Sorensen. The characteristics given are functions of the diodes only and do not include losses due to mismatch of the diode to the source, losses in the diode holder and assembly, or loss in sensitivity due to amplifier noise.

Sensitivity and dynamic range are given for frequencies much lower than the RF cut-off $f_o$ and for modulation frequencies much higher than the flicker noise corner $f_n$. For other carrier frequencies the sensitivity varies as $1 + (f/f_o)^2$. The variation in sensitivity due to flicker noise is proportional to $(1 + f_n / f)^{1/2}$. For convenience, sensitivity and dynamic range have been normalized for an amplifier bandwidth of 1 Hz. Sensitivity and dynamic range vary as $B^{1/2}$.

The upper square-law range given in Table 4.1 is defined as the input RF power that produces a 0.3 dB linearity error. The dynamic range is the difference in the upper square-law limit and the tangential signal level for which the mean
error due to noise is also 0.3 dB. A 0.3 dB detector error is generally excessive for antenna measurements, and in practice the useful normalized dynamic range will generally be significantly less than that given in Table 4.1. The dynamic range is also a function of the input resistance of the amplifier.

Data given are for point-contact and hot-carrier diodes biased at approximately 50 µa and 500-600 µa. As indicated, the point-contact diode is clearly superior to the hot-carrier diode for high microwave frequencies and wide band video modulation. On the other hand, the hot-carrier diode is superior for frequencies through 10 GHz when the modulation frequency is 1 KHz. The difference is due primarily to the lower noise corner of the hot-carrier diode.

Tunnel and Back Diodes - - - The familiar non-linear characteristic of tunnel and backward (back) diodes is caused by the tunneling phenomenon. These devices are essentially p-n junctions with a high impurity which produces a narrow junction and allows the electrons to easily tunnel across the junction. The resulting current flow is the sum of the tunneling current and the ordinary p-n junction current as shown in Figure 4.12.

Tunnel-diode detectors are ordinarily forward biased at a voltage somewhat lower than that required for the peak tunneling current. The detector can be biased at its peak current in order to obtain very high sensitivities. When biased at that point full-wave rectification occurs, but the bias point is critical and there is a loss in dynamic range due to the sharp non-linearity around the peak current point. Back diodes are special tunnel diodes that are designed to produce a low peak tunneling current. They are so named because they are operated in the reverse region of the diode current-voltage characteristic. The current-voltage characteristic for a back diode is given in Figure 4.12.

The RF equivalent circuit of a tunnel or back diode is the same as for the

* Tunneling is a process wherein a particle (obeying the laws of the quantum theory) can disappear from one side of a potential barrier and appear virtually instantaneously on the other side, even though it does not have enough energy to surmount the barrier. It is as though the particle can tunnel underneath the barrier. The basic conduction mechanism has a theoretical frequency limit of approximately 10^7 MHz which is several orders of magnitude higher than the frequency limit for the drift and diffusion mechanism involved in the operation of conventional diodes and transistors.
hot-carrier and point-contact diodes given in Figure 4.9. The dynamic resistance of tunnel diodes is usually very low, often of the order of 10 ohms. The series resistance may be as low as 5 ohms. Tunnel diodes often must be biased near the current peak in order to obtain an impedance match to a 50 ohm source. Back diodes can be made to have 50 ohms total resistance with little or no forward bias, and are usually designed to operate without bias. The capacitance is high compared to point-contact diodes, but the tunneling types have good high frequency response because of the low series resistance.

The flicker noise characteristics appear to be better for tunnel diodes than point-contact types, but not as good as planar Schottky-barrier diodes. Back diodes exhibit very low 1/f noise for moderate bias levels with noise corners typically less than 1 KHz.

![Diagram of current-voltage characteristics of tunnel and back diode detectors](image)

**FIGURE 4.12** Current-voltage characteristics of tunnel and back diode detectors.


**TABLE 4.1**

**COMPARISON OF TYPICAL POINT-CONTACT, HOT-CARRIER, BACK, AND TUNNEL DIODES**

(From Cowley and Sorensen, IEEE Trans.)

<table>
<thead>
<tr>
<th>Diode</th>
<th>Bias (µA)</th>
<th>R₁ (Ω)</th>
<th>R₂ (Ω)</th>
<th>Cₖ (pF)</th>
<th>fₑ (GHz)</th>
<th>fₑ⁺ (KHz)</th>
<th>β</th>
<th>TSS (dBm)</th>
<th>Range (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-noise point contact</td>
<td>5-500</td>
<td>5-600</td>
<td>20</td>
<td>0.1</td>
<td>20</td>
<td>10</td>
<td>300</td>
<td>≈13</td>
<td>≈-89.5</td>
</tr>
<tr>
<td>hot carrier</td>
<td>5-600</td>
<td>5-600</td>
<td>20</td>
<td>0.2</td>
<td>30</td>
<td>5000</td>
<td>5</td>
<td>-83</td>
<td>59.5</td>
</tr>
<tr>
<td>hot carrier</td>
<td>5-600</td>
<td>5-600</td>
<td>10</td>
<td>0.9</td>
<td>5.8</td>
<td>0.2</td>
<td>19</td>
<td>-91</td>
<td>59.5</td>
</tr>
<tr>
<td>hot carrier</td>
<td>5-600</td>
<td>5-600</td>
<td>5</td>
<td>0.9</td>
<td>8.2</td>
<td>0.2</td>
<td>19</td>
<td>-86</td>
<td>59.5</td>
</tr>
<tr>
<td>backward</td>
<td>0</td>
<td>90</td>
<td>10</td>
<td>1.0</td>
<td>5.5</td>
<td>25</td>
<td>100</td>
<td>-89</td>
<td>53.5</td>
</tr>
<tr>
<td>tunnel</td>
<td>1000</td>
<td>45</td>
<td>5</td>
<td>1.0</td>
<td>11.2</td>
<td>50</td>
<td>100</td>
<td>-93.5</td>
<td>50</td>
</tr>
</tbody>
</table>

*Current sensitivity for f ≪ fₑ
**Tangential sensitivity for f ≪ fₑ, f ≫ fₑ, and B = 1 Hz
***Difference between maximum level for 0.3 dB nonlinearity and TSS.

**Bolometer Detectors - - -** A bolometer is a device for detecting RF energy. Bolometers depend upon a change in resistance due to heating produced by the application of RF energy. A barretter is a particular form of bolometer consisting of a fine metal wire that acts as a load to the RF source. A thermistor is another form of bolometer. It consists of a small bead of semiconductor material supported between two fine wires. Thermistors usually have a high negative temperature coefficient of resistance. They are not ordinarily used in making antenna pattern measurements because of their relatively long thermal time constant and sensitivity to ambient temperature, and, therefore, are not discussed further in this chapter. Barretters are probably more often referred to simply as bolometers, and will be so called in these chapters.

A simplified schematic illustrating how a bolometer is used to demodulate an RF signal is shown in Figure 4.13. The signal to be measured is modulated, typically at a 1 KHz rate, and fed to the bolometer. A direct current supplied to the bolometer causes a change in voltage to be developed by the change in resistance of the bolometer element. The bolometer bias may be obtained from a constant-current dc supply.
Bolometers usually operate at a resistance of 200 ohms. The resistance of commercial bolometers made with a fine platinum wire element is almost directly proportional to the power dissipated and temperature of the element. The output voltage of the bolometer is directly proportional to input power provided the RF power is small compared to the dc bias power. Bolometers maintain this square-law characteristic over a wide dynamic range. Characteristics of representative commercial bolometers are given in Table 4.2.

<table>
<thead>
<tr>
<th>TYPE</th>
<th>R</th>
<th>I</th>
<th>ΔR/ΔP</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(OHMS)</td>
<td>(MA)</td>
<td>(OHMS/MW)</td>
<td>(GHz)</td>
</tr>
<tr>
<td>821B</td>
<td>200</td>
<td>8.75</td>
<td>.45</td>
<td>0 - 12.4</td>
</tr>
<tr>
<td>610B</td>
<td>200</td>
<td>8.75</td>
<td>.45</td>
<td>0 - 18.0</td>
</tr>
<tr>
<td>614</td>
<td>200</td>
<td>4.5</td>
<td>10.0</td>
<td>12.4 - 26.5</td>
</tr>
<tr>
<td>617</td>
<td>200</td>
<td>4.5</td>
<td>10.0</td>
<td>18.0 - 40.0</td>
</tr>
</tbody>
</table>

TABLE 4.2 Characteristics of representative commercial microwave bolometers.

The signal $ΔV$ which exists across the bolometer is $IΔR$. The bias current $I$ is normally kept constant to maximize the output voltage. The change in resistance $ΔR$ is
\[ \Delta R = \frac{dR}{dP} \Delta P = \frac{dR}{dP} \left( I_{r} + I^2 \Delta R \right), \]  

(4.41)

and

\[ \Delta V = \frac{I\beta}{1 - I^2 \beta} \Delta P_{rf}, \]  

(4.42)

where \( \beta = \Delta R/\Delta P \). Typical values for \( \beta \) are given in Table 4.2.

The signal is usually square-wave modulated. If the modulation frequency is low enough so that the output voltage is a square wave, and if a narrowband amplifier is used, the rms value of the fundamental component is \( \sqrt{2}/\pi \) times the peak-to-peak output. The rms output voltage \( v \) is

\[ v = \frac{\sqrt{2}}{\pi} \left( \frac{I\beta}{1 - I^2 \beta} \right) \Delta P_{rf}, \]  

(4.43)

where \( \Delta P_{rf} \) is the peak-to-peak change in signal power.

The threshold level and sensitivity of a bolometer have been solved theoretically in terms of the physical constants of the bolometer element. In references 5 and 6 the heat flow equation is set up and the temperature along the element is found in terms of the dimensions of the wire, heat capacity, radiation coefficient, thermal conductivity, and resistivity of the wire material. The sensitivity is found by then solving for \( \Delta R/\Delta P \), the change in resistance due to a change in input power.

In order to calculate the threshold level or tangential sensitivity the noise generated by the bolometer must be determined. Noise is attributed to two independent causes: (1) Spontaneous random fluctuations in the temperature of the bolometer due to interchange of heat with the surroundings (temperature fluctuation noise). (2) Johnson (thermal) noise.

Temperature fluctuations produce fluctuations in the resistance of the bolometer and, consequently, fluctuations in the output voltage. The Johnson noise is the
familiar resistance noise discussed previously. For simplicity, Weinschel assumes that the total noise is equal to that of a 200 ohm resistor at an elevated temperature. The noise is also approximately equal to that of a 300 ohm resistor at room temperature. For this assumption the output noise voltage is $2.2 \times 10^{-9}$ volts for a 1 Hertz bandwidth.

The tangential sensitivity can be calculated from (4.43), data in Table 4.2, and for an output noise voltage of $2.2 \times 10^{-9}$ volts. (Recall also that a tangential signal at the input of a square-law detector is 2.5 times the input noise power.) Thus for a 610B bolometer

$$P_{\text{RSS}} = 2.5 \frac{\pi}{\sqrt{2}} \left( \frac{1 - I_0^2}{I_0} \right) v = 2 \times 10^{-10} w.$$  (4.44)

or approximately -67 dBm.

From data given by Weinschel, the maximum RF level should not exceed about $1/4$ to $1/2$ the dc bias power in order to limit non-linearity errors to 0.1 to 0.25 dB. For a 610B bolometer, this would imply an upper limit of approximately 5.8 dBm. The dynamic range is the difference in this level and the tangential signal level, or approximately 72.8 dB for a 1 Hz noise bandwidth. For other bandwidths the dynamic range and tangential sensitivity decrease by a factor of $B^{1/2}$ where $B$ is the bandwidth of the output amplifier.

The above data indicate that bolometer detectors are capable of square-law operation over a greater dynamic range than are microwave diode detectors. On the other hand, the open-circuit voltage sensitivity and tangential signal sensitivity are much greater for diode detectors than for bolometers. The open-circuit voltage sensitivity of a diode may range from approximately 1 to 20 mv/μw. Bolometers typically produce 20 to 30 μv rms for 1 μw square-wave change in input power. The tangential sensitivity may be on the order of 20 dB better for a diode than for a bolometer detector. However, because of the greater stability, accuracy, and dynamic range of bolometer detectors they are generally used for detecting the 1 KHz modulation in the output of antenna measurements receivers.
4.4 MICROWAVE MIXERS

Superheterodyne receivers depend upon mixers for converting the RF signal to an IF signal. Frequency translation is accomplished by combining the RF signal with the output of a local oscillator (LO) in a non-linear element. The mixing action produces a series of frequencies harmonically related to the RF and LO frequencies. The general expression for this relationship is

\[ mf_{RF} = nf_{LO} \pm f_{IF} \] (4.45)

where m and n are integers and \( f_{RF} \), \( f_{LO} \), and \( f_{IF} \) are the signal, LO, and intermediate frequencies. If the local oscillator voltage is large compared with the amplitude of the signal, the conversion may be made linear and the output voltage will be linearly proportional to the input voltage.

In most applications m and n are unity and \( f_{RF} = f_{LO} \pm f_{IF} \). This process of heterodyning is called fundamental mixing. Furthermore, the receiver may be designed to favor one of the two RF responses given by the ± sign of \( f_{IF} \); the other response is called the image response or image frequency.

Receivers designed for antenna measurements should cover a broad range of frequencies. The practical frequency range for a local oscillator is about an octave, depending, of course, on the specific frequency band. In order to cover the broad frequency range without multiple local oscillator units, harmonic mixing is used. In this case \( f_{RF} = nf_{LO} \pm f_{IF} \). A harmonic of the local oscillator frequency is used to heterodyne with the signal frequency. Another advantage of harmonic mixing is that operation is possible at millimeter and submillimeter wavelengths using simple microwave oscillators at a lower frequency. With harmonic mixing, reception has been extended to 420 GHz using a local oscillator frequency of 70 GHz. Harmonic mixing is discussed briefly in later paragraphs.

The purpose of harmonic mixing is to convert the RF signal to an intermediate frequency. The efficiency of this conversion is measured by the conversion loss. Conversion loss (\( L_c \)) is defined as the ratio of the signal power available at the desired intermediate frequency (at the output terminals of the mixer) to the input signal power available, or
The actual loss encountered will depend on the match of the source to the mixer and the mixer to the IF amplifier. In microwave antenna measurements receivers the mixer is usually located at the antenna and connected to the receiver by a long cable. For minimum loss the mixer and the IF amplifier are matched to the impedance of the coaxial cable. In cases where the mixer can feed the IF amplifier directly the output impedance should be transformed to the impedance which minimizes the noise figure of the IF amplifier.

When the signal and image terminations are equal, the mixer is termed broadband. The conversion loss of broadband mixers is higher because part of the signal power is dissipated in the termination for the image and harmonic frequencies. Terminating the image in either a short or an open circuit will prevent this loss of IF or signal power. The conversion loss of harmonic mixers is discussed briefly in a later paragraph.

The sensitivity of a superheterodyne receiver is determined by its noise figure and bandwidth. For receivers without amplification preceding the converter the noise figure is given by

\[ F = L_c(t + F_{IF} - 1) , \]  

(4.47)

where \( t \) is the noise temperature ratio of the mixer, \( F_{IF} \) is the IF amplifier noise figure, and \( L_c \) is the conversion loss. From (4.47) we see that the mixer acts as an attenuator to the signal and adds some noise of its own.

The noise temperature ratio of a mixer is defined as

\[ t = \frac{N_o}{N_R} , \]  

(4.48)

where \( N_o \) is the noise power available from the diode or converter, and \( N_R \) is the available thermal noise power from an equivalent resistor at room temperature. The available thermal noise power was given previously (4.4) as \( kT_R \). The noise temperature ratio of an ideal diode mixer is unity. That is, the
only noise would be thermal noise. A noise temperature greater than unity is attributed to excess noise. Excess noise arises from shot noise and 1/f noise in the diode caused by the flow of rectified current when excited by the local oscillator.

**Principles of Operation** - It is beyond the scope of this chapter to give a mathematical treatment of the operation of mixers. The theory of operation of converters has been covered by other writers who assume a mathematical model for the diode (see Figures 4.7 and 4.9) on a non-linear characteristic in the form of a Taylor Series. Instead, mixer operation will be discussed with the aid of the simple phasor diagrams that follow. An important property of mixers is the correspondence between the phase of the IF signal and the phase of the RF signal relative to the LO, and simple diagrams illustrate this principle. This property makes possible the use of superheterodyne receivers for measuring the phase of RF signals.

Figure 4.14 illustrates the addition of two RF sinusoidal signals: a strong LO signal and a RF signal at a relatively small difference frequency (the intermediate frequency). The phasors are shown at some arbitrary time \( t = 0 \). The sum of the two signals produces an amplitude-modulated resultant. Modulation of the envelope occurs at the difference frequency. Note that the modulation is not pure AM since only one sideband is present.

Figure 4.14 also shows the effect of a 180 degree change between the phase of the LO and RF signals and the phase of the envelope. Note that it is the relative phase of the LO and RF signals that is important; a 180 degree change in the LO phase would have produced the same result.

The addition of two signals is illustrated by Figure 4.14, but no intermediate frequency is present. In a mixer, the local oscillator output and input signal are added as illustrated and applied to a non-linear element, usually a diode. The composite signal is rectified as illustrated in Figure 4.15. The rectified output is passed through a bandpass filter to obtain the IF signal.

Figures 4.14 and 4.15 illustrate how the mixing process results in a linear conversion of the RF signal to an IF signal. Three important properties of this process are:
FIGURE 4.14  Illustration of phase correspondence of IF and relative phase of LO and signal.

FIGURE 4.15  Output after rectification by mixer.
(1) the amplitude of the IF signal is directly proportional to the amplitude of the RF signal,

(2) a change in phase of the RF signal produces an equal change in phase of the IF signal, and

(3) the RF signal and noise frequency spectrum is translated to an identical IF spectrum.

Harmonic mixing is covered in less detail in the literature than fundamental mixing, and is more difficult to illustrate because of the complex current waveforms involved. An exaggerated simplification of the harmonic mixing process is illustrated in Figure 4.16. Here the diode mixer is symbolized by a switch. The switch is analogous to an ideal diode, and is closed during the time the diode is driven into conduction by the local oscillator. Many cycles of the RF signal may occur during the time the switch is closed depending on the conduction angle of the LO current and the harmonic number.

The net current during the LO period determines the amplitude of the output signal since the IF amplifier responds only to the low-frequency component of current. Since the average current for an integral number of cycles of the signal is zero, the net current is the current contributed by the fractional cycle as illustrated in Figure 4.16. As the phase of the RF signal relative to the LO harmonic changes the amplitude and polarity of the net current changes to produce the IF signal. Note that as the harmonic number increases the net current per LO cycle becomes proportionately less. As a result, the conversion loss is approximately proportional to the harmonic number \( n \). This causes the theoretical sensitivity of the receiver to decrease approximately 6 dB each time the harmonic number is doubled.

The conversion loss of a harmonic mixer can be minimized by applying a reverse bias to the diode and increasing the local oscillator output to optimize the conduction angle. This has the effect of increasing the harmonic content of the mixer current. Note that harmonic mixing occurs even though the local oscillator itself may not contain harmonics.

Sampling is a special case of heterodyning. It is related to harmonic mixing, but the techniques and circuitry are different. If the conduction angle of a harmonic mixer is decreased until the angle is small compared with the
period of the highest signal frequency, the mixer can be called a **sampler**. As the name implies, the RF signal is sampled at the LO rate. The sampling frequency is adjusted so that the signal frequency and a harmonic of the sampling frequency differ by the intermediate frequency. The output of the sampler is a series of short pulses whose envelope varies at the IF rate.

![Simplified analogy of harmonic mixing.](image)

**FIGURE 4.16** Simplified analogy of harmonic mixing.

The basic difference between sampling and harmonic mixing is that the sampling pulse is short compared to the period of the RF signal. As long as this relationship holds, the efficiency of the sampler is, in principle, the same for each harmonic, whereas the efficiency of a harmonic mixer varies inversely as the harmonic number. However, the efficiency of a fundamental mixer is much greater because a sampler produces relatively short pulses with low energy content. Like the mixer, the sampler produces a linear frequency
translation of the signal spectrum to an IF spectrum with amplitude linearity and phase coherence maintained.

Unlike the case of harmonic mixing, nearly absolute phase differences between two closely spaced RF input ports can be achieved by samplers by synchronizing the timing of the sampling pulses at the two mixers. This technique is used in swept frequency complex impedance measuring equipment at frequencies through X-Band.

Because of variations in the conduction angles of typical diodes, the IF signals of harmonic mixers involve a phase delay, which is unknown but constant for constant local oscillator frequency and drive. Most antenna phase measurements are made at fixed frequencies, and this unknown phase delay is either of no importance or can be accounted for by a calibration procedure. The sensitivity of harmonic mixers, the capability of separating the mixers at large distances from the local oscillator, and the higher frequency range capabilities make the technique of harmonic mixing more adaptable to the antenna measurements problem.

4.5 RESPONSE TO TIME-VARYING SIGNALS

Errors can occur in a measuring system because of restrictions in the system bandwidth. Antenna patterns are generated by rotating the antenna test positioner, usually at a constant rate, and plotting the received signal level as a function of positioner angle. The antenna pattern can be considered as a function of time, and the transient response errors of the system can be estimated for a particular antenna pattern input.

The response of a network to an input function can be calculated by classical Fourier or Laplace transform methods if analytic expressions for the input function and network characteristics are known. The response can also be determined by numerical or graphical means if the input and network characteristics are known quantitatively. The convolution-integral method is appropriate for solving the network response by either analytical or numerical techniques. The output \( f_o(t) \) given by the convolution integral is

\[
 f_o(t) = \int_0^t f_i(\tau) h(t - \tau) \, d\tau
\]  

(4.49)
where $f_1(\tau)$ is the input function and $h(\tau)$ is the network response to a unit impulse. The impulse response is the Fourier transform of the frequency characteristic of the network.

Usually an analytic expression for the antenna pattern is not known and a complete analysis by classical network theory is neither practical nor justified. An analysis can be made by approximating the antenna pattern in the region of interest by an analytic expression. If some simple approximations are made, the results can be easily estimated with reasonable accuracy.

Some general observations and statements can be made regarding the transient error in measuring antenna patterns. If the pattern is typical of a narrow-beam antenna, the distortion in the measured pattern appears primarily as (1) a filling in of sharp nulls, and (2) a delay or shift in the recorded pattern.

The convolution integral can be used to estimate the error as the input signal passes through a null. According to (4.49) the output signal is given by the average value of the input weighted by the impulse response. An approximation of the impulse response can be used to obtain an estimate of the transient error.

The delay or shift in the recorded data is caused by the envelope delay of band-pass amplifiers or phase delay of the low pass filters in the receiving and recording system. The time delay of a symmetrical bandpass amplifier is given by

$$T(\omega) = \frac{\phi(\omega)}{\omega} \quad (4.50)$$

where $\phi$ is the phase shift of the lower sideband relative to the carrier. For sidebands close to the carrier frequency the phase shift is approximately linear and the time delay is constant. For reasonably small transient response errors the frequencies contained in the input signal should be small compared with the system bandwidth. Under these conditions the output will be essentially equal to the input signal shifted by a constant time delay.

A direct relationship between the envelope delay and bandwidth of an amplifier does not exist. The time delay of cascaded stages adds directly, whereas the shrinkage in overall bandwidth depends on the transfer characteristic of each
stage as well as the number of stages. For example, the time delay of n cascaded synchronously tuned stages is

\[ T = \frac{n}{\pi \Delta f} \]  

(4.51)

where \( \Delta f \) is the bandwidth of each stage. The overall bandwidth \( B \) is approximately

\[ B = \frac{\Delta f}{1.2 \sqrt{n}}. \]  

(4.52)

The filter which determines the recording system bandwidth can usually be represented by a simple network. If the pattern is plotted with a conventional antenna pattern recorder, the bandwidth is limited by the servo system. If the data are recorded electronically, the system bandwidth is usually determined by the filter in a crystal-bolometer amplifier. For example, the bandwidth of a typical crystal-bolometer amplifier is 30 Hz, and the filter can be represented by two cascaded synchronously tuned stages. The time delay should be approximately 0.01 second. The bandwidth of a recorder servo system may be of the order of 5 Hz.

The servo error for an antenna pattern recorder can be partially compensated for by adjusting the time constants of the pen and chart servos to be nearly the same. When properly adjusted the difference seen when recording antenna patterns with opposite directions of positioner rotation is reduced significantly.

*The discussion of noise bandwidth given on page 4-4 assumes that the filter contributes no noise of its own. This is the case if the filter is a lossless network. If the filter is actually a bandpass amplifier, such as an IF amplifier, then it is assumed that sufficient preamplification exists to cause the amplifier noise to be negligible. For a lossless filter with thermal noise input, the noise at the output is

\[ F = kTB |H(f_0)|^2. \]

The noise at the input terminals in bandwidth \( B \) is given by (4.4).
CHAPTER 4
REFERENCES


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CHAPTER 5
COORDINATE SYSTEMS AND ANTENNA POSITIONERS
S. F. Hutchins

5.1 INTRODUCTION

Antenna measurements consist, for the most part, of measurements of the fields radiated by the antenna under test as a function of either position in space or direction from an origin. The antennas involved can evidently have a variety of parameters to serve a variety of applications. The purpose of this chapter is to acquaint the reader with the basic methods of defining and measuring angles and distances which are associated with antenna measurements.

Because of the nature of the antenna problem, the chapter will be concerned principally with spherical coordinate systems, with introductory material and references to related coordinate systems. The subsequent material is divided into three categories, operational coordinate systems, coordinate systems and positioners for antenna measurements, and errors in direction measurements. An appendix is included defining Euler angle transformations, which are required in calculating relative orientations between coordinate systems.

5.2 BASIC COORDINATE SYSTEMS

Position in three-dimensional space is usually described by one of the three sets of orthogonal* coordinates shown in Figure 5.1. These are rectangular or Cartesian coordinates, cylindrical coordinates and spherical coordinates.

*A three-space coordinate system is mathematically orthogonal if, at every point, the three-component vector directions are mutually orthogonal.
Coordinate systems having orthogonal axes, X, Y and Z, are defined to be right-handed if rotation from X toward Y would produce right-hand screw motion in the positive Z-direction. For a discussion of generalized, orthogonal, curvilinear coordinates, see References 1 and 2.

Direction from an origin in three-dimensional space is usually defined by the spherical coordinates \((\phi, \theta)\) of Figure 5.2a. For certain types of problems, direction angles or direction cosines, Figure 5.2b, become the logical means for describing direction. The position of a point in space can also be defined in terms of direction cosines by multiplying each direction cosine by the radius from the origin to the point.

Direction angles occur naturally in certain interferometer angle measurement systems, for example, in the Gemini rendezvous radar. See page 5-7.
FIGURE 5.2 Direction in space defined (a) in spherical coordinates, (b) in terms of direction angles or direction cosines. In (b) the position of a point P can also be defined by multiplying the direction cosines by the radius to the point.

Euler Angles -- The definition of direction and the measurement of angles often require calculation of the orientation of one coordinate system relative to another coordinate system. The relative orientation of two three-dimensional, orthogonal coordinate systems can be described by the three Euler angles \( \alpha \), \( \beta \) and \( \gamma \) of Figure 5.3. In this manner, any relative orientation of two coordinate systems can be achieved by three successive rotations about the coordinate axes.

*The limited number of Greek letters customarily used for angles leads us to use the same letters for different angles in different contexts.
The rotations from a coordinate system $\tilde{X}$, $\tilde{Y}$, and $\tilde{Z}$ to a system XYZ of Figure 5.3 are (1) rotation about the $\tilde{Z}$ axis through $\alpha$ to $X''$, $Y''$, $Z''$, (2) rotation about the $Y''$ axis through $\beta$ to $X'$, $Y'$, $Z'$, (3) rotation about the $Z'$ axis through $\gamma$ to XYZ.

Equations defining Euler angle transformations in terms of spherical coordinates are developed in Appendix 5A.

### 5.3 OPERATIONAL COORDINATE SYSTEMS

Many types of physical systems utilize coordinate systems to describe position or direction in space. The basic coordinate systems of the preceding paragraphs are used for operational systems, but the terminology is often changed to suit the environment.

Topocentric coordinates are related to systems which have the coordinate axes defined with respect to the earth. Figure 5.4 shows the altazimuth system. The two orthogonal axes normally identified as XY are in the horizontal plane tangent to the earth's surface. The Z axis is the direction of the zenith as defined by local gravity. Azimuth angle is measured from a defined direction, usually North, in the direction towards East. By the standards described in Section 5.2, the altazimuth coordinate system is left-handed.

![FIGURE 5.4 The altazimuth coordinate system used by surveyors.](image)

Elevation angle is measured from the horizontal plane, either up (positive) or down (negative). The altazimuth coordinate system may be utilized to describe a point by its range, azimuth and elevation or direction by azimuth and elevation. Surveyor's transits and many radar tracking systems employ altazimuth coordinates.
The equatorial system is another left-handed coordinate system. The Z axis (Figure 5.5) is aligned parallel to the earth's rotational, or polar, axis. Declination is measured either north or south from the celestial equator, which is perpendicular to the Z axis.

Angles about the polar axis of the equatorial coordinate system are called hour angles because the angles are associated with either solar or sidereal time and are traditionally measured in hours, minutes and seconds (24 hours per 360 degrees). Hour angles are usually referenced either to the earth or to the celestial sphere. If the reference is to earth, the angle is called local hour angle and is measured by left-handed rotation about the OZ axis from the meridian. The meridian is the plane established by the OZ axis and the zenith as defined by local gravity. If the reference is to the celestial sphere, the angle becomes the sidereal hour angle and is measured from the First Point of Aries.

Another topocentric system is the spherical coordinate system associated with certain positioners of radar or telemetry tracking systems, which are called "X Y" positioners (or more properly, traverse over elevation positioners).

The spherical coordinate system of the XY positioner has the OZ axis parallel with the horizontal plane as shown by Figure 5.6. Although Figure 5.6 shows the OZ axis oriented with south, the polar axis can be oriented in any direction in the horizontal plane.
For convenience in specifying direction or position relative to an aircraft, a three-axis coordinate system is usually defined as illustrated in Figure 5.7. The orientations of these axes are established when an aircraft is designed, and the established orientation is used thereafter throughout the life of the aircraft for lofting, fabrication, assembly and if necessary, armament orientation. The coordinate system thus defined must be considered rigidly referenced to the aircraft structure, and the axes do not change their defined position relative to the aircraft's structure due to subsequent motion or altitude changes of the aircraft in flight.

The Inter-Range Instrumentation Group of the Range Commanders Council has defined a coordinate system specifically for use with rockets, missiles and space vehicles. The coordinate axes (Figure 5.8) may be arbitrarily defined, but the reference, once established, remains fixed to the vehicle through the entire flight, even though the vehicle tumbles or becomes re-oriented with respect to its velocity vector.
Systems which are associated with naval fire-control employ a specific set of coordinates and a quite involved set of standards for coordinate transformation which are required to account for the roll and pitch of the ship and for its heading relative to the target and to true north. Specification of position and direction of targets relative to the ship employ Cartesian, cylindrical and spherical coordinates. Coordinate transformation between angles referenced to the horizontal plane and those referenced to the deck plane are performed by a rather complicated and specialized system which is peculiar to this particular discipline.

Special purpose coordinate systems are often defined to solve particular problems. Figure 5.9 illustrates the direction angle coordinate system utilized by the rendezvous radar employed in the Gemini program.

The Gemini rendezvous radar operated as a two-channel phase monopulse radar or interferometer to measure the direction of arrival of the received signal. The phase differences between the signals received by the two orthogonal pairs of antennas were measured by the radar. The electrical phase differences measured by the receiver were related to the direction angles $\alpha$ and $\beta$, as illustrated in Figure 5.9, defining the direction to the target as the intersection of two cones. The ambiguity resulting from the two lines of intersections of the cones, one in the hemisphere forward of the spacecraft and the other in the hemisphere to the rear of the spacecraft, was resolved by the directivity of the radar antennas.
5.4 POSITIONERS FOR ANTENNA MEASUREMENTS

The purpose of this section is to describe positioners and techniques which are employed in antenna measurements. The bulk of the section will be concerned with positioners for measurements in spherical coordinates. Description of a rectangular coordinate positioner for measurement of the phase and amplitude of the fields over a planar aperture is also given.

Definition of Antenna Coordinate System -- The conventional practice of defining a coordinate system to which the structural parts of an aircraft are referenced during design, fabrication and testing was discussed in Section 5.3. Similarly, it is good practice to define a coordinate system which can be used as a frame of reference for the design, fabrication, testing and operational orientation of an antenna. Sometimes this coordinate system has been specified by the system of which the antenna is a component. In some cases, it may be necessary to define a coordinate system, specifically for testing the antenna, which is new or different from the one employed by the associated system.

FIGURE 5.10 Standard spherical coordinate system employed in antenna measurements.
Figure 5.10 shows the standard spherical coordinate system which is employed for virtually all antenna pattern measurements. * If permissible, this coordinate system should be oriented with respect to the antenna in a manner that is most convenient to the antenna under consideration. A frame of reference that is permanent and invariable will be provided if the coordinate axes are defined with respect to a mechanical or geometric reference of the antenna.

The coordinate axes are usually defined in terms of a plane, which contains two of the coordinate axes, and the normal to the plane, which defines the third axis. Mathematically, a plane is defined by a set of three points, as shown in Figure 5.11. In practice the three points are typically represented by a mechanical means much as three tooling balls, or small, precision spot faces which are separated sufficiently to provide precise definition of the plane and its normal.

*The radiation characteristics of an antenna are defined in terms of the gain or the directivity of the antenna as functions of \( \psi \) and \( \theta \), which are independent of distance from the antenna. On the other hand, the measurements are made at some distance \( r_s \) from the antenna. This distance should ideally remain constant. In addition, \( r_s \), \( \psi \) and \( \theta \) are defined as if the antenna were electrically centered at the origin of the coordinate system. In practice this is not usually the case, and a certain amount of parallax (see Chapter 11) is introduced into the measured data. In the basic discussion presented here, it will be assumed that parallax is negligible and that \( r_s \) is constant.
The directions of the two axes in the reference plane are often defined by two precision locator pins, which can be used with a clinometer to define one axis directly and define the third axis by the requirement for mutual orthogonality between axes.

In applications where extreme precision is not required, the reference plane may be defined by a machined surface and the direction of one of the reference axes in the plane by a scribed line on the surface, a mark on the periphery of the antenna, or even by the orientation of the feed.

**Orientation of the Coordinate System Relative to Antenna Under Test and Definition of Basic Cuts** — When defining a spherical coordinate system with respect to a directional antenna, a variety of relative orientations may be used. An antenna that directs most of its radiated energy into a relatively narrow beam can have its coordinate system defined so that the beam will be in or near the equator (Figure 5.12). This places the Z-axis 90 degrees from the direction where the beam maximum is expected. The direction of the beam maximum can be oriented in any desired direction in the equator, usually it is oriented toward \( \phi = 0 \), but it may be oriented toward a direction such as \( \phi = 180^\circ \) to provide for continuity of positive \( \phi \) angle integers in the principal area of interest.

**FIGURE 5.12** Spherical coordinate system with pencil beam antenna.
An antenna having a fan-shaped beam, which is wide in one plane and narrow in the orthogonal plane, is usually oriented in the direction which is most convenient from the viewpoint of suppressing reflections from the range surface. This may lead to the configuration of Figure 5.13 or to that of Figure 5.14 depending on the orientation of the coordinate system relative to the range surface.

![Figure 5.13](image1.png)  
**Figure 5.13** Fan beam oriented with fan in plane of equator.

![Figure 5.14](image2.png)  
**Figure 5.14** Fan beam oriented with fan normal to plane of equator.

Radiation characteristics of an antenna are usually determined by sampling or measuring continuously the relative radiation intensity along lines of constant $\phi$ or constant $\theta$. Figure 5.10 shows that the loci of constant $\phi$ directions are great circles (actually semi-circles) and that loci of constant $\theta$ directions describe cones. Measurements made with $\theta$ variable and $\phi$ as a parameter are sometimes called *great-circle cuts*. Note, however, that the *conical cut* for $\theta = 90^\circ$ degenerates to a great-circle cut.

*Principal-plane* cuts are generally understood to represent orthogonal *great-circle cuts* which are through the axis of the beam. The beam axis must lie in the equator or at one of the poles for this definition to hold.

Alignment of the beam of a narrow-beam antenna along the Z axis is usually avoided because of the nature of the $\phi$ and $\theta$ cuts in this region. Note that the
\( \phi \) cut for \( \theta = 0 \) reduces to a polarization pattern and that \( \phi \) cuts near \( \theta = 0 \) involve radical changes in the direction of polarization of the incident field relative to the spherical coordinate system, except when the incident field is identically circularly polarized.

**Spherical-Coordinate Antenna Positioner Configurations** -- Practical antenna positioner configurations have evolved for achieving the relative motions required for making antenna measurements over solid angular sectors of \( \phi \) and \( \theta \) which occur in antenna measurement problems. In the following paragraphs, we will develop the basic configurations which are employed.

Two orthogonal axes of rotation are required to permit making antenna measurements as functions of \( \phi \) and \( \theta \). These axes are designated the \( \phi \) axis and the \( \theta \) axis in Figure 5.15. The \( \phi \) axis, which permits cuts in \( \phi \) with \( \theta \) as the parameter, is the \( OZ \) axis of the coordinate system. The \( \theta \) axis, which provides for \( \theta \) cuts with \( \phi \) as the parameter, is coincident with a line \( OA \) drawn through the origin normal to the line \( OT \), defining the direction to the source antenna. These axes are employed in all spherical-coordinate positioner configurations.

Some of the figures employed in this development will be purposely simplified. The coordinate systems and positioners will be generally shown in simple elevation views, and the \( Z \) axis and equator of the coordinate system will be designated as in Figure 5.16.
Figure 5.16a shows an arrangement which might be employed, but is not to our knowledge because other methods are more practicable. The antenna is shown fixed at the origin of the coordinate system. Variation in $\theta$ is accomplished by motion of a source antenna along a circular track, whose center of curvature is at the origin.

![Diagram](image)

**FIGURE 5.16** Hypothetical positioner configurations used for purpose of illustration. The indicated configurations permit measurement of radiation patterns of antenna under test in $\phi$, in $\theta$ and in polarization. These configurations are shown to illustrate the sampling problem. More practical positioners are generally used in practice.

*If the measurements are made with the antenna under test employed as a transmitting antenna, our source antenna will be a receiving antenna. We will assume that the antenna under test is reciprocal and is employed on receiving. We will call the other antenna the source antenna. This is done for convenience and because most measurements are made on receiving in practice. It makes no difference in the discussion.*
Motion in $\phi$ is accomplished by rotation of the $\theta$ track about the OZ axis by means of a base positioner. The source antenna is shown mounted on a small rotator called a polarization positioner, which permits measurements to be made of the polarization of the antenna under test.

The arrangements of Figure 5.16a have evident disadvantages. These lead to configurations which are actually employed in practical ranges.

In Figure 5.16b, as a step in the evolution of practical positioner configurations, the antenna under test has been placed on a single-axis base positioner called an azimuth positioner. (See Figure 5.17.) The base positioner provides nearly 360 degrees of relative rotation in $\phi$ by rotation of the antenna under test rather than the source antenna track. The source antenna track is mounted on the ground rather than on the base positioner. This configuration is more practical than that in Figure 5.16a in that the heavy track for the source-antenna is not supported by the base positioner.

FIGURE 5.17 Azimuth positioner.
In Figure 5.18 the source track has been modified to reduce the coverage in \( \theta \) to 90 degrees, but to eliminate interference in \( \phi \), since the base positioner can rotate through 360 degrees. The antenna under test has been mounted on a positioner which permits reversal of the +Z and -Z axes. Complete coverage in \( \theta \) can be achieved by making measurements in \( \theta \) from zero through 90 degrees and by then rotating the antenna under test through 180 degrees about its axis to provide coverage in \( \theta \) from 90 degrees through 180 degrees. There are a number of problems associated with the arrangement of Figures 5.16 through 5.18. One is in the difficulty of fabricating a track which is precisely circular; the configuration does not lend itself to making phase measurements for this reason and also because there is no convenient means for obtaining a phase reference from the movable source antenna. The configuration of Figure 5.19 overcomes this problem in cases, such as in measurements of primary feeds, where the required distance \( r_0 \) is not so great as to make it impracticable. The source track has been replaced with a gantry arrangement, which can provide a nearly constant separation between source antenna and antenna under test if the gantry is made sufficiently rigid. The positioner and other critical reflecting surfaces are usually covered with microwave absorbing material to reduce reflections, thus enhancing the precision of the measurements. When used in making phase measurements, rotary joints are employed on all three axes.

\*See Chapters 9 and 10 on measurement of phase and polarization.
Figure 5.20 is a photograph of a similar positioner configuration, but designed for operation over a ground plane. In one installation of this type, the rotation of the polarization positioner on the gantry arm, the $\theta$ motion of the gantry, and the $\phi$ motion of the base turntable are synchronized. This provides continuous rotation in polarization and produces a spiral sampling path over the sphere rather than cuts which are constant in $\phi$ or $\theta$.

FIGURE 5.19 Positioner configuration in which source antenna is supported by gantry to provide rotation in $\theta$. 
FIGURE 5.20 Positioner configuration for testing small antennas over ground-plane.
In the positioner configuration of 5.16a the antenna under test was held fixed in position, while the source antenna was moved in \( \phi \) and \( \theta \) to permit measurements over the sphere. In the arrangements of Figures 5.16b through 5.20, the antennas under test were rotated to produce coverage in \( \phi \), while the source antennas were moved to produce coverage in \( \theta \).

The discussion which follows relates to coordinates for conventional antenna test ranges, in which the line of sight OT between a source antenna and an antenna under test remains fixed in direction (usually horizontal or nearly so) while the antenna under test is oriented to simulate movement of the line of sight in the test coordinate system.

Two positioner configurations, commonly referred to as azimuth-over-elevation and elevation-over-azimuth, are employed in ranges of this type. Both accomplish the same function but differ in the physical orientation of the coordinate system and in the practical limits of rotation in \( \phi \) and \( \theta \). In addition to the two axes of the basic configurations, one or more axes are often employed to provide versatility in making antenna measurements. These configurations are also discussed.

Azimuth-over-elevation and elevation-over-azimuth positioners are shown in Figures 5.21 and 5.22. The azimuth-over-elevation arrangement would be more aptly designated the horizontal \( \theta \)-axis configuration, while the elevation-over-azimuth arrangement would be called the vertical \( \theta \)-axis configuration. The designations azimuth-over-elevation and elevation-over-azimuth are quite descriptive of the positioners which are employed but lead to confusion in defining the axes of the spherical coordinate system. While we will employ the azimuth-elevation terminology in describing positioner types, we will refer to the positioner axes as they are employed in the spherical coordinate system.

In both arrangements, the line OT is fixed in direction; the axis OA (the \( \theta \) axis) is also fixed in direction and must be set normal to OT, as in Figure 5.15. Line OZ (the \( \phi \) axis) is normal to OA and therefore describes a plane as the lower (\( \theta \)) axis rotates. This plane contains the line OT.

Study of Figures 5.15, 5.21 and 5.22 indicates the following requirements for any antenna positioner in a test configuration with the direction OT fixed in space:
(1) The positioner must have two orthogonal axes if true $\phi$ and $\theta$ cuts are to be made.

(2) One axis must be fixed in direction normal to $OT$. This axis is the $\theta$ axis and is the lower of the two axes of the basic positioner.

(3) The upper axis, which is mounted on the lower axis, is the $\phi$ axis.

**FIGURE 5.21** Coordinate system defined by azimuth-over-elevation positioner. The $\theta$ axis is defined by the lower positioner, while the $\phi$ axis is defined by the upper positioner. The $\theta$ axis thus remains fixed in direction while the $Z$ axis moves in the plane containing $OT$. 

5-19
When the geometry is established, the axis of the upper turntable becomes the Z-axis of the spherical coordinate system. It should be understood that the Z-axis of the coordinate system is established by the geometry of the two-axis positioning system and cannot be arbitrarily changed.

![Coordinate system defined by elevation-over-azimuth positioner.](image)

**FIGURE 5.22.** Coordinate system defined by elevation-over-azimuth positioner. As in the azimuth-over-elevation configuration, the θ axis is defined by the lower positioner while the φ axis is defined by the upper positioner. The θ axis thus remains fixed in direction, while the Z axis moves in the plane containing OT.

Consideration of the positioners shown in Figures 5.21 and 5.22 also shows that, other than the orientation of the coordinate system, the major practical difference in the two configurations is in the relative coverage of the sphere which is afforded. For example the azimuth-over-elevation configuration provides coverage of 360 degrees in φ, but incomplete coverage in θ because of the interference of the side-members which support the elevation shaft. The elevation-over-azimuth configuration gives 360 degrees of rotation in θ, but limited coverage in φ for the same reason. In many cases it is necessary to
isolate the test antenna from nearby sources of reflection. This is particularly true of model-antenna measurements which are concerned with antennas of low directivity. This requirement leads to the use of the model-tower configuration. A model tower normally consists of a dielectric mast with a rotary "head", mounted on an azimuth turntable as illustrated in Figure 5.23. The coordinate system used with a model tower is the vertical \( \theta \)-axis system (elevation-over-azimuth) and is identical to that illustrated in Figure 5.22 except that complete, rather than partial, coverage in \( \phi \) is provided.

FIGURE 5.23 Model tower and associated coordinate system. The model tower is an example of the elevation over azimuth or vertical \( \theta \) positioner.
Auxiliary Axes -- To provide versatility and increase the overall capability of an antenna test facility, it is often desirable to have one or more auxiliary axes in addition to the two axes needed for the fundamental $\phi$, $\theta$ cuts. Figure 5.24 shows an auxiliary axis added to an azimuth-over-elevation positioner to provide a convenient method of rolling the antenna about its axis. On an outdoor elevated range, where ground reflections represent a problem, it is normally undesirable to direct the main beam of the antenna down where the sources of reflection are coincident with the main beam. The auxiliary axis permits examination of the lower hemisphere of the antenna pattern by directing the main beam upward; then a 180 degree roll by the polarization axis permits examination of the upper hemisphere of the pattern without the necessity of pointing the main beam toward the ground.

![Auxiliary Axis Diagram](image)

FIGURE 5.24 Auxiliary axis which provides a convenient method of rolling an antenna under test about its own axis.

The auxiliary positioner for this application is usually called a polarization positioner, although it does not generate a polarization pattern except when its axis is directed toward OT. It would be more properly designated an axis-reversal positioner. An additional use of this axis is for making principal plane cuts (See page 5-11).
Figure 5.25 is a simple sketch of a four-axis positioner. This positioner arrangement consists of an azimuth-over-elevation positioner mounted over a second azimuth axis, with the complete positioner mounted on a "tilt-axis" which has a travel of a few degrees. The positioner can be employed as an elevation-over-azimuth (vertical θ-axis) positioner using the lower-azimuth and elevation axes as shown in Figure 5.22. In this case the upper azimuth axis serves as a polarization axis. The tilt axis serves to orient the axis OA (the θ-axis) of Figure 5.22 normal to the line OT. In the sketch the source antenna is shown in two different locations, requiring different adjustments of the tilt axis.

Alternatively the positioner can be employed in the model-tower mode. The upper azimuth axis serves as the horizontal ϕ-axis to give 360-degree rotation in ϕ. The lower azimuth axis serves as the θ-axis. The elevation axis is adjusted to make the ϕ and θ axes orthogonal and thereafter remains fixed; thus it does not serve as one of the axes defining the coordinate system. The tilt axis is employed as in the previous paragraph.

Finally, the positioner can be employed as an azimuth-over-elevation positioner to define the coordinate system shown in Figure 5.21. In this case the lower azimuth axis is used (1) to orient the elevation axis (axis OA of Figure 5.21) perpendicular to the direction OT, and (2) to increase the θ-coverage of the sphere provided by the azimuth-over-elevation positioner configuration. This latter is accomplished by changing the orientation of the lower-azimuth axis through 180 degrees, such that OA is reversed in direction relative to OT.
Rectangular Coordinate Positioners -- Measurement of the amplitude and phase of fields over planar surfaces such as the apertures of narrow-beam antennas requires a positioner of the general type shown in Figure 5.26. Positioners of this type usually have two sets of precision guide rails to provide motion of the probe along orthogonal axes in the plane.

In the positioner shown, motion along both axes is provided by d-c motors and the position of the probe is indicated by means of synchros. Travel of the probe is 8 feet along each axis.

The probe is attached to the small carriage. Since it is convenient to have the positioning device rigidly mounted, adjustments in the Z direction are usually accomplished by moving the antenna under test in the direction normal to the XY plane generated by the movement of the two carriages. A positioner system described above to measure phase requires special attention to the signal cable which must transmit the signal from the moving probe to the non-moving instrumentation equipment. Rigid coaxial cables or waveguide mounted on a scissors or four-bar linkage with coaxial or waveguide rotary joints at the intersections have been found to be acceptable.

5.5 COORDINATE SYSTEMS FOR RADOME BORESIGHT TESTING

Measurements of radome boresight shift are made to determine the angular deviation of a radar beam as it passes through a radome. Radome boresight testing requires careful consideration of coordinate systems in order to perform the test in accordance with system requirements and existing standards. The test fixture must be constructed so that the correct physical relationship between the antenna and the radome is maintained while the radome electrical parameters are being measured. If the proper axis order is not provided by the test positioner, discrepancies in the simulation of the relative orientation of the radome and its associated antenna will result.

Techniques employed in radome boresight testing are described in Chapter 12. The discussion in this section will be concerned particularly with the coordinate systems and positioners which are employed in the measurements.
FIGURE 5.26 Rectangular coordinate positioner for phase, amplitude measurements. The scissors linkage carries rigid coaxial line and rotary joints.
Antenna pattern measurements positioners change the relative orientation between the test antenna and a source antenna. Radome boresight measurements are performed by changing the relative orientation between the radome and the line of sight between a fixed test antenna inside the radome and a source antenna.

![Diagram of aircraft search and navigation radar](image)

**FIGURE 5.27** Typical aircraft search and navigation radar, enclosed in radome.

The axes of aircraft search and navigation radars are usually identified as azimuth and tilt as shown in Figure 5.27. The system shown is similar to an inverted altazimuth tracking system (see Figure 5.4) with the coordinate system fixed relative to the airframe and radome so that the tilt and azimuth axes measure spherical coordinate angles $\theta$ and $\phi$, respectively. Search and navigation radars usually have asymmetrical radiation patterns of the cosecant-squared variety to provide heavy illumination of the ground in normal flight attitudes.

In making boresight measurements on such systems it is desirable to keep the main beam from illuminating the range surface. To accomplish this the radome and the reflector are usually inverted. A typical positioner for testing radomes for systems of this type is shown in Figure 5.28.
FIGURE 5.28  Aircraft radome mounted in inverted position on radome boresight positioner. Reflector is fixed in position and supported from the floor by an independent tubular structure inside the radome support column to isolate the beam from deflections as radome is rotated.
The determination of a positioner coordinate system which faithfully reproduces the relative motion between the radome and antenna is sometimes a source of confusion and error in radome measurements. Two basic steps are required in the definition of the proper coordinate system. These are (1) establishment of the correct axis order between the axis of the antenna beam and the radome, and (2) establishment of the correct physical positions of the axes relative to the antenna and radome. The schematic representation of Figure 5.29 illustrates the relationship of the operational system to the test system. In the operational system (a) the radome is the fixed reference and the antenna changes in direction with azimuth and tilt angles. In the test system (b) the antenna is fixed in position and the radome changes in orientation to simulate the azimuth and tilt angles. In the aircraft system the azimuth ($\phi$) axis is fixed parallel with the yaw axis of the airframe and the tilt ($\theta$) axis is normal to both the beam axis and the $\phi$ axis.

From the fixed reference the axis order in the operational system (Figure 5.29a) is (1) radome, (2) azimuth, (3) tilt, and (4) beam axis. Examination of the boresight positioner system of Figure 5.29 shows that from the fixed reference the axis order is (1) beam axis, (2) tilt, (3) azimuth, and (4) radome.

FIGURE 5.29  Schematic illustration showing relationship of radome boresight positioner to operational system. Both systems are shown in inverted positions.
Although the axis order from the radome to the beam axis is the same in both systems, the axis order from the fixed reference is inverted. This is because the fixed reference is the radome in the operational system while it is the beam axis in the test system. The above satisfies requirement (1). Requirement (2) must be satisfied in the mechanical design of the positioning system. The design must also allow for exercise of the radome through the required azimuth and tilt angles.

Some aircraft radar installations have the two-axis gimbal system installed on a third axis to provide an additional degree of stabilization of the coordinate system relative to earth. Such a system would require an additional axis of rotation in the radome boresight fixture. The roll ring shown on the boresight fixture, Figure 5.30, permits the radome to be rolled about its longitudinal axis. (See also Chapter 12.)
FIGURE 5.30  Radome boresight test positioner with roll ring.
As was discussed in Section 5.2, the relative orientation between two orthogonal, three-space coordinate systems is commonly described in terms of three Euler angles, which are usually designated \( \alpha, \beta, \) and \( \gamma \); the Euler angles are derived in Appendix 5A. Definition of the antenna range coordinate systems of Figures 5.21 and 5.22 was made in the previous section simply with reference to two orthogonal axes other than the XYZ axes of the coordinate system. These were the \( \theta \)-axis and the line OT. Reference to Figure 5.31 shows that these two axes, with a third orthogonal axis \( \overline{OZ} \), define an earth-fixed coordinate system. We will designate this system \( X \, Y \, Z \), where \( OX \) is identical with \( OT \) and \( OY \) is identical with \( OA \).
The angles \((\phi, \theta)\) describing the orientation of OT in the XYZ coordinate system (defined by the antenna under test in its position on the antenna positioner, as shown in Figure 5.21) are described in terms of the relative orientations of the XYZ and \(\bar{X} \bar{Y} \bar{Z}\) coordinate systems.

Consider the equations

\[
\theta = \cos^{-1}\left[ \cos \bar{\theta} \cos \beta + \sin \bar{\theta} \sin \beta \cos (\phi - \alpha) \right] \tag{5.1}
\]

and

\[
\phi = \tan^{-1}\left[ \frac{\sin \bar{\theta} \sin (\phi - \alpha)}{\sin \bar{\theta} \cos \beta \cos (\phi - \alpha) - \cos \bar{\theta} \sin \beta} \right] - \gamma \tag{5.2}
\]

which are derived in Appendix 5-A. Inspection of Figure 5.31 shows that the line OT (OX) is in the direction \((\phi = 0, \phi = 90)\) of the \(\bar{X} \bar{Y} \bar{Z}\) coordinate system and that \(\alpha = 0\), since \(X^{\prime}\) is coincident with \(\bar{X}\). Substituting these values in equations (5.1) and (5.2) gives the corresponding values of \(\phi, \theta\):

\[
\theta = \cos^{-1}[\sin \beta] \tag{5.3}
\]

or

\[
\theta = \beta - 90^\circ
\]

and

\[
\phi = -\gamma \tag{5.4}
\]

In the following section the Euler angle approach is employed in analyzing errors which occur from misalignment in antenna measurement systems.

5.7 ANGLE ERRORS IN ANTENNA MEASUREMENTS

Angle measurement errors from a number of sources must be considered in determining the accuracy of an antenna range. For the purposes of this analysis, the error sources can generally be included in one of the following classifications:

(1) Geometric Error,
(2) Shaft-Position Error,
(3) Deflection Error.
Additional direction errors which can be caused by phase and amplitude variations in the field over the test aperture, reflections, and parallax are discussed in other chapters.

**Geometric Error** -- If the positioners of Figures 5.21 and 5.22 were geometrically perfect, the earth-fixed positioner axis OA would be exactly normal to the positioner axis OZ, and the direction (OT) from the positioner to the source antenna would be exactly normal to OA. The geometrically perfect antenna test system would also have an antenna installed on the positioner turntable so that the coordinate system of the antenna would be exactly aligned with the coordinate system of the positioner.

An actual antenna range consists of physical components that may approach the above requirements, but geometric and mechanical errors will always exist. Every antenna measurement system will have three separate geometric errors, which can be identified and described as follows:

1. **Coordinate axis alignment error** -- Improper alignment of the antenna coordinate system with the antenna positioner coordinate system.
2. **Orthogonality error** -- Non-orthogonality of the two motion axes of the antenna positioner.
3. **Collimation error** -- Non-orthogonality of the θ-axis (OA) with the direction OT to the signal source.

The analysis in this section will not present arguments relating to the interdependence of one geometric error upon the others. For most measurement applications sufficient accuracy will be achieved by calculating the three geometric-error effects separately, and the following calculations will treat them as if they were independent functions.

**Coordinate Axis Alignment Error** -- In order to calculate the errors that are caused by coordinate axis misalignment it is convenient to stipulate that the positioner and range be mechanically perfect in all other respects. That is, it will be assumed that the two motion axes are exactly orthogonal and that the direction to the source antenna is exactly orthogonal to the OA axis.
Examination of Figure 5.21 shows that the polar axis of the antenna coordinate system is to be coincident with the $\phi$ axis of the positioner. Since the antenna coordinate system is defined with respect to a mechanical reference on the antenna while the $\phi$ axis is mechanically related to the positioner, some misalignment between the two will always exist in practice. The misalignment can be described and the errors analyzed by the use of Euler angles (see Appendix 5A).

Figure 5.32 illustrates an antenna range installation in which an antenna coordinate system (XYZ) is positioned and fixed to a positioner turntable such that a misalignment error exists. The direction $\phi, \theta$ is the direction to the source transmitter in the antenna coordinate system (XYZ).

![Figure 5.32](image)

**FIGURE 5.32** A direction described in two coordinate systems illustrating misalignment error.
The polar (φ) axis of rotation is OZ of the positioner turntable coordinate system X Y Z, and the direction φ, θ is the direction of the source transmitter in the positioner coordinate system.

The antenna coordinate system XYZ is misaligned from positioner coordinate system X Y Z by the Euler angles α, β, and γ.

In the typical antenna measurements situation signal amplitude is required as a function of φ, θ. However, the antenna range equipment (positioner, position indicators, recorder) will measure signal amplitude as functions of φ, θ.

Given the Euler angles α, β, and γ and the direction φ, θ, the direction φ', θ may be calculated by equations (5A.9) and (5A.2) of Appendix 5A:

\[ \phi = \tan^{-1} \left( \frac{\sin \theta \sin(\phi - \alpha)}{\sin \theta \cos \beta \cos(\phi - \alpha) - \cos \theta \sin \beta} \right) - \gamma \]  

and

\[ \theta = \cos^{-1} \left[ \cos \beta \cos \theta + \sin \beta \sin \theta \cos(\phi - \alpha) \right] \]  

**Orthogonality Error** — A positioner with an orthogonality error is diagrammed in Figure 5.33. For this case it is stipulated that all other characteristics of the geometry of the system are perfect. The coordinate system of the antenna coincides perfectly with the positioner coordinate system, and the direction to the source antenna is exactly normal to the mechanical θ-axis, which will be designated OA'. The polar (φ) axis of the antenna positioner is OZ. The mechanical θ-axis OA', which should be coincident with OA, is displaced by an angle δ from the XY plane.

If the geometry were perfect, the antenna coordinate system XYZ could be rotated about OA so that the direction OT to the source antenna would be in the plane OMZ. However, due to the orthogonality error, rotation about the θ axis will position OT in the plane OMZ'.

The angles or directions which will be indicated by the instrumentation equipment are φ, θ, where φ is the rotation about the positioner polar (φ) axis and θ is the rotation about the positioner θ axis.
In order to position the direction OT as shown in Figure 5-33, OT can be considered to have moved from OX to OM by rotation about the OZ axis, then from OM to OT by rotation about the OA' axis. Therefore, the direction of OT is the angle between planes XOZ and MOZ.

Since $\bar{\phi}$ is a measure of the mechanical rotation about OA' it is an angle in plane MOZ', which is the plane of the great circle that is normal to OA'.

The spherical triangles utilized in the solution are shown in Figure 5.34. It can be seen that the spherical angles ZX, ZM, ZM' and Z'M are all equal to 90 degrees by definition or construction. Also the spherical angles ZZ'T and TM'M are equal to 90 degrees. Angle $\delta$ is the orthogonality error.

From the right spherical triangle Z'ZT

$$\theta = \cos^{-1} (\cos\bar{\phi} \cos\tilde{\phi})$$

and

$$\sin^2 \theta = 1 - \cos^2 \delta \cos^2 \bar{\phi}.$$ 

(5.5)  

(5.6)
From the right spherical triangle MM'T

\[ \cos \Delta \phi = \frac{\cos(90^\circ - \phi)}{\cos(90^\circ - \theta)} , \]

or

\[ \cos \Delta \phi = \frac{\sin \theta}{\sin \phi} . \]  (5.7)

Substituting from equation (5.6) gives

\[ \cos \Delta \phi = \frac{\sin \theta}{(1 - \cos^2 \delta \cos^2 \phi)^{1/2}} \]  (5.8)

It can be seen from Figure 5.34 that

\[ \phi = \bar{\phi} + \Delta \phi , \]

therefore

\[ \phi = \bar{\phi} + \cos^{-1} \left( \frac{\sin \theta}{(1 - \cos^2 \delta \cos^2 \phi)^{1/2}} \right) . \]  (5.9)

\[ \text{FIGURE 5.34 Spherical triangles of Figure 5.33.} \]

Since the evaluation of equation (5.9) is difficult when \( \delta \) is small, the substitution of identities provides a more convenient form. It can be shown that equation (5.9) is identical to

\[ \phi = \bar{\phi} + \sin^{-1} \left( \frac{\sin \delta \cos \theta}{(1 - \cos^2 \delta \cos^2 \phi)^{1/2}} \right) . \]  (5.10)

thus for small \( \delta \) where \( \cos^2 \delta \approx 1 \),

\[ \phi = \bar{\phi} + \sin^{-1} \left[ \sin \delta \cot \theta \right] . \]  (5.11)

Collimation Error - - Collimation error is that error which exists when the direction to the source antenna is not normal to the \( \theta \) axis. The effect of collimation error is shown in Figure 5.35. In this analysis the coordinate axes of the antenna are coincident with the coordinate axes of the positioner turntable and the positioner \( \theta \) axis (OA) is exactly normal to the positioner \( \phi \) axis. Collimation error is identified in Figure 5.35 as angle \( \epsilon \). When the antenna coordinate system XYZ is moved by rotation
of the \( \theta \) axis, OT sweeps a conical surface \( OZ'M' \). The arc \( Z'M' \) is a portion of a small circle which is parallel to the great circle (ZM) of axis OA. Arc \( TT' \) is a portion of a great circle that intercepts point A. It can be seen that an antenna positioner with collimation error cannot position OT coincident with the OZ-axis; \( OZ' \) is the closest approach of OT to the OZ-axis.

The zero position for \( \phi \) will be defined as that position of the mechanical \( \phi \)-axis that will align OT with the coordinate axis OX. This occurs when OA is at \( OA' \), an angle \( \epsilon \) from Y. The angle \( \theta \) is again defined as rotation of the axis OA; the zero position for \( \theta \) is that position which establishes coincidence of OT with \( OZ' \).

The spherical triangles of Figure 5.35 which are utilized for the solution of angles \( \phi, \theta \) are redrawn in Figure 5.36. By construction, \( TT'Z \) is a right spherical triangle with the 90-degree spherical angle at \( T' \), and \( MM''Z \) is a right spherical triangle with 90-degree spherical angles at both \( M \) and \( M'' \).

---

FIGURE 5.35 Antenna positioner with collimation error.
In Figure 5.36 it can be seen that

$$\epsilon = ZZ' = TT' = MM'$$

and

$$\phi = \overline{\phi} + MM''$$

or

$$\phi = \overline{\phi} + MM'' - \epsilon, \quad (5.12)$$

where \(\epsilon\) is positive when angle \(A'OT\) is less than 90°.

Further

$$\tan \sigma = \frac{\tan \epsilon}{\sin \theta},$$

and since \(\sigma = MM''\),

$$MM'' = \tan^{-1} \frac{\tan \epsilon}{\sin \theta}$$

which may be substituted in equation (5.12) to give

$$\phi = \overline{\phi} - \epsilon + \tan^{-1} \frac{\tan \epsilon}{\sin \theta}. \quad (5.13)$$

From the conventional relation between the sides and the angles of the right spherical triangle \(TT'Z\).

$$\theta = \cos^{-1} \left[ \cos \overline{\theta} \cos \epsilon \right]. \quad (5.14)$$

Summary of Geometric Error Calculation — The calculation for a direction \(\phi, \theta\) of an antenna having geometric misalignment errors are summarized as follows:

1. Coordinate Axis Misalignment (FIGURE 5.32)

   $$\phi = \tan^{-1} \left[ \frac{\sin \overline{\theta} \sin(\overline{\phi} - \alpha)}{\sin \overline{\theta} \cos \beta \cos(\overline{\phi} - \alpha) - \cos \overline{\theta} \sin \beta} \right] - \gamma \quad (5A. 9)$$

   $$\theta = \cos^{-1} \left[ \cos \overline{\theta} \cos \beta + \sin \overline{\theta} \sin \beta \cos(\overline{\phi} - \alpha) \right] \quad (5A. 2)$$
(2) Orthogonality Error (Figure 5.33)
\[
\phi = \frac{\overline{\phi} + \sin^{-1} \left( \frac{\sin \delta \cos \overline{\theta}}{1 - \cos^2 \delta \cos^2 \overline{\theta}} \right)}{2} \tag{5.10}
\]
\[
\theta = \cos^{-1} (\cos \delta \cos \overline{\theta}) \tag{5.5}
\]

(3) Collimation Error (Figure 5.35)
\[
\phi = \overline{\phi} - \epsilon + \tan^{-1} \left( \frac{\tan \epsilon}{\sin \theta} \right) \tag{5.13}
\]
\[
\theta = \cos^{-1} \left[ \cos \overline{\theta} \cos \epsilon \right] \tag{5.14}
\]

**Shaft Position Error** -- The shaft position angle of an antenna positioner is usually determined by synchro transmitters, resolvers or digital encoders. Shaft position error is the difference between the true shaft angle and the shaft angle as indicated by the encoder or synchro readout system. The angle measuring equipment of typical commercial positioners consists of geared synchro transmitters at ratios of 1:1 and 36:1 with respect to each axis. Over 360 degrees of axis rotation the readout error is typically 0.05 degree to less than 0.01 degree depending on the model. An improvement in accuracy usually results when a small angle is considered. At the current state of the art, direct-drive digital encoder systems are more accurate than geared synchro systems. The total shaft position error with an encoder installed consists of the encoder error, encoder housing deflection, encoder coupling error, and differential temperature effects.

Digital encoders are available with a wide range of accuracies and resolution and with a correspondingly wide range in price. Twenty-one bit encoders are in use in operational systems.*

* A resolution of 1 part in $2^{21}$ corresponds to a resolution of approximately 0.6 arc seconds.
Deflection Error -- Positioner deflection errors are caused principally by changes in the forces applied to the positioner turntables and by expansion and contraction of structural members due to differential temperatures. Temperature effects caused by uneven solar heating can be a major source of deflection error.

The magnitude of the solar radiation problem can be illustrated by the results of a series of tests that are described in reference 7. Precision, calibrated levels were utilized to measure the inclination of the top of a vertical steel cylinder as it deflected from unequal expansion due to solar heating. The walls of the steel cylinder were 3/8-inch thick, and the cylinder was approximately 240 inches long by 66 inches in diameter. The cylinder was rigidly fastened to a concrete foundation. The exterior surface was coated with red Rustoleum paint. The change in inclination across the north-south and east-west directions was measured at intervals during the day from sunrise and sunset, on a clear day, for which maximum temperature was 73 degrees Fahrenheit. As expected, the steel cylinder deflected away from the sun, leaning toward west in the morning, toward north at noon, and toward east in the afternoon. The maximum inclination, which occurred between 3 and 5 PM local time, was approximately 125 arc seconds. The maximum difference in inclination between morning and afternoon was approximately 225 arc seconds (over 0.06 degree).

To reduce the effect of errors due to solar deflection, insulated shields or barriers can be fabricated to cover the positioner support structure. Alternatively, electronic levels having remote reading indicators can be employed to monitor the magnitude of the deflections.

Large changes in the positioner bending moment applied at the antenna mounting surfaces will cause significant direction errors in a vertical plane. However, deflections caused by load stresses are predictable and can be calibrated to provide correction factors for the angles involved.
REFERENCES

CHAPTER 5


APPENDIX 5A
SPHERICAL COORDINATE TRANSFORMATION BY EULER ANGLES*

The relative orientation of two three-dimensional, orthogonal coordinate systems can be described by the three Euler angles $\alpha$, $\beta$, and $\gamma$. In the general case any relative orientation of the two coordinate systems can be achieved by three successive rotations about the coordinate axes. The rotations from a coordinate system $\overline{X\ Y\ Z}$ to a system $XYZ$ are (Figure 5A.1):

1. Rotation about the $\overline{Z}$-axis through an angle $\alpha$ to $X''Y''Z''$.
2. Rotation about the $Y''$-axis through an angle $\beta$ to $X'Y'Z'$.
3. Rotation about the $Z'$-axis through an angle $\gamma$ to $XYZ$.

FIGURE 5A.1 Coordinate axes rotated through the Euler angles $\alpha$, $\beta$, $\gamma$.

*The material presented in this appendix was developed under Contract AF30-(602)-3425, and was contained in Rome Air Development Center Report RADC-TR-65-534 dated February 1966.
Let the spherical coordinates $\phi$, $\theta$ designate a direction OP in the coordinate system XYZ of Figure 5A.2, and let the spherical coordinates $\overline{\phi}$, $\overline{\theta}$ designate the direction OP in the coordinate system XYZ. The direction $\phi$, $\theta$ may then be described in terms of $\overline{\phi}$, $\overline{\theta}$ and the Euler angles $\alpha$, $\beta$, and $\gamma$.

**FIGURE 5A.2** A direction described in two coordinate systems.

Figure 5A.3 shows the spherical triangles which are used in the solution of

$$\phi, \theta = f(\overline{\phi}, \overline{\theta}, \alpha, \beta, \gamma).$$

From the spherical triangle $Z\overline{Z}P$ and the law of cosines,

$$\cos \theta = \cos \overline{\theta} \cos \beta + \sin \overline{\theta} \sin \beta \cos (\overline{\phi} - \alpha),$$

(5A-1)
FIGURE 5A.3 Spherical triangles from Figure 5A.2.

or

\[ \theta = \cos^{-1}[\cos \bar{\theta} \cos \beta + \sin \bar{\theta} \sin \beta \cos (\bar{\phi} - \alpha)] . \]  

(5A-2)

And also by the law of cosines

\[ \cos \bar{\theta} = \cos \theta \cos \beta + \sin \theta \sin \beta \cos n . \]  

(5A-3)

It can be seen that

\[ n = \pi - (\phi + \gamma) , \]

and

\[ \cos n = \cos [\pi - (\phi + \gamma)] , \]

or

\[ \cos n = -\cos (\phi + \gamma) , \]  

(5A-4)

and

\[ \sin n = \sin (\phi + \gamma) . \]  

(5A-5)

Equation (5A-4) may be substituted into (5A-3) to give

\[ \cos \bar{\theta} = \cos \theta \cos \beta - \sin \theta \sin \beta \cos (\phi + \gamma) , \]  

(5A-6)
or

\[ \sin \theta \cos (\phi + \gamma) = \frac{\cos \theta \cos \beta - \cos \delta}{\sin \beta} \quad . \quad (5A-6) \]

From the spherical triangle \( Z \bar{Z} P \) and the law of sines,

\[ \frac{\sin (\phi - \alpha)}{\sin \theta} = \frac{\sin \pi}{\sin \theta} \quad . \]

Substituting from (5A-5) and rearranging gives

\[ \sin \theta \sin (\phi + \gamma) = \sin \theta \sin (\phi - \alpha) \quad . \quad (5A-7) \]

Dividing (5A-7) by (5A-6) gives

\[ \tan (\phi + \gamma) = \frac{\sin \theta \sin \beta \sin (\phi - \alpha)}{\cos \theta \cos \beta - \cos \delta} \quad . \quad (5A-8) \]

Substituting (5A-1) into (5A-8) and rearranging gives

\[ \phi = \tan^{-1} \left[ \frac{\sin \theta \sin \beta \sin (\phi - \alpha)}{\cos \beta [\cos \theta \cos \beta + \sin \theta \sin \beta \cos (\phi - \alpha)] - \cos \beta} \right] - \gamma \quad , \]

which may be simplified to

\[ \phi = \tan^{-1} \left[ \frac{\sin \theta \sin (\phi - \alpha)}{\sin \theta \cos \beta \cos (\phi - \alpha) - \cos \theta \sin \beta} \right] - \gamma \quad . \quad (5A-9) \]
6.1 BASIC CONSIDERATIONS

The antenna pattern is a graphical representation of the radiation properties of an antenna as a function of direction. Of the typical far field measurements made — directivity, gain, phase, polarization and pattern — the latter is the most commonly measured parameter. The other measurements are discussed in subsequent chapters.

In the far zone* of an antenna the field vectors are transverse to the direction of propagation, and their amplitudes vary inversely with distance from the source. The normal measurement procedure is to resolve these vector fields into two orthogonal components whose amplitude and phase can be described separately, at a fixed separation between source and test antennas. Those orthogonal components are then related by specifying their ratio as a function of angular coordinates. This ratio is a phasor, whose modulus gives the relative amplitude of the two components, and whose argument gives their relative phase. **

A measurement of the magnitude of the electric field intensity $E(\theta, \phi)$ of an electromagnetic field in free space is equivalent to a measurement of the magnitude of the magnetic field intensity $H(\theta, \phi)$ and vice-versa, since the two quantities are directly related by the expression $E = (120\pi)H$. Therefore the pattern could equally well be given in terms of $E$ or $H$.

* See Chapter 14.

** See Chapter 3.
If the radiation pattern is plotted in terms of the field strength in electrical units, such as volts per meter, or the power density in watts per square meter, it is called an absolute pattern. An absolute pattern in terms of constant-field-strength contours plotted on a geographic map, for example, is a most useful representation for a radio broadcast station, since it defines the geographic regions within which various received-signal levels are available to listeners.

Most often, however, antenna patterns are plotted in relative terms; i.e., the field strength or power density is represented in terms of its ratio to some reference value, generally the peak of the beam or maximum-field-strength. The normalized radiation pattern is a function of two variable angles $\theta$ and $\phi$, characterizing the angular distribution of the field produced in the far zone about the antenna.

Accurate measurement of radiation patterns involves the consideration of a number of factors, such as the physical and electrical size of the antenna, its operational frequency band, and the environment in which it is to operate. The most common environmental condition to be simulated is that of semi-infinite separation between transmitting and receiving antennas. Free-space measurements, in which the far field conditions for plane wave operation are simulated, are correspondingly the most common pattern measurements. Less often, probe antennas are stationed in the field about very large antenna installations, for which the surrounding terrain or environment makes it impractical to simulate the operational condition or to employ scale-model measurements. The concentration of interest in this chapter will be on free-space measurements.

The term free-space pattern describes a hypothetical concept, since in practice the measurement technique is influenced by the source-antenna characteristics, source and test supports, positioning equipment, cabling and nearby objects, in particular the range surface. However, as discussed in Chapter 14, the effects of such factors can be kept to an acceptable level by careful design of the test environment, or accounted for in the data interpretation.

6.2 PATTERN FORMATS

The manner in which measured antenna pattern data are presented for
observation and analysis represents one of the most important aspects of the study of techniques for antenna measurements. This is true primarily because of the large quantity of data which is typically required to adequately describe the performance of an antenna. In the following paragraphs various presentation techniques are described which have application to the antenna pattern measurements problem. For purposes of the discussion, an antenna pattern is considered to be a function of orientation angle \((\theta, \phi)\) for a single polarization, frequency, and set of static environmental conditions.

6.2.1 Three-Dimensional Graph – – – Because of the three-dimensional nature of an antenna radiation pattern, presentation of the data in solid form has often been suggested. Such a presentation suffers from several obvious disadvantages: (a) The difficulty of transmitting the data in report form and of storing the patterns, (b) the lack of adaptability to quantitative measurements, and (c) the problem of rapidly and accurately generating the data surface. Because of these disadvantages, three-dimensional graphs are seldom used and are not considered feasible for the general measurements problem.

6.2.2 Cuts of Relative Gain Versus Angle – – – An antenna pattern can be described by means of a family of two-dimensional graphs. The graphs are normally obtained by recording relative gain versus one of the angular coordinates. The orthogonal coordinate is held fixed during each cut and varied in increments between cuts. Measurements are made over angular regions defined by the application.

Since the pattern in a plane or appropriate conical surface involves only one variable angle, the pattern may be plotted in polar form (Figure 6.1) or rectangular form (Figure 6.2). Note that the polar plot permits a convenient visual interpretation of the spatial distribution of the radiation lobes. The rectangular display is often chosen for narrow beam antennas since the pattern can be expanded easily by changing the ratio of the chart speed with respect to the rate of rotation of the antenna positioner.

The three most commonly used ordinates in pattern formats are relative power, relative field, and logarithmic relative gain. * Each format has distinct

*See Chapter 2 for a discussion of decibel notation.
FIGURE 6.1  Polar logarithmic plot of a normalized radiation pattern.

FIGURE 6.2  Rectangular logarithmic plot of normalized radiation pattern.
advantages and disadvantages from the viewpoint of performance analysis, as discussed below.

Relative Power Patterns - - - Such patterns illustrate the variation of power density at a fixed distance from the antenna as a function of angular coordinates (Figure 6.3a). Power patterns are most useful in assessing small power variations between 100% and 10% of the peak value. Power variations in the range below 10% of full-scale are difficult to resolve without changing gain levels. Power patterns are used in calculating directivity, and sometimes in making measurements to determine half-power beamwidths of antennas because of the increased resolution afforded at the higher signal levels. Obviously, it would not be useful in measurement problems where low sidelobe levels and high front-to-back ratios are of interest.

Relative Field Patterns - - - This format shows the variation of the electric field intensity at a fixed distance from the antenna as a function of angular coordinates (Figure 6.3b). Field patterns provide greater sidelobe resolution while still displaying small variations in field level near the maximum. The half-power level on this format is 0.707 times the full-scale value.

Logarithmic (Decibel) Relative Gain Patterns - - - The value of the use of decibels in antenna pattern recording is largely based on two factors. First, if $n_1$ and $n_2$ are power ratios whose values in dB are $N_1$ and $N_2$, the product $n_1n_2$ is represented by $[N_1 + N_2]$ dB, and $n_1/n_2$ is represented by $[N_1 - N_2]$ dB. This permits the handling of products and quotients of large power ratios simply by the operations of addition and subtraction. Second, the decibel scale represents a compression of the power ratio scale, which is extreme for large ratios. This permits the display of very large power ratios on a single graph with equal resolution at all power levels (Figure 6.3c).

In addition, the normalization reference for a given pattern is arbitrary; the recording technique automatically normalizes the data to the maximum value for each individual pattern. Initial gain adjustments are not critical (other than precautions to allow for linear system response over the expected dynamic

*See Chapter 7.
range of the data; see Chapter 15), and known step changes in gain during a recording do not destroy the resolution, or alter the basic reference.

FIGURE 6.3 Power, field, and decibel plots of the same antenna pattern.
6.2.3 Contour Plot -- A complete radiation pattern can be presented on a single page by connecting points of equal signal level with lines to form isolevel contours in the manner employed for topographic contour maps. Key levels are numerically identified to permit interpretation of the graph.

The major problem in the use of this type of presentation is that it is extremely difficult to obtain the data for automatic preparation of the charts. It is not practicable to servo drive the antenna positioner to trace out the contours, and attempts to make contour plots by a raster-scanning process, in which data are printed at selected increments of signal level, have not been successful because of the difficulty in identification of the contours from the printed dots. Identifying signal levels with numbers is not practicable because of the crowding together and over printing of the numbers in regions of rapid signal level changes.

6.2.4 Radiation Distribution Table -- The radiation distribution table provides the same basic information as the contour plot, but accomplishes it in a manner which overcomes the major disadvantages described above. A portion of a radiation distribution table is shown in Figure 6.4. Signal levels are printed numerically in decibels at preselected intervals of \( \theta \) and \( \phi \). The graph shown was made by scanning in \( \phi \) and stepping in \( \theta \) after each \( \phi \) cut. In the figure, the angular increments are 0.5 degree in \( \theta \) and \( \phi \), and the dynamic range is 40 decibels. Signal levels are sensed by an encoder coupled to the servo-driven logarithmic potentiometer in an antenna pattern recorder, and printed on the data form by means of a modified typewriter.

The contour effect is obtained by printing the even values of signal level and omitting the odd values. The signal-level resolution is 1 decibel in the example shown; a printed level of 4 decibels indicates a level between 3.5 and 4.5 decibels. It should be noted that contours can be drawn which are generally more accurate than 0.5 decibel by interpolating in the regions where the pattern changes slowly. There are two major advantages of the radiation distribution table: (a) it provides data for assessment of the general configuration of a radiation pattern over a selected region on a single page, and (b) the table can be readily obtained by programming the antenna positioner through \( \phi \) cuts at successive increments of 0.
FIGURE 6.4 Radiation distribution table recorded by scanning in $\phi$ and stepping in $\theta$. Data are presented at 0.5-degree increments, over a 40-decibel dynamic range.
It should be noted, however, that (a) the resolution is discrete, (b) the fine details of deep, sharp nulls are not recorded, and (c) the configurations of individual signal level versus angle cuts are not as immediately evident from the radiation distribution table as they would be directly from a two-dimensional graph of a single cut.

A newer, experimental technique employs photographic techniques to record the radiation distribution table. The major advantages of the technique are its increased speed and resolution, and compact data storage on film.

For either of the radiation distribution table techniques, the coding of the signal levels and angular coordinates provides for a simple interface with paper tape or magnetic data storage devices. Subsequent computer analyses of antenna characteristics are thus convenient. Tape storage of data is particularly attractive in such test problems as directivity measurements (Chapter 7) and multiple-component polarization analysis (Chapter 3 and 10), and in design modification analyses. In production tests of antennas, pertinent specified parameters may be compared with pre-stored data.

Because of the importance of the problem of transferring data between missiles, or space vehicles, and land-based antennas, and between space vehicles, standards have been established by the Inter-Range Instrumentation Group of the Range Commanders Council for coordinate systems and data formats for antenna patterns, including radiation distribution tables, paper tape and magnetic tape formats.

6.3 FRESNEL REGION MEASUREMENTS

While there is no clear cut boundary between the Fraunhofer and Fresnel regions, the commonly accepted range-length criterion of \( \frac{2D^2}{\lambda} \) represents a reasonable far-zone distance for most pattern measurements, where \( D \) is the maximum dimension of the antenna aperture and \( \lambda \) is the operating wavelength. In practice, an unobstructed, open space suitable to make accurate pattern measurements with a length of \( \frac{2D^2}{\lambda} \) is not always available, or even practical. This leads to the problem of measurement of far-field patterns in the near field.

* See Chapter 14.
Two techniques are available. The first is by calculation—Fraunhofer radiation patterns may be calculated from phase and amplitude distributions measured in the Fresnel region. Accurate measurement of the amplitude and phase distribution over an antenna aperture requires extreme care in order not to cause untenable perturbation in the aperture distribution by the sampling probe in the field, and typically corresponds to stringent mechanical tolerances.

The second method of measuring Fraunhofer radiation patterns in the Fresnel region is by simulating the requirements of the Fraunhofer region in the Fresnel region. For this discussion attention will be restricted to reflector-type antennas where energy is focused on a primary feed. It is possible to focus energy in the near field by displacing the primary feed from the true focal point of the reflector.

The radiation pattern of a focused antenna, as measured in the Fresnel region, represents the sum of the Fraunhofer region pattern and a certain secondary field whose magnitude is zero in the principal direction (0°). It is this additional term that differentiates the radiation pattern measured in the Fresnel region from the radiation pattern measured in the Fraunhofer region. This additional term will be smaller the farther from the aperture the pattern is measured. We can represent the situation by the expression

\[ E' = E + \Delta E \]  \hspace{1cm} (6.1)

where \( E \) is the Fraunhofer pattern and \( \Delta E \) is the error term. For pencil-beam antennas it can be shown that \( \Delta E \) is proportional to \( \lambda/R \), where \( R \) is the test separation. For such antennas, the focused antenna radiation pattern measured in the Fresnel region is practically identical with the radiation pattern of the antenna measured in the Fraunhofer region or far-field. The patterns have been shown to be practically identical not only in the region of the main lobe, but also in the region of the minor lobes. (See Chapter 14, and references 3–9 at the conclusion of this chapter.)

Several methods of calculating the amount of displacement that is required to focus antennas in the Fresnel region have been suggested in the literature.

The Geometrical Approach - - - It is a well known fact that the best radiation
pattern from a paraboloid can be obtained in the far-field if the feed is located at the focal point. Geometrically, this is equivalent to equal path lengths from the feed to all points in an aperture plane by virtue of the inherent property of a focused paraboloid. If the field point is far enough away from the reflector, the path lengths from the aperture points to the field point will be again approximately equal, resulting in an optimum additive effect. When the field point lies in the Fresnel region, the path length differences from the points in the aperture plane to the field point must be compensated in some way in order that the measured radiation pattern may approach the true Fraunhofer pattern. This is done by slightly defocusing the source (feed) along the reflector axis in the direction away from the reflector. Since the amount of on-axis defocus is the only adjustable variable, one cannot expect to achieve equal path lengths for all points in the aperture plane.

For simplicity, one approach is to make the path lengths from the source (feed) to the field point by way of the apex of the paraboloid equal to the path length by way of the points on the edge of the reflector. The above requirement is equivalent to movement of the feed away from the reflector through a distance

$$\epsilon = \frac{f^2}{R} \left[ \left( \frac{R}{R-f} \right)^2 + \left( \frac{D}{4f} \right)^2 \right] , \quad (6.2)$$

where $f$ is the focal length of the paraboloid.

When $(f/R)^2 \ll 1$, (6.2) may be approximated as

$$\epsilon = \frac{f^2}{R} \left[ 1 + \frac{f}{R} + \left( \frac{D}{4f} \right)^2 \right] . \quad (6.3)$$

The amount of defocus needed is seen to increase when $R$ decreases and when $D$ increases. As $R$ approaches infinity, $\epsilon$ correctly goes to zero.

**Aperture Phase Approach** - - - The defocusing problem can also be approached from a consideration of the phase distribution in an aperture plane of the reflector together with the diffraction integral for the field at a point in space. When the point under consideration is in the quasi-near zone of a paraboloidal reflector, at a distance $R$ from the aperture, and when the primary source (feed)
is displaced from the focus of a paraboloidal reflector along the reflector axis in the direction away from the reflector, the relative path-length variation over the aperture can be approximated satisfactorily by

\[ \delta = 2\epsilon \left[ 1 - \frac{\rho^2}{(4f)^2} \right], \]  

(6.4)

where \( \rho \) is normalized radial dimension in the aperture. Equation (6.4) is exact for \( r = 0 \) (center) and \( r = 1 \) (edge of aperture). By forcing the argument of the integrand in the diffraction integral to vanish, this approach yields

\[ \frac{\epsilon}{f} = \frac{f}{R} \left[ 1 + \left( \frac{D}{4f} \right)^2 \right], \]  

(6.5)

which agrees with equation (6.3) when \( \frac{R}{f} \gg 1 \). It should be pointed out that for \( R = 2D^2/\lambda \), appreciable defocus is still necessary.

Ellipsoidal-Reflector Approach - - - The purpose of defocusing the primary source in the case of a paraboloidal reflector is to simulate far-zone radiation patterns at points in the Fresnel zone. In terms of geometrical optics, it is quite easy to see that this could be achieved by means of an ellipsoidal reflector. If the primary source is placed at one of the two foci of an ellipsoidal reflector, the reflected rays will converge at the other. For foci at \( f_1 \) and \( f_2 \), as measured from the vertex, with \( f_1 \) near the reflector and \( f_2 \) at the test separation, the ellipsoidal reflector approximates a paraboloid of focal length

\[ f = \frac{f_1 f_2}{f_1 + f_2}, \]  

(6.6)

for \( \frac{D^2}{4f_1 f_2} \ll 1 \). Therefore, for reflected rays to converge at \( R = f_2 \), the primary source should be placed such that

\[ \epsilon = (f_1 - f) = \frac{f^2}{R - f}. \]  

(6.7)
Equation (6.7) should be compared with both equations (6.3) and (6.5).

**Comparison of Defocusing Methods** - Curves plotting ε/f vs R/f based upon equations (6.3), (6.5), and (6.7) from the three different approaches discussed above are shown in Figure 6.5. It is seen that except for small values of R/f, the required ε/f from the geometrical approach and from the aperture-phase approach are nearly the same, both of which increase with increasing D/f. The required ε/f from the ellipsoidal-reflector approach is the smallest of the three methods and is independent of D/f.

It is believed that ε in equation (6.7), derived from the ellipsoidal-reflector approach, gives the most nearly correct results because the approximation implied by equation (6.6) is very good; it does not restrict its correctness only to the edge of the reflector.

It should be pointed out that the required amount of defocus in all three methods is independent of the operating frequency and that diffraction phenomena are neglected.

### 6.4 Modeling Techniques

The main motives for scale modeling antennas are to obtain improved control over the conditions of measurement, or cut measurement costs. Antennas for ships, aircraft and space vehicles often fall into this category. Physically large antennas are also often modeled.

Although the model is usually smaller than the original, a model may be either larger or smaller provided that the following requirements for exact simulation are satisfied:

\[ \epsilon_\mu \frac{f_m^2}{f_r^2} = n_\epsilon \epsilon_f \mu_f f_r^2 \]  \hfill (6.8)

and

\[ \sigma_\mu \frac{f_m^2}{f_r^2} = \sigma_\epsilon \epsilon_f \mu_f f_r \]  \hfill (6.9)

where
FIGURE 6.5 Focusing displacements of primary feed for paraboloidal reflector tested at finite range.
\( \epsilon \) = dielectric constant
\( \sigma \) = conductivity
\( \mu \) = permeability
\( f \) = frequency
\( n \) = an arbitrary constant which determines the size of the model.

Subscript \( M \) refers to the parameters of the model, and \( F \) to the parameters of the full-scale system.

The quantities \( \epsilon_M, \sigma_M, \mu_M, \) and \( n \) for the model may be chosen at will, provided the above equation is satisfied and the media are linear. If the full-scale system and the model system are to be measured in air then

\[ \epsilon_M = \epsilon_F \]

and

\[ \mu_M = \mu_F. \]

Therefore,

\[ f_M = nf_F \]

and

\[ \sigma_M = n\sigma_F. \]

It is impossible to satisfy the requirements of the model conductivity if the full-scale system is constructed of good conductors like copper or aluminum. However, the effects of inaccurate simulation of model conductivity will be small if good conductors are used in both the full-scale and model system. Mott and others \(^{14}\) have investigated some aspects of inexact modeling techniques.
REFERENCES
CHAPTER 6


2. (a) "Missile Antenna Data Requirements," AFETRM 80-5; June 1965.
(b) "IRIG Standard Coordinate System and Data Formats for Antenna Patterns," IRIG Document 111-65, AD 637-189; May 1966.


CHAPTER 7
MEASUREMENT OF DIRECTIVITY
J. S. Hollis and R. E. Pidgeon, Jr.

7.1 INTRODUCTION

The directivity $D(\phi, \theta)$ of an antenna measures the distribution in space of the energy which is radiated by the antenna. The directivity is defined as the ratio of the radiation intensity $\Phi(\phi, \theta)$ (the power radiated per unit solid angle in a given direction) to the average radiation intensity. Thus,

$$D(\phi, \theta) = \frac{\Phi(\phi, \theta)}{\Phi_{av}} .$$  \hspace{1cm} (7.1)

The equation for directivity may also be written

$$D(\phi, \theta) = \frac{P_t'}{P_t} (\phi, \theta) ,$$  \hspace{1cm} (7.2)

where $P_t$ is the total power radiated by the test antenna, and $P_t'$ is the power that would have been radiated by an isotropic source to give the same radiation intensity as that exhibited by the test antenna in the direction of interest $(\phi, \theta)$. The two definitions are equivalent; we will employ that represented by 7.1.

*See Chapter 2. Also see Chapter 5 for definition of spherical coordinate system.

**In this discussion as in the remainder of this text the unit of solid angle will be taken as the steradian.
The gain of an antenna is the product of the directivity times the antenna efficiency \( \eta \), that is,

\[
G(\phi, \theta) = \eta D(\phi, \theta),
\]

(7.3)

where the antenna efficiency \( \eta \) is the ratio of the power radiated into free space to the power accepted at the antenna terminals.

To obtain a meaningful measurement of the radiation characteristics of an antenna it is essential to duplicate those features of the actual operating environment which can significantly affect the characteristics in question. Certain types of antennas, especially low-gain antennas mounted on air frames or space frames, are not suited for the direct measurement of radiation characteristics. These antennas and their mounts must be simulated by models of a reasonable size. Proper interpretation of radiation measurements which are made on a correctly scaled model antenna can lead to an accurate evaluation of the characteristics of a full-scale system.

Pattern integration is a convenient method for making the necessary measurements to determine the gain of a model antenna. By performing pattern integration over a sphere surrounding the test model, one may gather sufficient data to compute the directivity of the antenna system. It is then possible to use the measured directivity of the model and the calculated efficiency of the full-scale system to compute the gain of the actual system. This process eliminates the need for compensating for the mismatch of the model antenna and the difference in efficiency between the model and the full-scale system, which are necessary in gain comparison methods.

7.2 EVALUATION OF THE DIRECTIVITY BY PATTERN INTEGRATION

To evaluate (7.1) for any \((\phi, \theta)\) direction it is convenient to determine \(D(\phi, \theta)\) for some reference direction \((\phi_r, \theta_r)\) and to determine the directivity in any general direction from the directivity pattern of the antenna. The directivity in the reference direction will be called \(D_r\). Usually \(D_r\) will be chosen in a direction such that \(D(\phi, \theta)\) is maximum. It will then be referred to as \(D_s\).
Since there are $4\pi$ steradians of solid angle over the sphere, (7.1) can be written for the reference direction

$$D_r = \frac{\Phi_r}{P_t/4\pi} = \frac{\Phi_r}{\int_0^{\pi} \left[\int_0^{2\pi} \Phi(\phi, \theta) \, d\phi\right] \sin \theta \, d\theta}.$$  \hspace{1cm} (7.4)

Since the radiation intensity may be contained in any two orthogonal polarizations, it is necessary to write (7.4):

$$D_r = \frac{\Phi_{r1}}{\frac{1}{4\pi} (P_{t1} + P_{t2})} + \frac{\Phi_{r2}}{\frac{1}{4\pi} (P_{t1} + P_{t2})} = D_{r1} + D_{r2},$$  \hspace{1cm} (7.5)

where

$$P_{t1} = \int_0^{\pi} \left[\int_0^{2\pi} \Phi_1(\phi, \theta) \, d\phi\right] \sin \theta \, d\theta$$  \hspace{1cm} (7.6)

and

$$P_{t2} = \int_0^{\pi} \left[\int_0^{2\pi} \Phi_2(\phi, \theta) \, d\phi\right] \sin \theta \, d\theta$$  \hspace{1cm} (7.7)

and where $D_{r1}$ and $D_{r2}$ are partial directivities (see Chapter 2).

To determine $D_r$, it is necessary to evaluate $D_{r1}$ and $D_{r2}$. This is accomplished by evaluating (7.6) and (7.7) and substituting in (7.5). Note that both (7.6) and (7.7) must be evaluated to determine either $D_{r1}$ or $D_{r2}$. The discussion which follows will be in terms of (7.4) for conciseness with the understanding that it applies to each component indicated in (7.5).

An initial step in applying equation (7.4) to an actual test situation is to derive from the antenna under test a signal which bears a direct relationship to the radiation intensity for a selected antenna polarization. Since it is not possible to measure $\Phi(\phi, \theta)$ continuously over the sphere, the integral must be approximated by sampling techniques in which the process of integration is replaced by a process of finite summation. The integral can be evaluated by digital,
electronic-analog or mechanical methods.

Since (7.4) represents a ratio of radiation intensities, it is not necessary to determine the value of $\phi$ in watts per steradian because $k\Phi$ can be substituted for $\Phi$ in both numerator and denominator, where $k$ is a constant. Recording numerator and denominator in a normalized form such as $\Phi_r/\Phi_z$ and $\Phi(\phi,\theta)/\Phi_z$, respectively, is tantamount to setting $k$ equal to $1/\Phi_z$.

The most commonly used procedure for obtaining the required test data is to position the antenna at successive equally spaced $\theta$ angles, and to make a revolution in $\phi$ for each $\theta$ angle, so that the data are taken in $\phi$ cuts. Cuts are made at equally spaced $\theta$ angles until the imaginary sphere surrounding the antenna has been covered, or until all regions of appreciable power flow have received complete coverage. Measurements are made in this manner for a selected antenna polarization and the orthogonal polarization.

A common technique for reducing the data, in cases where the volume of data involved does not make it prohibitive, is to integrate the measured $\phi$ patterns with a planimeter to determine numbers proportional to the integrals within the brackets of (7.6) and (7.7).

If the $\theta$ interval from 0 to $\pi$ is divided into $M$ equal sectors, and $\phi$ cuts are made at intervals of $\theta_i$ where $\theta_i = \pi i/M$ radians, (7.4) becomes

$$D_z = \frac{4\pi}{\sum_{i=1}^{M} \int_{\theta_0}^{\theta_1} \frac{\Phi(\phi,\theta)}{\Phi_z} d\phi} \sin \theta_i \Delta \theta \quad (7.8)$$

where $\Delta \theta = \pi/M$, giving

$$D_z = \frac{4M}{\sum_{i=1}^{M} \int_{\theta_0}^{\theta_1} \frac{\Phi(\phi,\theta)}{\Phi_z} d\phi} \sin \theta_i \quad (7.9)$$

*See page 7-8.
** See Appendix 7A.
where $D_z$ has been chosen in the direction of maximum directivity.

In evaluating the integral within the brackets with a planimeter, a multiplier constant must be employed which makes the integral equal $2\pi$ when $\Phi(\phi, \theta)$ is constant and equal to $\Phi_z$.

The pattern $\Phi(\phi, \theta)/\Phi_z$ can be recorded in rectangular coordinates or in polar coordinates. If rectangular coordinates are employed, the ordinate is linear in power. If polar coordinates are employed, the radius is linear in voltage. This latter is required because the area under the polar graph is proportional to the square of the radius, and the integral is proportional to $\int \Phi d\phi = K\Phi d\phi$.

Electronic integrators are available to evaluate the integral within the brackets of (7.9) or to evaluate the complete denominator automatically. Integrators of the latter type are called spherical-coordinate integrators. Electronic integrators usually operate digitally, and $\Phi(\phi, \theta)$ is sampled at discrete angular intervals which are independent of the speed of rotation of the antenna under test. In this case, equation (7.9) is written

$$D_z = \frac{(2/\pi) MN}{\sum_{i=1}^{M} \sum_{j=1}^{N} \frac{\Phi(\phi_i, \theta_j)}{\Phi_z} \sin \theta_j} \quad (7.10)$$

where

$$\theta_j = \frac{\pi j}{M}$$

and

$$\phi_j = \frac{2\pi j}{N}$$

In terms of the digital quantities employed in the integrators, (7.10) is written

$$D_z = \frac{0.63662 MN C_{\text{int}}}{\sum_{i=1}^{M} \left[ \sum_{j=1}^{N} C_{ij} \right] \sin \theta_j} \quad (7.11)$$
where $C_{ij}$ is the digital readout or count proportional to $\Phi(\phi_j, \theta_i)$ at each sample point, $C_n$ is the count per sample in the direction of maximum directivity, and where $0.63662 = \frac{\sqrt{2}}{\pi}$. The directivity $D(\phi, \theta)$ in any other direction is given by

$$D(\phi, \theta) = \frac{C(\phi, \theta)}{C_n} D_n,$$

(7.12)

where $C(\phi, \theta)$ is the count proportional to $\Phi(\phi, \theta)$ in the direction of interest.

Conventional integrators evaluate only the summation within the brackets of (7.11). Weighting of the individual sums by sin $\theta_1$, summing over $i$ and multiplication by the necessary constant multiplier are accomplished manually. However, integrators are available which evaluate the complete denominator of (7.11) automatically and incorporate the necessary multiplier to give a number which is the reciprocal of $D_n$.

**Sample Problem** - - - The problem given below is presented to indicate the application of (7.11) to determination of the directivity of an antenna.

**Problem:** Determine the directivity of a short dipole by the pattern integration method.

**Solution:** If the dipole is aligned with the Z axis, the radiation intensity is given by

$$\Phi(\phi, \theta) = \Phi_n \sin^2 \theta.$$  

(7.13)

Thus the pattern is constant in $\phi$ and has a toroidal shape with the axis of the toroid along the Z axis. The directivity could be determined from a single $\theta$ cut because $\Phi$ is independent of $\phi$, but we will assume a family of $\phi$ cuts, spaced at intervals of 10 degrees in $\theta$, for purposes of illustration. Further, let us assume that a sample proportional to $\Phi(\phi_j, \theta_i)$ is taken at 100 equally spaced intervals of $\phi$ on each $\phi$ cut and that $\Phi(\phi_j, \theta_i)$ is measured by a digital integrator whose count $C_{ij}$ per sample is 1000 for $\phi = \Phi_n$.
The directivity is determined from (7.11) with \( M = 18 \) and \( N = 100 \). In accordance with (7.13) the count per revolution in \( \phi \) for each \( \theta_i \) is given by

\[
NC_{1i} = 10^5 \sin^2 \theta_i .
\]

(7.14)

The results of the computations are given in Table I.

<table>
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<th>( i )</th>
<th>( \theta_i^\circ )</th>
<th>( NC_1/10^5 )</th>
<th>( NC_2/10^5 )</th>
<th>( NC_1 \sin \theta_i/10^5 )</th>
<th>( NC_2 \sin \theta_i/10^5 )</th>
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<td>100-180*</td>
<td></td>
<td></td>
<td></td>
<td>3.3197</td>
</tr>
</tbody>
</table>

Total for each orthogonal polarization

Total for two orthogonal polarizations

* See text.

The subscripts 1 and 2 are used to indicate the \( \phi \) and \( \theta \) polarizations, respectively. Since the dipole is linearly polarized in the \( \theta \) direction, \( C_1 \) is zero for all \( ij \). The pattern is symmetrical about the equator of the coordinate system; thus the counts for the lower hemisphere are identical with those for the upper hemisphere.

The directivity \( D_\theta \) is given by **

\[
D_\theta = \frac{0.63662(18)(10^2)(10^3)}{0 + (7.6394)(10^5)} \frac{0}{0 + (7.6394)(10^5)} = 1.500
\]

(7.15)

This value is the same as that given by Kraus in reference 1.

** See (7.5), page 7-3.
Integration by Means of $\theta$-cuts -- Equation (7.9) is given for the case in which $D_\theta$ is evaluated by means of $\phi$-cuts at fixed intervals of $\theta$. If $\theta$ cuts are to be employed at fixed intervals of $\phi$, $D_\theta$ is given by

$$D_\theta = \frac{1}{\frac{1}{4\pi} \sum_{j=1}^{N} \int_{0}^{\pi} \frac{\Phi(\phi_j, \theta)}{\Phi_\theta} \sin \theta d\theta \Delta \phi}, \quad (7.16)$$

where $\Delta \phi = 2\pi/N$. Thus,

$$D_\theta = \frac{2N}{\sum_{j=1}^{N} \int_{0}^{\pi} \frac{\Phi(\phi_j, \theta)}{\Phi_\theta} \sin \theta d\theta}, \quad (7.17)$$

If $D_\theta$ is to be evaluated by use of (7.17), multiplication of $\Phi(\phi_j, \theta)/\Phi_\theta$ by $\sin \theta$ must be accomplished during the process of integration, and the integrator must contain means for performing this multiplication. Sine potentiometers, which are slaved to the $\theta$ axis of the test positioner, are commonly employed for this purpose.

If $\Phi$ is to be sampled in $\theta$ by a digital integrator, then (7.11) is employed and is rewritten,

$$D_\theta = \frac{0.63662MNC_\theta}{\sum_{j=1}^{N} \sum_{i=1}^{M} C_{ij} \sin \theta_i}, \quad (7.18)$$

indicating the order of summation, and indicating, in addition, that each sample must be multiplied by $\sin \theta$ before being summed.

Integration of Patterns with Rotational Symmetry -- If the directivity pattern of the antenna under test has rotational symmetry about an axis, which we will
define to be the Z axis, (7.4) can be written

\[
D_n = \frac{1}{\frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \frac{\Phi(\theta)}{\Phi_n} \sin \theta \, d\theta}
\]

(7.19)

where \(D_n\) in (7.4) has been chosen to be in the direction of maximum directivity. This reduces to

\[
D_n = \frac{2}{\int_0^\pi \int_0^{2\pi} \frac{\Phi(\theta)}{\Phi_n} \sin \theta \, d\theta}
\]

(7.20)

Inspection of (7.20) shows that a single \(\theta\) cut is sufficient to determine \(D_n\). As in the preceding section, a \(\sin \theta\) multiplier must be employed in performing the integration.

In digital form (7.20) is written

\[
D_n = \frac{2}{\sum_{i=1}^M \frac{\Phi(\theta_i)}{\Phi_n} \sin \theta_i \Delta \theta}
\]

(7.21)

where \(\Delta \theta = \pi / M\),

giving

\[
D_n = \frac{0.63662MC_n}{\sum_{i=1}^M C_i \sin \theta_i}
\]

(7.22)

It will be recognized that in every case the indicated equations apply to each of two orthogonal polarizations, which must be handled in the manner indicated for (7.5).

**REFERENCE**

APPENDIX 7A

A NOTE ON THE TRAPEZOIDAL RULE FOR APPROXIMATION OF THE DEFINITE INTEGRAL

The approximations to the integrals given in this chapter are special cases of the trapezoidal rule for evaluation of definite integrals of the form

\[ I = \int_a^b f(x) \, dx \quad (7A.1) \]

by the finite summation

\[ I \approx \sum_{i=0}^{k-1} f(x_i) \Delta x \quad (7A.2) \]

where \( \Delta x = (b-a)/k \), \( x_i = i\Delta x \), and where the points \( x_0 \) and \( x_k \) are identified with the end points \( a \) and \( b \), respectively. For the uniform spacing defined by (7A.2) the trapezoidal rule gives

\[ I \approx \Delta x \left\{ \frac{1}{2} \left[ f(x_0) + f(x_k) \right] + \sum_{i=1}^{k-1} f(x_i) \right\} \quad (7A.3) \]

For the special cases involving integration in \( \theta \) from 0 to \( \pi \) used in Chapter 7, the values of \( f(0) \) and \( f(\pi) \) are zero since the integrand contains a factor \( \sin \theta \). Therefore, (7A.3) reduces to

\[ I \approx \Delta x \sum_{i=1}^{k-1} f(x_i) \quad (7A.4) \]

For simplicity we have written this in the form

\[ I \approx \Delta x \sum_{i=1}^{k} f(x_i) \quad (7A.5) \]

since \( f(x_k) = f(\pi) = 0 \).
For the cases involving integration in φ, the interval (a, b) closes on itself so that \( f(a) = f(b) \), and the trapezoidal rule is given identically by (7A. 5).

REFERENCE

The IEEE standard definition of gain was presented in Chapter 2. The form of this definition which is most useful for visualization purposes is as follows: The gain of an antenna in a given direction is the ratio of the power radiated per steradian by the antenna in that direction to the power which would be radiated per steradian by a lossless isotropic radiator with the same power accepted through its input terminals. The term gain is often taken to mean the maximum gain of the antenna, hence the gain at the peak of the main lobe.

Since an isotropic radiator does not exist in practice, the obvious choice of measuring the gain of a test antenna by comparing it to a lossless isotropic radiator is immediately ruled out. The next most obvious choice is to compare the test antenna to an antenna whose gain is known. This is in fact usually done.*

This gain comparison technique requires that an antenna be provided whose gain is known. This antenna is usually a dipole or standard horn antenna whose gain has been calibrated in a previous measurement or has been calculated from the antenna geometry. Calculated gains suffice for some comparison measurements where high accuracy is not required. However,

*This comparison is usually made while receiving a signal radiated by a third antenna as opposed to radiating to a third antenna, but through the reciprocity theorem implicit to Maxwell's equations, we know the results to be identical.
for precision measurements a gain standard which has been previously cal-
ibrated must be utilized. The accurate calibration of this gain standard can be accomplished by the three antenna method or the two antenna method, both of which make use of the Friis transmission formula for power transfer between two antennas. In these methods as in any extremely precise measure-
ment, the utmost attention must be given to obtaining a proper test environ-
ment and to processing the errors from various sources to insure that the measured gain is indicative of the true gain of the antenna.

In cases of some extremely high-gain large-aperture antennas, test ranges are not available which will allow their gain to be accurately measured. In such cases, radio stars can be utilized. Four of the stronger of such stars are Cassiopeia A, Cygnus A, Taurus A, and Virgo A. The sun is also a strong radio emitter and can be utilized for gain measurements. However, it is very sporadic, especially during periods of sunspot activity and during such periods it would be of doubtful value as a calibration source.

In the sections that follow, the above procedures will be discussed in detail. Special attention will be given to accuracy in the various measurements because many sources of error are involved. These individual error sources in themselves may be quite small, but when combined may result in an error incompatible with the gain accuracy desired. The techniques discussed in the following sections will refer to the calibration of the maximum gain of the antenna. In Sections 8.1 through 8.4, it is assumed that the antennas under test are ideally matched to the transmission lines unless otherwise indicated. Effects of mismatch are covered in Section 8.5.

8.1 Calibration of a Gain Standard

The calibration of gain standards for use in comparison measurements is most frequently accomplished by the two antenna method or the three antenna method. A third method, called the mirror method, will be mentioned briefly in passing but is of little practical value for reasons that will become apparent to the reader. The calibration of horn type antennas will be discussed before calibration of dipoles or other broadbeam antennas because while many of the techniques used in the horn calibration will also apply to the broadbeam case, the latter is more complex.
Calibration of Horn-Type Antennas -- Consider two antennas A and B separated by a distance \( R \) as seen in Figure 8.1. The power transfer between the two, assuming them to be polarization matched, is given by

\[
P_r = P_o G_A G_B \left( \frac{\lambda}{4\pi R} \right)^2
\]  

(8.1)

\[ P_\text{T} \]
\[ P_\text{R} \]
\[ \text{TRANSMIT} \]
\[ \text{RECEIVE} \]
\[ \text{A} \]
\[ \text{CL} \]
\[ \text{B} \]

\[ \lambda \]

\[ g_A \]
\[ g_B \]

\[ \lambda \]

\[ L_r = L_o + (g_A)_R + (g_B)_R - 20 \log \frac{4\pi R}{\lambda} \]  

(8.2)

Figure 8.1 Test configuration for the two antenna and three antenna methods of gain determination.

which may be written in logarithmic form as

In the above equation,

- \( L_r \) = power level at the output terminals of the receive antenna in decibels relative to a convenient reference such as 1 milliwatt,
- \( L_o \) = power level at the input terminals of the transmit antenna expressed in the same units as \( L_r \),
- \( (g_A)_R \) = apparent maximum decibel gain of antenna A at the test separation \( R \),
- \( (g_B)_R \) = apparent maximum decibel gain of antenna B at the test separation \( R \), and
- \( \lambda \) = wavelength of the transmitted wave.

The gain terms \( (g_A)_R \) and \( (g_B)_R \) are the apparent gains at the separation \( R \) and differ from the true gains \( g_A \) and \( g_B \) of the antennas because the wave illuminating the receive antenna is not a plane wave of uniform
amplitude. The greater the separation distance between these antennas, the more closely this condition is approached, and the more closely the apparent gains approximate the true gains of the antenna. When extreme accuracy is desired, however, calculated correction factors based on the antenna geometry, the frequency of operation, and the test separation must be added to the apparent gains to obtain the true gains of the antennas. Calculated data relating these correction terms to antenna geometry are available in the literature.\(^1\)\(^-\)\(^3\) The quantities \(L_r\), \(L_o\) and \(R\) in equation (8.2) can be measured and the wavelength \(\lambda\) can be determined from a measurement of the frequency. The sum of the apparent gains is therefore determined and is given by the equation

\[
(g_A)_r + (g_B)_r = 20 \log \frac{4\pi R}{\lambda} - (L_o - L_r) .
\]  

(8.3)

In the two antenna method, the antennas selected are assumed to be identical. On this assumption the gains are given by

\[
(g_A)_r = (g_B)_r = 0.5 \left[ 20 \log \frac{4\pi R}{\lambda} - (L_o - L_r) \right] .
\]  

(8.4)

This apparent gain is then corrected to the true gain by adding the corrective terms discussed previously.\(^1\)

The three antenna method does not make use of the assumption that \((g_A)_r = (g_B)_r\). It is assumed that there can be enough difference between the relative gains of the two antennas to contribute an error greater than is compatible with the accuracy required of the measurement. Equation (8.4) does not hold in such cases. Instead, a third antenna \(C\) replaces antenna \(B\) and the measurements are repeated, thus resulting in an equation similar to equation (8.3) but which relates the gains of antennas \(A\) and \(C\). A third set of measurements is then made with antennas \(B\) and \(C\), and a third equation is obtained. These three equations are of the form

\[
(g_A)_r + (g_B)_r = \Sigma_1
\]  

(8.5)
which can now be solved simultaneously resulting in the apparent gains of all three antennas. The apparent gains are then converted to the true gains as before. Some of the measurement problems associated with both the two antenna method and the three antenna method are discussed in the following paragraphs.

The first consideration must be a test environment compatible with the measurements to be made. The test range must be of sufficient length to adequately suppress interaction between antennas and to permit the apparent gain to be corrected to the true gain with a negligible error associated with the additive correction factors. Generally, these corrections must be extrapolated between two curves or tables and corresponding extrapolation errors occur. The greater the test separation the smaller will be the corresponding correction factors and extrapolation errors between them.

With sufficient length established, sufficient test heights must be obtained to reduce reflections from surrounding objects to an acceptable level. Regardless of the other precautions taken and the precision of the measurements, the results may be worthless if the reflected energy levels are so high as to contribute errors in excess of those allowed in the error budget. The levels of these extraneous signals are best evaluated by means discussed in Chapter 14. Care should also be taken to place the antennas at the same level so as to reduce problems associated with measuring their separation.

Referring once again to equation (8.3), we see that the difference in transmitted power level and received power level, \( L_o - L_r \), must be measured. The most obvious approach is to measure the input power and received power with a power meter, subtract the two, repeat the measurements many times to get a statistical average of source drift, meter drift, thermal effects, etc. and assume the result to be the actual value of \( L_o - L_r \). One is then limited by the accuracy of the power meter.
A more satisfactory procedure is to insert a calibrated directional coupler between the generator and the transmit antenna whose coupling coefficient is such that the output at the auxiliary arm and \( L_r \) are approximately equal. The power meter can then be used to compare these two levels and thus measure a very small power difference as opposed to two widely separated levels. By so doing, the calibration of the power meter is not a critical factor since the differences it will be measuring are small fractions of a decibel.

The determination of the required coupling coefficient can be accomplished in two different ways. The gains of the antenna can be calculated from their geometry and substituted into equation (8.3) to determine the value of \( L_o - L_r \). The coupler can then be set to this value and calibrated accurately in the laboratory. The range length can be adjusted slightly until the power levels concur. This would require that one of the antennas be so mounted that it could be moved longitudinally to correct the range length. Another method is to have a padded precision variable attenuator in the auxiliary arm of the coupler and adjust the attenuation until the power levels are equal. The attenuator would then be locked in this position and the entire coupling network, attenuator included, would be returned to the laboratory for calibration. A block diagram of the above described system is shown in Figure 8.2.

The quantity \((L_o - L_r)\) can be measured by the above method, the wavelength can be determined very accurately from the frequency counter shown in Figure 8.2, and the only remaining quantity to be measured is the separation, \( R \), between the antennas. This can be measured very accurately by suspending a plumb line from each antenna and measuring the separation between these plumb lines. This separation should be measured horizontally with a steel surveying tape. These tapes are calibrated at a given temperature and corrections to this calibration at other temperatures due to thermal expansion are provided. Wind effects on the plumb line can be minimized by using very heavy plumb bobs suspended in a container of 90 weight motor oil or other viscous liquid. If the antennas are at different heights, the measurement problem is somewhat more complex.
FIGURE 8.2 Block diagram of gain calibration system for three-antenna method.
Possible error in this gain sum measurement due to interaction between antennas can be determined by moving one antenna along the line of sight while recording the received signal. Antenna interaction will evidence itself as a cyclic perturbation of the received signal with a period of $\lambda/4$.

The above techniques complete the two antenna method except for correction of the apparent gains to the true gains by adding the correction terms. For the three antenna method, the procedure must be twice repeated with a third antenna as was discussed previously. With the three antenna method, accuracies which exceed $\pm 0.05$ decibels are possible. In any gain calibration, the analysis of the errors involved in the measurement technique is of paramount importance. These errors must be evaluated using standard statistical techniques to arrive at a stated level of confidence for a given tolerance.

The mirror method of gain measurement utilizes only one antenna operating in the monostatic mode as shown in Figure 8.3. A reflecting plane is used to reflect the transmitted energy back to the antenna under test. For an ideal planar reflector the apparent gain of the antenna at a test separation $R$ from its image is given by

$$g_R = 0.5 \left[ 20 \log \frac{4\pi R}{\lambda} - (L_o - L_r) \right]$$

(8.8)

This apparent gain is then corrected to the true gain.

Problems involved with this method for precision gain measurements are numerous. For example, no such reflecting plane would be lossless, and the losses in this plane would decrease the indicated gain of the antenna. The plane would be of finite extent and in most practical cases would present diffraction problems from the edges of the plane. Any deviations from a plane surface would distort the image antenna, thus contributing perturbations to the reflected wave. The measurement problems associated with the mirror method make it of little practical value in precision measurements.
Calibration of Wide Beam Antennas -- The calibration of wide beam antennas such as dipoles or log periodic arrays is complicated by unavoidable reflections from the ground. It is convenient in this case to utilize the reflections from the ground rather than attempting to suppress them; that is, to calibrate the antennas in the ground reflection mode of operation. (See Chapter 14.) The basic test configuration is that shown in Figure 8.4. The field at the receive antennas is assumed to be the vector sum of the field due to the direct path contribution and that due to the contribution from the reflected path.

**FIGURE 8.3** Test configuration for the mirror method of gain determination.
FIGURE 8.4 Range configuration for calibrating the gain of a wide beam antenna.

The amplitude of the field at the receive antenna due to the direct path signal is given by

$$E_0 = K \left[ P_0 (K_1 G_t) (K_2 G_r) \left( \frac{\lambda}{4\pi R_0} \right)^2 \right]^{\frac{1}{2}}$$

(8.9)

where

- $P_0$ = power into the terminals of the transmit antenna,
- $G_t$ = peak gain of the transmit antenna,
- $K_1 G_t$ = gain of the transmit antenna in the direction of the receive antenna,
- $G_r$ = peak gain of the receive antenna,
- $K_2 G_r$ = gain of the receive antenna in the direction of the transmit antenna,
\[ K \quad = \quad \text{constant of proportionality, and} \]
\[ R_d \quad = \quad \text{direct path separation between antennas} = \left[ R_0^2 + (h_r - h_t)^2 \right]^{\frac{1}{2}}. \]

Equation (8.9) is the field equivalent of the Friis transmission formula which was previously expressed in equation (8.1). The amplitude of the ground-reflection field at the receive antenna is given by

\[ E_R = K \left[ P_G G_r \left( \frac{\lambda}{4\pi R_R} \right)^2 r^2 \right]^{\frac{1}{2}}, \quad (8.10) \]

where \( R_R \) = the effective path length between the receive antenna and the transmit antenna image and \( r^2 \) = an effective gain factor which accounts for the transfer of energy by means of reflection from the range surface.

Although the transmit antenna image is distorted due to the finite wavelength of the radiation and the irregularities in the range surface, it is convenient to define \( R_R \) as \( \left[ R_0^2 + (h_r + h_t)^2 \right]^{\frac{1}{2}} \). The factor \( r^2 \) is thus a function of the electrical and geometrical properties of the range surface, the radiation patterns of the antennas, the frequency and polarization of the transmitted wave, and the geometry of the test range.

We will assume the antennas to be linearly polarized. It is sometimes necessary to determine the gain of an antenna which is circularly or elliptically polarized, but in such cases its gain is usually determined by the method of partial gains using a linearly polarized gain standard as a comparison.*

When the antenna heights are adjusted such that the two field contributions arrive in phase at the receive antenna, the total field at that point is

\[ E_T = E_0 + E_R \quad (8.11) \]

*See Chapter 2.
or

$$E_r = K \left[ P_o G_t G_r \left( \frac{\lambda}{4\pi R_o} \right)^2 \right] \left[ (K_1 K_2)^{\frac{3}{2}} + \frac{R_n}{R_s} \right].$$ \hspace{1cm} (8.12)$$

The total received power for this in-phase case is therefore

$$P_r = P_o G_t G_r \left( \frac{\lambda}{4\pi R_o} \right)^2 \left[ (K_1 K_2)^{\frac{3}{2}} + \frac{R_n}{R_s} \right].$$ \hspace{1cm} (8.13)$$

Expressing (8.13) in logarithmic form, we have

$$L_r = L_o + g_t + g_r - 20 \log \left( \frac{4\pi R_n}{\lambda} \right) + 20 \log \left[ (K_1 K_2)^{\frac{3}{2}} + \frac{R_n}{R_s} \right].$$ \hspace{1cm} (8.14)$$

The gain sum $g_t + g_r$ is obtained by measuring the remaining quantities in the above equation. The only quantity which presents a significant measurement problem is the effective gain factor $r^2$. This factor may be determined through a procedure similar to the following.

First specify a height for the receive antenna such that

$$h_r \geq 4D \hspace{1cm} (8.15)$$

and

$$h_r \geq 4\lambda \hspace{1cm} (8.16)$$

In the above equations,

- $h_r$ = the height of the receive antenna,
- $\lambda$ = the wavelength of operation, and
- $D$ = the maximum aperture dimension of the receive antenna.

Satisfaction of these criteria ensures that the amplitude taper in the vertical plane across the receive antenna can be made less than 0.25 decibel and further ensures that mutual coupling between the receive antenna and its image in the range surface is suppressed by at least 40 decibels. These conditions will be adequate for most calibration measurements of this type.

Once this height is specified, the range length $R_o$ should be of sufficient

8-12
length that

\[ R_0 \gg 2h_r \]  \hspace{1cm} (8.17)

This requirement permits a small grazing angle (the complement of the angle of incidence) for the reflected wave. A low grazing angle is desirable for reasons that will be presented later.

In order to satisfy the in-phase criterion of (8.11) it is necessary that

\[ h_t = \frac{(2n-1)\lambda R}{4h_r} \]  \hspace{1cm} (8.18)

where \( n \) is a positive integer corresponding to the interference lobe which is peaked on the receive antenna and \( h_t \) is the height of the transmit antenna. (See Chapter 14.) The transmit antenna should be placed at the lowest position that satisfies both (8.18) and the mutual-coupling criterion

\[ h_t \geq 4\lambda \]  \hspace{1cm} (8.19)

With the transmit antenna in this position, the total received power is given by equation (8.13). This received power should be recorded.

The transmit antenna should then be moved to the lowest position which satisfies both (8.19) and the relation

\[ h_t = \frac{m\lambda R}{2h_r} \]  \hspace{1cm} (8.20)

where \( m \) is an integer. This corresponds to the location of a minimum in the interference pattern at the receive antenna. The field at the receive antenna is then given by

\[ E_\parallel = E_\parallel' - E_\parallel^1 \]  \hspace{1cm} (8.21)

which produces a received power of

\[ P_r = P_0 G_t G_r \left( \frac{\lambda}{4\pi R_0} \right)^2 \left[ \left( K_1 K_2' \right)^\frac{1}{2} - \frac{R_{p_0}'}{R_0} \right] \]  \hspace{1cm} (8.22)
The primed quantities are defined for the new position as were the unprimed quantities for the original position. This received power should be recorded for comparison with $P_r$.

The effective gain factor $r^2$ was assumed to be the same in equations (8.13) and (8.22). It in fact differs slightly between these orientations since the angle of incidence differs slightly. The quantity measured will represent an average of the values between these two positions. It is therefore important that the range configuration be such that the grazing angle of the reflected wave change as little as is practical between these two measurement configurations. For the criteria of equations (8.15) through (8.20), it is seen that the geometry which produces the smallest grazing angle also produces the smallest change in grazing angle. The limitation in choosing the smallest grazing angle is usually one of economics in the selection of range length. It is also more desirable to test at horizontal polarization than at vertical since the effective gain factor changes less rapidly with grazing angle at horizontal polarization.

Division of (8.13) by (8.22) yields

$$\frac{P_r}{P_r'} = \left(\frac{R_0}{R_0'}\right)^2 \frac{(K_1 K_2)^\frac{1}{3} + \frac{R_0}{R_0'} r}{(K_1' K_2')^\frac{1}{3} - \frac{R_0'}{R_0'} R_0 R_0'}$$

from which,

$$r = \frac{R_0 R_0'}{R_0' R_0} \left(\frac{P_r}{P_r'}\right) ([P_r' / P_r])^\frac{1}{3} R_0 - (K_1 K_2)^\frac{1}{3} R_0' \right) / \left( ([P_r' / P_r])^\frac{1}{3} R_0 + R_0' \right)$$

The antenna directivity quantities $K_1$, $K_1'$, $K_2$, and $K_2'$ should be taken from measured pattern data, based on the test geometry. Here also the low grazing angle tends to give greater accuracy because the $K_1$, being near the peak of the beam, vary slowly with angle.

To accurately determine the various range terms $R_0$, the locations of the phase centers of the two antennas must be known. Knowing their locations, one then measures their heights and the horizontal separation $R_0$ and calculates the direct and reflected path lengths for each configuration. Since
the power terms $P_r$ and $P_t$ are measured quantities, the factor $r^2$ is now calculable from (8.24).

Returning now to the in-phase configuration corresponding to (8.14), the received and transmitted power levels $L_r$ and $L_t$ are determined with power meters just as in the case of the horn type antennas. Here again, a calibrated coupling network similar to the one described in the section on horn antennas is recommended in order to reduce calibration errors associated with the power meters.

If the above procedure is repeated three times for three antennas in their respective transmit-receive configurations, their gains are determined by solving the three resulting simultaneous equations, just as was done in the case of the horn antennas. The only difference is that the equations are somewhat more complex, as are the measurements. The two antenna method can also be used here as in the case of the horn antennas.

8.2 Gain Transfer Measurements

The most frequently used technique for calibrating the gain of an antenna is the transfer technique. This technique necessitates having a gain standard antenna to which the gain of the test antenna can be compared. It will be assumed here, as is usually the case, that the gain standard is linearly polarized. The problems associated with the transfer measurement are much less complex than those associated with the calibration of the gain standard and as a result, accurate transfer measurements can be made much more quickly than gain standard calibration. As in all antenna measurements, a test range of a quality compatible with the measurements to be made is mandatory.

When possible the test antenna and the gain standard are positioned such that the outputs at their terminals can be compared when their positions relative to a fixed transmit antenna are interchanged. One such configuration might be as shown in Figure 8.5. The test antenna and the gain standard are shown aligned with the peaks of their beams pointing in opposite directions, and with their phase centers at equal distances from an axis of rotation. A panel of microwave absorbing material is shown
between the two antennas. This is advisable in a case such as the one illustrated where one of the antennas is considerably larger than the other, in order to eliminate reflections from behind the smaller antennas which can contribute error to the measurements.

![Diagram](image)

**FIGURE 8.5** Possible test configuration for gain transfer measurements.

With such a configuration, the output signal level at the terminals of the unknown test antenna would first be measured. This output in logarithmic form is given by

\[
L_u = L_o + (g_t)R + (g_u)R - S_R
\]

(8.25)

where
- \( L_u \) = the output signal level at the terminals of the unknown test antenna,
- \( L_o \) = the input signal level at the terminals of the transmit antenna,
\((g_t)_R\) = the apparent gain of the transmit antenna at the test separation \(R\),

\((g_u)_R\) = the apparent gain of the transmit antenna at the test separation \(R\), and

\(S_R = 20 \log \frac{4\pi R}{\lambda}\).

The entire configuration shown in Figure 8.5 is then rotated until the gain standard is aligned with the transmit antenna. The output signal level at the terminals of the gain standard is then measured. This output in logarithmic form is given by

\[ L_s = L_0 + (g_t)_R + (g_s)_R - S_R \quad , \quad (8.26) \]

where \(L_s\) = the output signal level at the terminals of the gain standard,

\((g_s)_R\) = the apparent gain of the gain standard at the test separation \(R\), and

\(L_0\), \((g_t)_R\), and \(S_R\) are the same quantities defined following equation (8.25).

Subtracting equation (8.26) from (8.25)

\[ L_u - L_s = (g_u)_R - (g_s)_R \quad , \quad (8.27) \]

\[ (g_u)_R = (g_s)_R + (L_u - L_s) \quad . \quad (8.28) \]

\((g_s)_R\) can be found from the true gain, \(g_s\), by reversing the procedure described in Section 8.1 for converting the apparent gain to the true gain.

The determination of the quantity \((L_u - L_s)\) can be accomplished by measuring each level directly with power meters and subtracting. In cases where the gain of one antenna is significantly greater than that of the other, the measurement accuracy can be enhanced by use of a padded variable attenuator on the antenna with the larger gain. This is essentially the same procedure as that suggested in the case of the coupling network described in Section 8.1. The advantage of this procedure is to reduce
the errors associated with measuring widely different power levels with a power meter. A correction term must now be added to the apparent gain, \((g_u)_a\), to obtain the true gain \(g_u\) of the unknown test antenna.\(^{1-6}\) If the unknown antenna is not linearly polarized, the above described measurements must be performed for two orthogonal orientations of the unknown antenna. These partial gains are added to find the total gain of the antenna.

8.3 Use of Extraterrestrial Sources

In the case of some extremely large aperture antennas, test ranges are not practical for the gain measurements as described previously because the distances and tower heights required to maintain an acceptable test environment are not practical. In such cases, gain comparison measurements can be made utilizing various extraterrestrial radio sources to replace the transmit antenna. Extremely sensitive receiving systems, such as radiometers, must be employed due to the low energy levels from these radio stars.

Four of the most useful of these radio sources are: Cassiopeia A, Cygnus A, Taurus A, and Virgo A. The celestial coordinates\(^*\) and angular dimensions of these sources are given in Table 8.1.\(^7\)

<table>
<thead>
<tr>
<th>Source</th>
<th>Right Ascension</th>
<th>Declination</th>
<th>Angular Diameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taurus A</td>
<td>05°31'31&quot;</td>
<td>42°59.0'</td>
<td>3' by 4.5'</td>
<td>Radiation from the Crab Nebula; elliptical shape with major axis at 140&quot;</td>
</tr>
<tr>
<td>Virgo A</td>
<td>12°28'17&quot;</td>
<td>+12°39.9'</td>
<td>core 0.6' halo 6'</td>
<td>Halo contributes 40 percent of flux density at 1420 MHz, 55 percent at 400 MHz, 75 percent at 100 MHz, and almost all below 30 MHz</td>
</tr>
<tr>
<td>Cygnus A</td>
<td>19°57'44&quot;</td>
<td>+40°37.4'</td>
<td>each &lt; 0.7'</td>
<td>Double source; components separated by 1.8' on axis at 110°</td>
</tr>
<tr>
<td>Cassiopeia A</td>
<td>23°21'11&quot;</td>
<td>+58°32.8'</td>
<td>4'</td>
<td>Circularly symmetric, probably ring shaped; flux density decreases one percent per year</td>
</tr>
</tbody>
</table>

\(^*\) A description of celestial coordinate systems is presented in Chapter 5. The azimuth angle in Chapter 5 is given as sidereal hour angle; 360 degrees of sidereal hour angle correspond to exactly 24 hours (sidereal time) of right ascension.
Absolute gain can also be determined from these sources without the use of a gain standard. This can be accomplished since the flux densities of these sources are known. The effective area $A_e$ of the antenna whose gain is being determined is given by

$$A_e = \frac{2KT_A}{S}, \quad (8.29)$$

where $T_A$ is the measured antenna temperature due to the source above that due to the sky background, $K$ is Boltzmann's constant, and $S$ is the flux density* of the radio source. Since the gain of an antenna is related to its effective area by

$$G = \frac{8\pi K T_A}{\lambda^2 S}, \quad (8.30)$$

then

$$G = \frac{8\pi K T_A}{\lambda^2 S}, \quad (8.31)$$

The flux densities of the above listed radio sources are presented in Table 8.2.6

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Flux Density (W m$^{-2}$ Hz$^{-1}$ x 10$^{-26}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cassiopeia A</td>
</tr>
<tr>
<td>100</td>
<td>17300</td>
</tr>
<tr>
<td>150</td>
<td>12800</td>
</tr>
<tr>
<td>200</td>
<td>10300</td>
</tr>
<tr>
<td>300</td>
<td>7700</td>
</tr>
<tr>
<td>400</td>
<td>6140</td>
</tr>
<tr>
<td>600</td>
<td>4550</td>
</tr>
<tr>
<td>900</td>
<td>3700</td>
</tr>
<tr>
<td>1000</td>
<td>3100</td>
</tr>
<tr>
<td>1500</td>
<td>2330</td>
</tr>
<tr>
<td>2000</td>
<td>1850</td>
</tr>
<tr>
<td>3000</td>
<td>1380</td>
</tr>
<tr>
<td>4000</td>
<td>1100</td>
</tr>
<tr>
<td>6000</td>
<td>820</td>
</tr>
<tr>
<td>8000</td>
<td>640</td>
</tr>
<tr>
<td>10000</td>
<td>500</td>
</tr>
</tbody>
</table>

*The factor of 2 in (8.29) accounts for the fact that the receiving antenna is of a single polarization while the radio source is randomly polarized.
From 200 MHz to 5 GHz, the absolute accuracy is about 3 percent for Cassiopeia A, 5 percent for Taurus A and Cygnus A, and from 5 to 10 percent for Virgo A. The absolute accuracy over the range from 5 to 10 GHz is about 5 percent for Cassiopeia A and Taurus A and about 10 percent for Cygnus A and Virgo A.

8.4 Swept-Frequency Gain Measurements

The gain measurement techniques described in previous sections were primarily based on the measurement of gain at fixed frequencies. It is often desirable to determine the gain of an antenna at many frequencies within its operating frequency band, thus requiring an extensive measurement process if single-frequency measurements are accomplished. The use of swept-frequency techniques can greatly reduce the time and effort required and, with proper care, can yield very accurate results.

Several authors have investigated the application of swept-frequency techniques to the measurement of antenna gain. An in-depth discussion of this technique is not presented in this text, and the reader is encouraged to consult the referenced publications. The general concept of swept-frequency antenna gain measurements and the various techniques which may be employed are discussed in the following paragraphs.

Swept-frequency techniques may be used for an absolute gain measurement or for transfer measurements between a gain standard and the antenna under test. In either case, the basic measurement process is the same as that previously described with only minor exceptions. The most obvious difference is the measuring and recording instrumentation. The swept frequency measurement requires instrumentation which is commonly used in laboratory swept-frequency measurements of attenuation and gain of microwave components. The swept-frequency measurement is further complicated by the increased complexity of evaluating electrical mismatch.
Absolute Gain Measurements

The most common methods for absolute gain measurements are the two antenna method and the three antenna method described in Section 8.1. The basic configuration for the instrumentation necessary to perform a swept-frequency calibration by either of these methods is shown in Figure 8.6. Note that the differences between this measuring system and that shown in Figure 8.2 for fixed frequency measurements is the addition of a swept signal source and x-y recorder and the elimination of all tuners. Since, in a practical measurement situation, neither the antenna nor the generator can be matched across the frequency band with the use of tuners, the effects of mismatch on the measurement must be determined separately. These effects are discussed in Section 8.5 and must be determined by accurately measuring the various reflection coefficients and determining their mismatch contribution.

The method of obtaining a transmitted power reference shown in Figure 8.6 is the same as that described in Section 8.2. The coupler-attenuator combination between the signal generator and the load must be calibrated, with the total attenuation of the combination chosen to approximate the total signal reduction in the transmission path between the antennas. Another possibility for obtaining this reference is to remove the antennas and to physically join the transmitting and receiving components. The gain calibration then essentially becomes an insertion loss measurement. This technique is used by the Bureau of Standards. However, a convenient means of longitudinal movement is necessary for this technique and incorporating such a capability must be weighed against the calibration of the coupling network in the previously described technique. This insertion loss method might also require a calibrated attenuator to make the two levels (transmit reference and receive) more nearly equal and to thus reduce the required dynamic range of the receiving system.

The receiving system may consist of either a diode detector or a receiver which is phase-locked to the swept-frequency oscillator. The use of a detector is acceptable if the dynamic measurement range is small and frequency discrimination is not required to eliminate out-of-band signals which might be received by the receiving antenna.
Once a calibrated transmit reference level is available, the measurement technique involves recording of this reference as the frequency is swept and then superimposing the signal level received through the transmission path between the antennas. The recorded difference between these two measurements, when compensated by coupler and attenuator values, mismatch effects, and free-space attenuation, will yield the sum of the antenna gains. This process may be repeated when utilizing the three antenna method to derive the antenna gain, or if the antennas are assumed identical, as in the two antenna method, the antenna gain is one half of the measured sum.

As in the case of fixed-frequency measurements, free-space conditions must be closely approximated by the antenna range to obtain accurate measurements. Multipath interference, near field conditions, and improper aperture illumination can result in significant measurement errors if not considered carefully. Bowman and Fitzgerrel have investigated the effects of multipath interference and near field corrections and suggested methods for minimizing the resulting measurement error.
The requirement of maintaining an approximately uniform aperture distribu-
tion is particularly difficult when making swept frequency measurements
on a ground reflection antenna range. As previously discussed in Section 8.1,
the height of the transmit antenna on a ground reflection range is a function
of the wavelength, \( \lambda \). Thus, if swept-frequency measurements are to be
performed, the height of the transmitting antenna must be programmed to
maintain the proper aperture illumination at the receiving antenna location
as the frequency is swept.

Gain Transfer Measurements

The technique for calibrating antenna gain by swept-frequency transfer
measurements is essentially the same as that described in Section 8.2. As
in the case of absolute gain calibration the major differences are (1) the
instrumentation required and (2) the more complex method required to
compensate for the effects of electrical mismatch.

The basic equipment configuration for swept-frequency gain transfer measure-
ments is shown in Figure 8.7. Knowing the gain of the standard antenna,
sequential swept frequency measurements of the power received by the standard and test antennas would allow computation of the test antenna gain.

If the standard and test antennas have greatly different gains, the precision variable attenuator may be utilized to limit the required dynamic range of the receiving system. This also allows an expansion of the Y axis (signal level) of the recorded data and thus permits greater resolution of measured difference in gains.

8.5 Effects of Electrical Mismatch

The definition of gain given in Sections 2.7 and 8.1 are based on power accepted by the antenna from its generator. It is therefore necessary to determine the power actually accepted by an antenna under test. One technique for accomplishing this is to match the antenna under test to the input transmission line so that the reflected power is zero and then to measure the incident power. An alternative procedure is to measure the voltage standing wave ratio looking into the antenna terminals, to measure the incident power, and to calculate the amount of reflected power.

If the measurements are to be made at a single frequency, either technique can be used, although each has its problems. In the former case the loss of the tuning element must be determined. In the latter, the mismatch between the generator and the transmission line complicates the problem of calculating the reflected power, although this mismatch can be eliminated by tuning or padding the signal source.

If the measurements are to be made on a swept frequency basis, neither the antenna nor the generator can be identically matched over the frequency band and the problem becomes extremely complicated.

From reciprocity considerations the problem of matching the receiving antenna which is used in the calibration procedure to the load is of equal importance to that of matching the transmitting antenna.
Much work has been done at the National Bureau of Standards on the subject of mismatch and the results of this work have been published. For an in depth discussion of mismatch, the reader is encouraged to consult the referenced publications. Some of the results of this work as specifically apply to the problem at hand are discussed in the following paragraphs.

In order to assess the mismatch error in such a measurement, let us consider the configuration in Figure 8.9. The reflection coefficients at the junction are $\Gamma_0$ and $\Gamma_1$ looking into the generator and load respectively. It can be shown$^{10}$ that the mismatch factor associated with this configuration is given by

$$M = \frac{(1 - |\Gamma_0|^2)(1 - |\Gamma_1|^2)}{|1 - \Gamma_0 \Gamma_1|^2}.$$  
(8.32)

The power $P_L$ delivered to the load in this configuration is given by

$$P_L = P_{L0}M$$  
(8.33)

where $P_{L0}$ is the power that would be delivered to the load if the signal generator and load were matched.

\[Signal\ Generator\]  
\[Load\]  
\[\Gamma_0\]

\[\Gamma_1\]

\[FIGURE 8, 9\ Block diagram illustrating mismatch between a signal generator and a load.\]
Let us now consider the case where the load is the transmitting antenna used in making a gain calibration and the generator is a calibrated coupling network such as that shown in Figure 8.2. The mismatch factor at this junction is given by

$$M_1 = \frac{(1 - |\Gamma_\lambda|^2)(1 - |\Gamma_\tau|^2)}{|1 - \Gamma_\lambda \Gamma_\tau|^2}$$  \hspace{1cm} (8.34)

where $\Gamma_\tau$ is the reflection coefficient of the transmit antenna. To indicate an approximate magnitude of this factor, let us assume VSWR values of 1.2:1 and 1.5:1 for the coupling network and transmit antenna respectively. This corresponds to

$$|\Gamma_\lambda| = 0.09,$$ \hspace{1cm} (8.35)

$$|\Gamma_\tau| = 0.199.$$ \hspace{1cm} (8.36)

These values would result in values of $M_1$ ranging from 0.919 to 0.988, depending on the relative phase of the reflection coefficients. In decibels, this mismatch factor ranges from 0.05 dB to 0.37 dB. Therefore, the phases of the reflection coefficients can be very important if accurate gain calibrations are to be performed. While it will sometimes be adequate to determine only the magnitude of the reflection coefficients, instruments are commercially available that can measure them in both phase and amplitude on a swept frequency basis.

Now, consider the generator in Figure 8.9 to be the receiving antenna with a reflection coefficient, $\Gamma_\lambda$, and the load to be a receiver with a reflection coefficient, $\Gamma_\lambda$. The receiving mismatch factor $M_2$ is given by

$$M_2 = \frac{(1 - |\Gamma_\lambda|^2)(1 - |\Gamma_\lambda|^2)}{|1 - \Gamma_\lambda \Gamma_\lambda|^2}.$$ \hspace{1cm} (8.37)

These existing mismatch terms therefore result in a modification to the expression for the actual power received in the case of the Friis transmission formula of equation 8.1. This power is given by:

$$P_r = P_0 G_A G_B \left(\frac{\lambda}{4\pi R}\right)^2 M_1 M_2,$$ \hspace{1cm} (8.38)
or expressed in logarithmic form as in equation 8.2,

\[ L_r = L_0 + (g_A)_R + (g_B)_R - 20 \log \frac{4\pi R}{\lambda} + 10 \log M_1 + 10 \log M_2 \] (8.39)

The power level measured at the output test point of the directional coupler in Figure 8.2 must be modified in a similar manner.

In the event single frequency measurements are being made and it is not convenient to make phase and amplitude measurements of the reflection coefficient at all frequencies, tuners can be employed on the signal generator and receiving system only. When tuned at each discrete frequency, \( \Gamma_g \) and \( \Gamma_l \) will be reduced to zero and, \( M_1 \) and \( M_2 \), to:

\[ M_1 = 1 - |\Gamma_l|^2, \quad \text{and} \]

\[ M_2 = 1 - |\Gamma_R|^2. \] (8.40) (8.41)

In order to evaluate these expressions, phase measurements are not required. Another means of obtaining a low reflection generator is by utilizing a generator that is leveled by means of a leveling signal derived from the nearest point to the output port. \( \text{15-16} \) This is commonly done by using a directional coupler for the output component of a generator and the output from the coupled port is fed back into a leveling circuit for the generator. The maximum possible reflection coefficient magnitude of the equivalent generator \( g \) is then

\[ |\Gamma_g|_{\text{max}} = |\Gamma_c| + 10^{-\left(\frac{d}{20}\right)} \] (8.42)

where \( \Gamma_c \) is the reflection coefficient looking into the output port of the directional coupler with the other two arms terminated with reflectionless loads, and \( d \) is the directivity of the coupler in decibels.
REFERENCES


9.1 INTRODUCTION

The increased complexity and sophistication of today's antenna systems require more complete measurements of all antenna parameters and at the same time demand greater accuracy in these measurements.

When considered from the viewpoint of measurement system error and instrumentation complexity, few antenna system measurements have been more difficult or time consuming than those involving measurement of phase of the radiated field.

Recent advances in the phase measurement instrumentation area have significantly reduced the magnitude of the measurement problem. This chapter will be concerned only with phase measurements of the radiated energy of the antenna system rather than the closed circuit case such as input impedance or component phase shift. These latter areas have been extensively treated in the literature and will not be repeated here. Before discussing phase measurement techniques and application, a brief review of basic definitions and phase terminology will be given.

9.2 BASIC CONCEPTS

Radian Measure of Plane Angles - - - By definition, the value in radians of the angle \( \theta \) subtended by an arc length \( s \) (Figure 9.1) is given by the dimensionless ratio of arc length to radius \( r \): that is,

\[
\theta = \frac{s}{r}
\]  

(9.1)
Thus since there are \(2\pi\) radians in a complete circle,

\[
1 \text{ radian} = \left(\frac{360}{2\pi}\right) \text{ degrees} \approx 57.3 \text{ degrees}, \text{ and}\n1 \text{ degree} = \left(\frac{2\pi}{360}\right) \text{ radian} \approx 17.4 \text{ milliradians}.
\]

If the angle \(\theta\) varies at a constant rate \(d\theta/dt\), then from a time \(t_1\) to a time \(t_2\) the change in the angle \(\theta\) is

\[
\Delta\theta = (t_2 - t_1) \frac{d\theta}{dt} . \tag{9.2}
\]

The angular velocity \(d\theta/dt\) is usually given the notation \(\omega\),

\[
\omega = \frac{d\theta}{dt} . \quad \text{(radians/unit time)} \tag{9.3}
\]

Thus,

\[
\theta = \int_{0}^{t} \omega \, dt \tag{9.4}
\]

By convention, when measuring plane angles as a function of time, a horizontal axis is chosen as the zero reference, and the angle is assumed to increase positively in the counter-clockwise direction.
If \( \theta \) has some particular value \( \phi \) at \( t = 0 \), the phase at any time \( t \) is given by

\[
\theta(t) = \omega t + \phi .
\]  
(9.5)

FIGURE 9.2 Geometry illustrating equation (9.5).

The phase difference between any two sinusoidally varying quantities, regardless of whether they are of the same frequency, is given by

\[
\theta_1 - \theta_2 = (\omega_1 - \omega_2) t + \phi_1(t) - \phi_2(t) .
\]  
(9.6)

When \( \omega_1 \) and \( \omega_2 \) are equal, (9.6) reduces to

\[
\theta_1 - \theta_2 = \phi_1(t) - \phi_2(t) .
\]  
(9.7)

The more general case defined by (9.6) is required in problems such as those involving heterodyning of two frequencies and is basic in phase measurements systems which make use of the heterodyne process. * The case described by

* See Chapter 4.

9-3
(9.7), defining the phase difference between two signals of the same frequency, and the special case of (9.7) for which either $\phi_1(t)$ or $\phi_2(t)$ is constant, are of specific interest in this chapter; for the latter case (9.7) becomes

$$\theta_1 - \theta_2 = \phi_1(t) - k$$

(9.8)

where $\phi_1(t)$ is the non-constant term. In (9.6) and (9.8) $\phi$ is usually dependent on time through some intermediate variable such as antenna position, orientation angle or the like. The angle $\phi_1(t_2) - \phi_1(t_1)$ is referred to as a phase shift, while the angle $\phi_1(t_1) - \phi_2(t_1)$ is called a phase difference.

**Phasors and Complex Numbers** - - - The phase angles given in the preceding discussion can be thought of as the arguments of circular functions $\left[ \sin(\omega t + \phi), \cos(\omega t + \phi) \right]$ or as the arguments of phasor quantities.

The word *phasor* is used to denote a quantity representing the amplitude (by the length of the phasor) and phase (by the angle measured from the positive real axis of the complex plane) of a harmonic function.

---

**FIGURE 9.3** Illustration of phasor in complex plane.
In complex notation, we write the phasor shown in Figure 9.3 as

\[
\tilde{A} = A \cos \theta + jA \sin \theta \quad (9.9)
\]

or, making use of the identity *

\[
e^{\pm j\gamma} = \cos \gamma \pm j \sin \gamma \quad (9.10)
\]

we may write

\[
\tilde{A} = Ae^{j\theta} \quad (9.11)
\]

The exponential phasor notation is used extensively in calculations and derivations dealing with sums and differences of harmonic functions, since

\[
\text{Re} \left\{ \tilde{A} \pm \tilde{B} \pm \tilde{C} \right\} = \text{Re} \left\{ \tilde{A} \right\} \pm \text{Re} \left\{ \tilde{B} \right\} \pm \text{Re} \left\{ \tilde{C} \right\} \quad (9.12)
\]

and

\[
\text{Im} \left\{ \tilde{A} \pm \tilde{B} \pm \tilde{C} \right\} = \text{Im} \left\{ \tilde{A} \right\} \pm \text{Im} \left\{ \tilde{B} \right\} \pm \text{Im} \left\{ \tilde{C} \right\} \quad (9.13)
\]

Use of the exponential notation in these cases simplifies the mathematics, while retaining a physical picture of the behaviour of the functions. The field or signal components to be described at the conclusion of such computations are real functions, and are easily extracted from a complex exponential expression by taking the real or imaginary part of the expression.

It is emphasized that straightforward use of phasors in summations of field or signal quantities is limited to propagation in linear media where the principles of reciprocity and superposition hold. The familiar use of complex notation in solving for impedance, power factor, etc. in linear circuits is possible because the time dependence of the various quantities cancels out

*See any standard calculus text treating Maclaurin's series.
in the associated ratios or products, or becomes a double-frequency term associated with power.

That is, the ratio

\[
\frac{A}{B} = A e^{j\phi_A} / B e^{j\phi_B} = (A/B)e^{j(\phi_A - \phi_B)}
\]  

(9.14)

has real and imaginary parts

\[
\text{Re}\left\{\frac{A}{B}\right\} = (A/B) \cos (\phi_A - \phi_B)
\]  

(9.15)

and

\[
\text{Im}\left\{\frac{A}{B}\right\} = (A/B) \sin (\phi_A - \phi_B)
\]  

(9.16)

while for the product

\[
\overline{AB} = A B e^{j(\phi_A + \phi_B)}
\]  

(9.17)

we have

\[
\text{Re}\left\{AB\right\} = AB \cos (\phi_A + \phi_B)
\]  

(9.18)

and

\[
\text{Im}\left\{AB\right\} = AB \sin (\phi_A + \phi_B)
\]  

(9.19)

Phasors are of particular value in problems involving interference between signals. For example consider Figure 9.4, which shows the effect that a voltage \(\overline{v_2}\) of random phase can have on a voltage \(\overline{v_1}\) of the same frequency.

It is evident that if \(\overline{v_2} \ll \overline{v_1}\), \(\Delta \phi\) has a maximum value given by

\[
\Delta \phi = \pm \frac{v_2}{v_1}, \quad \text{(radians)} \quad (9.20)
\]
which occurs when \( \bar{v}_2 \) and \( \bar{v}_1 \) are very nearly in quadrature.

\[
\begin{align*}
N_1(dB) &= 20 \log (1 + v_2/v_1) \\
&\leq 8.68 v_2/v_1
\end{align*}
\]

defines the maximum change in the level of \( v_1 \) in decibels for a ratio \( v_2/v_1 \ll 1 \), given in decibel form by

\[
N_2(dB) = 20 \log v_2/v_1
\]
For example, for $v_2/v_1 = 10^{-2}$, from (9.23) $N_2$ is -40 dB and $N_1 = 0.09$ dB from (9.22). From (9.20), $\Delta \phi$ is $\pm 0.01$ radian or about $\pm 0.57$ degree.

If a number of sinusoidal contributions are to be added, a graphical summation of phasors provides a valuable insight into the process of summation. This is illustrated in Figure 9.5.

![Figure 9.5](image_url)

**FIGURE 9.5** Summation of phasors representing sinusoidal contributions of angular frequency $\omega$.

In the limit, as the number of phasors approaches infinity and the amplitudes of the individual phasors approach zero, the phasor sum approaches a continuous smooth curve (Figure 9.6).

![Figure 9.6](image_url)

**FIGURE 9.6** Summation of a large number of small phasor contributions with incremental phase differences.
Classic examples are the Cornu spiral and vibration curve of optics and similar summations in antenna analysis.

9.3 INSTRUMENTATION

The discussion of phase measurement systems in this chapter will be limited to consideration of the heterodyne method of phase measurement. For a general discussion of methods of phase measurements, the reader is referred to an excellent treatment of the subject by Dr. John Dyson of the University of Illinois.

Heterodyne techniques in phase measurements are possible because of the one-to-one correspondence between the change in phase of the intermediate frequency signal to the change in phase of the radio frequency input signal. That is to say, a one degree change in the phase of the RF signal causes a corresponding one degree change in the phase of the IF signal. A discussion of phase correspondence in heterodyne systems is found in the literature.

The heterodyne process also results in linear conversion of the RF signal to the intermediate frequency. Thus, the amplitude of the IF signal is proportional to the amplitude of the microwave signal.

Figure 9.1 illustrates a single channel, double conversion heterodyne circuit as used in a typical microwave measurements systems. This circuit in the form shown makes use of harmonic mixing (see Chapter 4) to convert the input signal to an intermediate frequency of 45 MHz. The input signal is coupled directly to a crystal mixer, combined with the first local oscillator output and converted to the first intermediate frequency.

The second intermediate frequency is set at 1 KHz because this frequency is compatible with standard microwave output equipment. Heterodyning to such a low second intermediate frequency is made possible by the use of phase-lock techniques. (See Chapter 15, Section 15.3.)

*See Chapter 15 for discussion of choice of intermediate frequencies in antenna measurements receivers.
FIGURE 9.7 Basic heterodyne receiving system employing double conversion and phase locking.

The phase-lock circuit causes the intermediate frequency to be synchronous with (locked to) the frequency of a crystal oscillator. Since the crystal oscillator has a virtually pure spectrum, the signal intermediate frequency contains almost no frequency modulation components within the pass band of the phase-lock loop. This permits the IF signal to be converted to 1 KHz by means of a second crystal oscillator, which is in turn phase-locked so that its frequency differs from the 45-MHz reference oscillator by precisely 1 KHz. The bandwidth of the first IF amplifier is of the order of 10 MHz while the bandwidth of the second IF amplifier is of the order of 100 Hz.

The noise bandwidth of the double conversion system is approximately that of the narrow-band second IF amplifier. This is in contrast to the noise bandwidth of conventional measurements receivers, which employ square-law second detectors. The noise bandwidth for the latter is given by \( B = \sqrt{B_1 B_2} \) where \( B_1 \) is the bandwidth of the intermediate frequency amplifier and \( B_2 \) is the post-detection bandwidth (see Chapter 4, equation 4.26). The narrow noise bandwidth produces an increase in sensitivity of the order of 30 decibels over that
of conventional receivers employing square-law detection and with postdetection bandwidths equal to the second IF bandwidth of the phase-locked double conversion receiver.

A block diagram of a complete receiver which employs double conversion with phase locking for making microwave phase measurements is shown in Figure 9.6. This circuit is similar to that of Figure 9.5, except that it has two signal channels, designated A and B, which are fed by the same first and second local oscillators.

FIGURE 9.8 Dual channel, heterodyne receiving system designed for measurement of phase.

The output voltage of each channel is directly proportional to the input microwave voltage of that channel and the signal level in each channel can vary independently over the dynamic range of the system.

One of the primary areas of concern in a system of this nature is that of eliminating inter-channel interference. Using isolators, proper shielding and careful design the inter-channel isolation can be maintained greater than
90 decibels. An inter-channel isolation of this magnitude results in linearity errors over a 60 decibel dynamic range of less than ±0.25 dB.

After heterodyning of the two input signals to a final frequency of 1 KHz, phase measurement becomes essentially time measurement. Figure 9.8 shows a second output from each of the 1 KHz IF amplifiers feeding an axis crossing sensor. As the name implies each axis crossing sensor senses the time of the axis crossover of the 1 KHz signal. Specifically it senses each positive axis crossing. At that instant, each axis crossing sensor provides a narrow pulse output. Measurement of the time between axis crossing of the signal in Channel A and that of Channel B can be directly translated to degrees at 1 KHz and consequently to the input RF signal. With a system of this type, amplitude variations of the signals in the two signal channels and the phase difference between them can be measured simultaneously. Amplitude and phase readouts can be provided in either analog or digital form as required.

**Summary** - - - The heterodyne method of phase measurement, using phase-locked, double conversion receivers with harmonic mixing is ideally suited to the microwave antenna phase measurement problem for the following reasons:

1. The use of harmonic mixing permits wide physical separation between the test points at which the two test signals are sampled. Distances of several hundred feet are practicable.
2. The use of harmonic mixing permits phase measurements to be made over wide frequency ranges with a single receiver and without use of multiple RF heads.
3. The phase-locked receiver, combined with a phase-locked source provides a degree of frequency stability in the complete measurement system which permits accurate phase measurements even under the conditions of (1).
4. The phase-locked, double-conversion receiver provides a narrow noise bandwidth and consequently high sensitivity. This permits use of small probes and isolation of test devices.
by means of pads or directional couplers to reduce errors from reflections without requirement for high-level signal sources.

(5) Conversion of the RF signals to the output frequency rather than use of square-law detection results in a linear input-output amplitude relationship. This results in an amplitude dynamic range of greater than 60 decibels without special feedback arrangements and permits linear conversion of amplitude data to digital form.

(6) The heterodyne method permits continuous phase measurement over an unlimited range of phase with a linear relationship of phase readout to RF phase change. Measurement of phase by the differential time delay between two lower frequency pulses permits easy conversion of the output data to digital form.

9.4 APPLICATIONS

The Apollo flights are an excellent example of the increased emphasis being placed on complete antenna measurements. Virtually every antenna on the Apollo was designed and tested for proper polarization characteristics of its radiation pattern. The quality and reliability of its communications is testimony to how well this program was performed.

Measurement of the complete polarization characteristics of radiation patterns is probably receiving more attention than any other single antenna characteristic today. For this reason this subject has been treated as a separate chapter
and the reader is referred to Chapters 3 and 10 for a treatment of the theory of polarization as well as a discussion on application of phase measurements to determination of the polarization characteristics of antennas.

As an illustration of the use of phase measurements in antenna design, we will consider the primary-feed measurement problem.

The primary feed of a reflector is designed to produce a desired illumination of the reflecting surface. If the associated antenna is to generate a pencil beam, the feed is required to produce a planar phase front over the radiating aperture, with an illumination taper which is consistent with the required side lobe structure of the secondary pattern.

If the secondary pattern is to be properly focused, the "phase center" of the feed must be located at the focal point of the reflector. This is on the assumption that the feed has a unique phase center, although antennas in general do not rigorously satisfy this criterion. A unique phase center implies that the phase is constant over some sphere surrounding the antenna, and even for a well focused antenna this situation does not generally exist. On the other hand, over a major portion of the main lobe, the phase is usually relatively constant over some spherical segment, whose center is defined as the phase center of the antenna. The deviation of the phase front from spherical over a specified region of solid angle depends on the design of the feed and the location of the center of the spherical reference surface.

Before proceeding further let us consider the geometry of Figure 9.9. Let a small antenna be located at the origin O of a spherical coordinate system with its main lobe directed along the X axis, which is in the plane of the paper. Let the center of rotation of a probe antenna be at O', in the plane of the equator. If the probe is at an angle $\phi$ from the antenna axis, since $OX = O'P$,

$$r' + \Delta = \Delta \cos \phi + r' + \delta$$

(9.24)

Thus, $\delta$ is given by

$$\delta = (1 - \cos \phi)\Delta$$

(9.25)
The phase deviation of the measured phase front from constant phase is given by $2\pi \delta/\lambda$ radians. It is evident from (9.25) that the resolution in determination of $\Delta$ is dependent on the width of the angular sector of $\phi$ over which the measurements are made.

Center of phase measurements setups are usually arranged so that $\Delta$ can be varied. Evidently if $\Delta$ is zero, $\delta$ must be zero on the assumption of a circular phase front in the plane of exploration. If the field should be identically circular in the plane of exploration, the phase of the signal measured by the probe antenna would be constant with rotation in $\phi$. In practice, of course, $\delta$ will not be zero for all $\phi$. Figure 9.10 illustrates measurements of the amplitude and phase of a typical, small feed horn, made in a planar cut through the peak of the beam. The measurements were made in a setup similar to that illustrated in Figure 9.11 (discussed later in this section).
although they could have been made with a simpler setup consisting of only an azimuth positioner and a fixed sampling antenna.

FIGURE 9.10 Phase and amplitude patterns of a small feed for the $F_\theta$ position taken in the plane $\theta=90^\circ$.

(a) Center of rotation off axis and slightly in front of the phase center of the horn.
(b) **Center of rotation nearly on axis, but slightly in front of center of phase.**

(c) **Center of rotation nearly coincident with center of phase.**
The illustrated measurements describe the amplitude and phase patterns and location of the center of phase of a feed in one plane. One of the important problems of feed design is to determine the degree of astigmatism which characterizes the feed under test. Astigmatism is the wave front aberration which is produced by a difference in radius of curvature of the phase front in two orthogonal planes.

Measurements of astigmatism can be made in a set-up like that of Figure 9.11 by locating the center of phase in the plane of the equator ($\theta = 90^\circ$) as described above using the azimuth positioner and by then measuring the phase deviation in the orthogonal plane by measurements in $\theta$ using the gantry positioner. *

The positioner of Figure 9.11 permits, in addition, sampling of the phase and amplitude of fields as functions of $\phi$ and $\theta$ over the complete sphere. Measurements in $\phi$ for incremental $\theta$ angles of other than 90 degrees result in $\phi$-cuts, which are also called conical cuts. Measurements in $\theta$ for incremental $\phi$ angles result in $\theta$-cuts, which are great circle cuts. See Chapter 5.

![Diagram of positioner configuration](image)

**FIGURE 9.11** Positioner configuration in which sampling antenna is supported by gantry to provide rotation in $\theta$. Base positioner provides rotation in $\phi$.

*See next page for footnote.*
Figure 9.11 is schematic in nature. Not shown are height, axial and lateral adjustments, which are provided for accurately positioning the antenna to be tested at the point of coincidence of the azimuth and elevation axes, and coaxial cables, which deliver the source signal to the antenna under test and the sampled signal to the phase-amplitude measurement receiver. Phase shiftless rotary joints must be used on all axes to eliminate phase error due to cable flexing with rotation. Microwave absorber should be used on the positioner and gantry surfaces.

Bartlett and Moseley of Radiation, Inc. made extensive use of primary-feed phase measurements in their development of the Dielguide. A paper on their program was published in the December 1966 issue of Microwave Journal.4

Turning again to the test fixture shown in Figure 9.11, it must be understood that a degree of mechanical stability and precision commensurate with the allowable phase error and the operating wavelength is required of the support brackets and azimuth positioner. Without proper care the positioning system can be one of the greatest sources of measurement error. All coaxial cables should be securely clamped to avoid phase shift with movement.

The sampling antenna should be small enough to avoid integration of fine grain phase variation; a small electromagnetic horn or section of open waveguide is typically used. In addition, reducing the gain of the sampling antenna reduces interaction between the radiator and the probe caused by multiple reflections.

*(p. 9-18) Measurement of astigmatism can also be made with a source antenna located in a fixed position in the equator of Figure 9.11. The base positioner (φ axis) is employed, but the gantry and polarization rotator are not essential. E-plane and H-plane cuts are made with source and test antennas rotated together through 90 degrees in polarization between the E-plane and H-plane cuts.
\[ \Theta = \text{Elevation Angle} \]
\[ \Psi = \text{Azimuth Angle} \]

FIGURE 9-12  Measured far-field amplitude and phase patterns of a small horn probe.
Figure 9.12 is a set of phase and amplitude patterns of a small probe antenna which were made on a positioner of the type shown in Figure 9.11\textsuperscript{5} using a phase-locked receiver of the type described in this chapter.

REFERENCES

CHAPTER 9


The polarization of a reciprocal antenna is defined by the polarization of the wave radiated by the antenna, and measurement of the polarization of the antenna requires determination of the polarization of this wave. Sometimes the measurements are made directly; sometimes the response of the antenna to specific illuminating fields is measured as an alternate procedure. This chapter is concerned with techniques for making these measurements.

10.1 MEASUREMENT METHODS

The following are specific methods which can be employed to measure polarization:

(a) Measurement of the ratio of the amplitudes of two orthogonal linear polarization components of the field and their relative phase.

(b) Measurement of the ratio of the amplitudes of two orthogonal circular polarization components of the field and their relative phase.

(c) Measurement of the ratios of the amplitudes of two orthogonal circular polarization components, two orthogonal linear polarization components and a second set of orthogonal linear components which are displaced 45-degrees in tilt angle from the first set. This method does not require measurement of phase.

(d) Measurement of the axial ratio and tilt angle of the major axis of the polarization ellipse by means of the polarization pattern

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and determination of the sense of polarization by means of an auxiliary measurement with at least one antenna which is not linearly polarized.

(e) Measurement of the amplitudes of two orthogonal linear components, the tilt angle \( \tau \) and the sense of polarization. This method breaks down when the two linear components become equal and is not recommended.

Although this list is not exhaustive, it covers the more commonly used methods. These are summarized in Table 10.1 in terms of the nomenclature which was defined in Chapter 3.

### TABLE 10-1
SUMMARY OF METHODS OF MEASURING POLARIZATION

<table>
<thead>
<tr>
<th>Method</th>
<th>Measured Parameters</th>
<th>Poincare' Sphere Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( \rho_L, \delta )</td>
<td>( 2\alpha, \delta )</td>
</tr>
<tr>
<td>(b)</td>
<td>( \rho, \delta' ) or ( \tau )</td>
<td>( 2\gamma, \delta' = 2\tau )</td>
</tr>
<tr>
<td>(c)</td>
<td>( \rho_L, \rho_0, \rho )</td>
<td>( 2\alpha, 2\beta, 2\gamma )</td>
</tr>
<tr>
<td>(d)</td>
<td>( r, \tau, \text{sense} )</td>
<td>( 90^\circ - 2\gamma, 2\gamma, \text{upper or lower hemisphere} )</td>
</tr>
<tr>
<td>(e) **</td>
<td>( \rho_L, \delta' ) or ( \tau, \text{sense} )</td>
<td>( 2\alpha, \delta' = 2\tau, \text{upper or lower hemisphere} )</td>
</tr>
</tbody>
</table>

* \( \rho_L = E_2/E_1, \delta_0 = E_4/E_3, \rho = E_8/E_L \)

** Not recommended; see text, page 10-13.

Before proceeding to the discussion of these methods we will make some comments which apply to polarization measurements in general.

1. With reference to Table 10.1, the parameters listed in the right-hand column (a-d) are used to define the polarization state on the Poincare' sphere for purposes of visualization, while the corresponding parameters listed in the middle column are usually used to designate polarization of an antenna in practice. The polari-
zation is often indicated in communications and contractual documents by (d), that is \( r, \tau \) and sense.

(2) The measurement of polarization requires an understanding of the polarization of electromagnetic waves and of the power transfer between antennas. These subjects were treated in Chapter 3.

(3) The polarization of an antenna is a function of the angular coordinates \( \phi \) and \( \theta \) describing direction from the antenna. Often the polarization is required only at the peak of the beam, that is, for a particular \((\phi, \theta)\) direction. This is often the case for pencil-beam antennas, for example. On the other hand, the polarization of antennas which are omnidirectional in character, especially those used on airplanes and space-vehicles, is usually required as a function of direction and often over the complete region of solid angle about the antenna.

(4) Each of the methods indicated in Table 10.1 determines a unique point on the Poincare' sphere, which represents the polarization state of the wave. In many cases in practice, a unique location on the sphere is not determined, as for example, when the axial ratio and tilt angle are measured but the sense is not determined. Although this results in an ambiguity between conjugate points in the upper and lower hemispheres of the Poincare' sphere, the information may be adequate when the polarization of the antenna under test is nearly linear so that the two conjugate points lie close to the equator.

In certain cases the axial ratio and sense are determined, but the tilt angle is not measured because it is not of significant interest. This is particularly true of an antenna whose polarization is nearly circular. In this case the polarization lies near the pole of the Poincare' sphere with the result that the power transfer between the antenna and any other antenna is not very sensitive to the tilt angle (See Figure 3.30).

(5) It is not possible to determine the sense of polarization of an antenna without (1) making at least one measurement with a
sampling antenna * which is not linearly polarized or (2) making measurements of relative phase. Sometimes the sense of an antenna whose polarization is near circular will not be measured but will be deduced from the physical or electrical construction of the antenna. If this procedure is followed and if the using system is to communicate with another antenna, care must be exercised to avoid a mistake in the sense of polarization. Even when the sense is being measured there is substantial hazard of blundering.

(6) Rigorous determination of polarization must be made either by a three antenna method (See Section 10.4) or by use of a polarization standard which has been calibrated by a three antenna method. This fact is usually ignored in making polarization measurements, and the final "standard" is taken as a horn or linear dipole which is assumed to have precisely linear polarization from its construction.

(7) In addition to specification of the axial ratio of a polarization standard **, it is also necessary to specify its tilt angle relative to a defined reference direction. Because of the broad shape of the pattern near the maximum, the tilt angle is measured in practice by determining the direction of the null and adding ± 90 degrees to determine the direction of the peak. A three-antenna method for accomplishing this is described in Section 10.5, but here again rigorous determination of the tilt angle is often not made, and the tilt angle deduced from the construction of the "standard".

(8) Accurate measurement of polarization requires the same attention to range design and evaluation that is required in other types of antenna measurements. Antenna range design and evaluation are covered in Chapter 14.

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* It is assumed that the measurements are made on transmission. If the measurements are made on receiving this would be a source antenna.

** The axial ratio of the standard is presumed to be so large that the sense is of no consequence.
(a) **Linear Component Method, Figure 10.1.**

The angle $2\alpha$ of the Poincare' sphere is usually defined by the tangent of the half angle:

$$\tan \alpha = \frac{E_2}{E_1} = \alpha_l.$$  

It can also be defined by the sine or cosine of the half angle:

$$\sin \alpha = \frac{E_2}{(E_1^2 + E_2^2)^{1/2}} = \rho_l$$

$$\cos \alpha = \frac{E_1}{(E_1^2 + E_2^2)^{1/2}} = \rho_1$$  

$\rho_l = \frac{E_{2l}}{E_{1l}} = \frac{E_2}{E_1}$

$\delta$ is phase of $E_2$ relative to $E_1$.

**FIGURE 10.1** Polarization described in terms of linear components on Poincare' sphere.

*In Figures 3.10-3.12 the polarization of a wave is described by the letter W. In this chapter we are concerned with measurement of the polarization $A$ of an antenna, defined by the polarization of the wave radiated by the antenna.

**See page 3-17 for definition of subscribed dot.**
Note that \( \sin \alpha \) and \( \cos \alpha \) also define the amplitudes of the elements of the polarization matrix (See page 3-30 and Appendix 3G).

Finally, \( 2\alpha \) is uniquely defined by its cosine within its range of 0 to 180 degrees:

\[
\cos 2\alpha = \frac{E_1^2 - E_2^2}{E_1^2 + E_2^2} = \frac{F_1^2 - F_2^2}{E_1^2 + E_2^2} \tag{10.4}
\]

The angle \( 2\alpha \) is not uniquely defined by its sine because the sine is symmetrical about 90-degrees. Therefore the equation,

\[
\sin 2\alpha = \frac{2E_1E_2}{E_1^2 + E_2^2} = \frac{2F_1F_2}{E_1^2 + E_2^2} \tag{10.5}
\]

should not be used for this purpose.

The phase angle \( \delta \) is the electrical angle by which \( E_3(t) \) leads \( E_1(t) \). (See page 3-3).

(b) **Circular Component Method, Figure 10.2.**

The equations in terms of circular polarization components corresponding to (10.1) through (10.4) are:

\[
\tan \gamma = \frac{E_\alpha}{E_\ell} = \rho \tag{10.6}
\]

\[
\sin \gamma = \frac{E_\alpha}{(E_\ell^2 + E_\alpha^2)^{\frac{1}{2}}} = F_\alpha \tag{10.7}
\]

\[
\cos \gamma = \frac{E_\ell}{(E_\ell^2 + E_\alpha^2)^{\frac{1}{2}}} = F_\ell \tag{10.8}
\]

\[
\cos 2\gamma = \frac{E_\ell^2 - E_\alpha^2}{E_\ell^2 + E_\alpha^2} = \frac{F_\ell^2 - F_\alpha^2}{E_\ell^2 + E_\alpha^2} \tag{10.9}
\]
\[ r = \frac{E_R + E_L}{E_R - E_L} = \frac{\rho + 1}{\rho - 1} \]
\[ \rho = \frac{E_R}{E_L} = \frac{r + 1}{r - 1} \]

\( \delta' \) is phase of \( E_R \) relative to \( E_L \).

**FIGURE 10.2** Polarization described in terms of circular components on Poincaré sphere.

Equation (10.6) describes the circular polarization ratio, \( \rho \). The axial ratio \( r \) is related to \( \rho \) by (3.10) and (3.11) on page 3-8.

The phase angle \( \delta' \) is the electrical angle by which \( E_R(t) \) leads \( E_L(t) \). (See page 3-5). It is numerically equal to twice the tilt angle \( \tau \).

(c) The Multiple Amplitude Component Method.

This method requires measurements of only the relative amplitudes of the \((1,2), (3,4), \) and \((L,R)\) sets of orthogonal polarizations. These indicated relative amplitudes determine the polarization ratios, \( \rho \), \( \rho_L \), and \( \rho_D \), and as a result, the direction angles \( 2\alpha, 2\beta, \) and \( 2\gamma \) since:

\[ \tan \alpha = \rho_L = \frac{E_2}{E_1} \]  

(10.10)
\[ \tan \beta = \rho_0 = \frac{E_4}{E_3} \]  

(10.11)

\[ \tan \gamma = \rho = \frac{E_2}{E_1} \]  

(10.12)

Each of these direction angles uniquely defines a circle on the surface of the sphere centered on one of the axes (1, 2), (3, 4) or (L, R) and the intersections of these circles have only one point in common. This unique point is the polarization of the antenna or wave.

Since

\[ E_1^2 + E_2^2 = E_3^2 + E_4^2 = E_1^2 + E_2^2 \]  

(10.13)

it is necessary to make only one measurement from each orthogonal set and one additional measurement from any of the three sets. The other two remaining parameters can be calculated from (10.10). In this case, however, it is necessary that the gains of all the sampling antennas and their respective receiving networks be equal or known.

It is usually desirable to convert the data measured by the multiple component method to the format of one of the other methods. This is easily accomplished through use of the equations in Appendix 3G. For example, suppose one wishes to express the multiple component data in terms of \( \rho \) and \( \delta' \). It can be seen from equation (3.21) on page 3-11 that

\[ \delta' = \tan^{-1} \frac{E_3^2 - E_4^2}{E_1^2 - E_2^2} \]  

(10.14)

It can be shown that, in terms of the measured quantities \( \rho_0 \) and \( \delta_0 \), this can be expressed as

*See also Figure 3.16, page 3-27.
\[ \delta' = \tan^{-1} \frac{(1 + \rho_l)^2(1 - \rho_b^2)}{(1 + \rho_b)^2(1 - \rho_l^2)} \]  

(10.15)

This allows one to determine \( \delta' \) and hence the tilt angle \( \tau \). Note that an ambiguity of 180 degrees results when an angle is defined by its tangent alone, and this ambiguity in the inverse tangent results in a 90° ambiguity in the tilt angle \( \tau \). The ambiguity can be resolved by inspection of the signs of the numerator and denominator of (10.14) or (10.15). It is also helpful to consider these equations in relation to the Poincare' sphere by reference to Figure 3G. 1.

Table 10-2 resolves the location of \( \delta' \) and \( \tau \) in terms of (10.14) and (10.15).

<table>
<thead>
<tr>
<th>( \delta' = 2\pi )</th>
<th>Numerator</th>
<th>Denominator</th>
<th>( \rho_b )</th>
<th>( \rho_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>+</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1st Quadrant</td>
<td>+</td>
<td>+</td>
<td>(&lt;1)</td>
<td>(&lt;1)</td>
</tr>
<tr>
<td>90°</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2nd Quadrant</td>
<td>+</td>
<td>-</td>
<td>(&lt;1)</td>
<td>(&gt;1)</td>
</tr>
<tr>
<td>180°</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>(\infty)</td>
</tr>
<tr>
<td>3rd Quadrant</td>
<td>-</td>
<td>-</td>
<td>(&gt;1)</td>
<td>(&gt;1)</td>
</tr>
<tr>
<td>270°</td>
<td>-</td>
<td>0</td>
<td>(\infty)</td>
<td>1</td>
</tr>
<tr>
<td>4th Quadrant</td>
<td>-</td>
<td>+</td>
<td>(&gt;1)</td>
<td>(&lt;1)</td>
</tr>
<tr>
<td>Undefined</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Similar arguments can be made when it is desired to express the data in terms of \( \rho_l \) and \( \delta \) or in terms of \( \rho_b \) and \( \delta'' \).

(d) The Polarization Pattern Method.

The polarization of an antenna except for sense can be measured by rotating a linearly polarized sampling antenna about an axis joining the sampling
antenna and the antenna under test. The power received from the sampling antenna is recorded as a function of its angular orientation in either a rectangular format as shown in Figure 10.3 or a polar format as shown in Figure 10.3a.

FIGURE 10.3 Typical polarization pattern in a rectangular format.
FIGURE 10.3a Typical polarization pattern in a polar format.
If the polarization of the sampling antenna is identically linear, the axial ratio is the ratio of the maximum received amplitude to the minimum received amplitude in the case of a linear display, or the difference in the maximum and minimum power levels when the power is plotted in logarithmic form. This can be seen from the following analysis.

It is shown in Chapter 3* that the polarization efficiency between two arbitrarily polarized antennas is given by

$$\Gamma = \frac{1 + \rho_x \rho_y - 2 \rho_x \rho_y \cos \Delta}{(1 + \rho_x^2)(1 + \rho_y^2)}$$  (10.16)

where $\Delta$ is twice the physical angle in space between the polarization ellipses. When the sampling antenna is linearly polarized, $\rho_r = 1$, and (10.16) reduces to

$$\Gamma = \frac{1 + \rho_y^2 + 2 \rho_y \cos \Delta}{2(1 + \rho_y^2)}$$  (10.17)

The maximum and minimum values of $\Gamma$ occur when $\Delta = 0^\circ$ and $180^\circ$, respectively. For the case of the linearly polarized sampling antenna, the axial ratio $r_\phi$ is given by

$$r_\phi = \pm \left( \frac{\Gamma_{\text{max}}}{\Gamma_{\text{min}}} \right)^{\frac{1}{2}} .$$  (10.18)

Since $\Gamma$ in (10.17) is invariant to substitution of $1/\rho_\phi$ for $\rho_\phi$, the polarization pattern does not define the sense of rotation.

If sense is to be determined, at least one additional measurement is required with an antenna whose polarization is not linear and whose sense is known; usually two circularly polarized antennas of opposite sense are employed, and the ratio of the power received by the two from the test antenna determines the sense.

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* Page 3-40.
** See also Figures 3.21 and 3.28.
Practically speaking, if measurements are to be made for many \((\phi, \theta)\) directions about the antenna under test, and especially if sense is required, another method should be considered. The polarization pattern method is largely used where the axial ratio and possibly the tilt angle of a narrow-beam antenna are required only at the peak of the beam. In this case if the beam direction is coincident with a mechanical axis, the antenna under test can be rotated instead of the sampling antenna.

If the measurements are to be made by this method on a ground reflection antenna test range, the antenna under test must be rotated, or tedious calibration procedures must be employed to account for the difference in the reflection coefficient of the range surface for horizontal and vertical components of the field. (See page 10-26).

A variation of the method is often used to obtain the axial ratio as a function of direction about an antenna under test, especially where the antenna is nearly circularly polarized. Assume that the tests are made on transmission. A linearly polarized sampling antenna is rotated relatively rapidly in the field of the antenna under test while the latter antenna is slowly scanned through a desired angular sector. An example of the resulting pattern is shown in Figure 10.4. Superimposed on the pattern of the antenna is a high-pitched cyclic variation, synchronized at a 2:1 ratio with the rotation of the sampling antenna. The ratio of the amplitudes of adjacent maxima and minima represents the axial ratio of the antenna under test if the main pattern does not change appreciably during one-half resolution of the sampling antenna. It is evident that sense is not provided. The tilt angle could conceivably be obtained by a marker which would indicate the orientation of the sampling antenna, but this is usually not done in practice. The bandwidth of the measurement system must be adequate to give faithful reproduction of the polarization ripple at the speed of rotation of the sampling antenna.

(e) The following method has been mentioned in the literature:

Specification of the ratio of the magnitudes of two orthogonal linear components, the tilt angle \(\tau\) of the major axis of the polarization ellipse and the sense of rotation.
It breaks down when the magnitudes of the components are equal and is therefore not recommended.

As in method (a) the ratio of two orthogonal linear components defines a curve of constant $2\alpha$, that is, a latitude line with respect to the (1) axis. The tilt angle $\tau$ defines a meridian of longitude (constant $\delta'$), with respect to the LR axis. In the general case this line of constant $\tau$ or $\delta'$ intersects the curve of constant $2\alpha$ in two places, one in the L hemisphere and one in the R hemisphere as shown in Figure 10.5. Determination of the sense of polarization then determines the polarization uniquely. This is not the case, however, for $2\alpha = 90^\circ$, for at this position, the $2\alpha$ curve degenerates.
to a great circle coincident with the constant τ curve, and the ratio $E_R/E_L$ is indeterminate.

FIGURE 10.5 Poincare' sphere showing definition of unique polarization by means of (1) the ratio of two orthogonal linear components, defining $2\alpha$, (2) the tilt angle, defining the angle $2\tau$, and (3) the sense indicating the upper or lower hemisphere. This method breaks down when $\tau = 45$ degrees. This is evident from both the Poincare' sphere in (a) and from the polarization ellipses of (b); both of these show that when $\tau \approx 45$ degrees the axial ratio is not determined by the ratio $E_2/E_1$. 
Coordinate systems for antenna measurements are covered in Chapter 5. The coordinate systems required in definition of the polarization properties of waves and antennas are described in Chapter 3. In certain simple types of polarization measurements involving determination of a particular polarization characteristic, such as the axial ratio at the peak of the beam by rotation of a linearly polarized sampling antenna in the field, coordinates are not particularly important. In general, however, in making polarization measurements it is essential that the coordinate systems involved be understood and employed properly if gross errors are to be avoided.

The coordinate system problem tends to be confusing for two reasons:

(1) While the polarization of an antenna is defined by the polarization of the wave it transmits, the measurements may be made on receiving or transmitting.

(2) The polarization of the sampling antenna or of the source antenna (depending on whether the measurements are made on transmitting or receiving) is defined in a different coordinate system than that of the antenna under test.

The polarization \( A \) of the antenna under test is defined in the \( (\vec{u}_{1t}, \vec{u}_{2t}, \vec{u}_{pt}) \) coordinate system of Figure 10.6a where the \( \vec{u}_{1t} \) direction is defined by the measurements problem. \( A \) is also the polarization of the wave \( W \) which is incident on the aperture of the sampling antenna. However, the polarization of the sampling antenna is defined, in accordance with accepted standards, in the \( (\vec{u}_{1s}, \vec{u}_{2s}, \vec{u}_{ps}) \) coordinate system with \( \vec{u}_{ps} \) in the direction of propagation of a wave which it would transmit. To relate the coordinate systems, we show the \( \vec{u}_2 \) directions of all of the coordinate systems aligned; the results would be identical if the \( \vec{u}_1 \) directions were aligned.

The polarization of the sampling antenna and the antenna under test must be described in the same coordinate system before meaningful measurements can be made. The coordinate system of the sampling antenna is transformed to the coordinate system \( (\vec{u}_{1rs}, \vec{u}_{2rs}, \vec{u}_{prs}) \). In this coordinate system the
(a) Transmission by antenna under test.

(b) Reception by antenna under test.

(c) Alternate coordinate system conversion for reception by antenna under test.

FIGURE 10.6 Coordinate system conversions for use in polarization measurements.
polarization of the sampling antenna is described by its receiving polarization. (See Sections 3.5, 3.6, and Appendix 3D). The polarization of the incident wave is then measured in this coordinate system, which is the same as that of the antenna under test.

If the antenna under test is tested on receiving, it is usually convenient to transfer the polarization of the incident wave into the coordinate system of the antenna under test as in Figure 10.6b rather than transferring the coordinate system of the antenna under test to that of the incident wave as in Figure 10.6c (see the discussion on page 3-38 and 3-39). If the conversion of 10.6b is used, the measured polarization is in the coordinate system of the wave which is transmitted by the antenna under test and is therefore its defined polarization. If the conversion of 10.6c is used, while the incident wave is described in its proper coordinate system, the measured polarization is the receiving polarization, which must then be transferred to the polarization of the antenna under test as defined on transmitting.

The conversion of 10.6c has merit in testing non-reciprocal antennas, where the receiving polarization and the polarization are not necessarily related. In this case it is essential to specifically define whether the polarization measured on receiving defines the receiving polarization or a fictitious polarization which is the polarization of the wave which would be radiated by the antenna under test if it were reciprocal.

10.3 IMPLEMENTATION OF THE PHASE-AMPLITUDE METHODS OF MEASURING POLARIZATION

If the antenna whose polarization characteristics are to be measured is reciprocal or if it is a non-reciprocal transmitting antenna, polarization measurements which involve measurement of phase are most conveniently made on transmission. In this event the necessary data can be obtained in one measurement as a function of direction about the antenna and without
the necessity for rotation of a sampling antenna. This permits measurements to be made very rapidly. The measurements involve determination of the phase differences and the relative amplitudes of the two orthogonal outputs of a dual polarized sampling antenna.

A different technique is required if the antenna under test is a non-reciprocal receiving antenna. Since it has only one port, two sequential measurements are required to determine the relative amplitude and phase of the response of the antenna to two orthogonal illuminating fields. The phase measurements in this case are measurements of phase shift rather than phase difference (See page 9-4). The phase shift technique can also be used to advantage in testing reciprocal antennas, although it has certain disadvantages, which are pointed out, in addition to the requirement for two sequential measurements.

Reciprocal Antennas or Non-Reciprocal Transmitting Antennas.

Implementation of the technique for making polarization measurements by this method is shown schematically in Figure 10.7. The antenna under test is mounted on a test positioner so that it can be oriented over the required angular sector in $\phi$ and $\theta$. (See Chapter 5). The antenna is operated on transmission, and a dual polarized sampling antenna is located in the far-zone radiated field. Figure 10.7 illustrates method (a) in which the two sampled polarizations are linear, specifically in the $u_1$ and $u_2$ directions.

* The multiple component method (Method (c), pages 10-1 and 10-7) permits continuous measurements to be made as a function of direction about the antenna under test. It does not involve measurement of phase, but requires at least three sequential measurements unless a relatively complicated measurement system is employed.
FIGURE 10.7  Schematic illustration of polarization measurement by method (a), using the phase-amplitude technique. The arrangement shown uses a dual polarized sampling antenna with orthogonal linear polarization outputs.

A reference antenna of known polarization must be employed in place of the antenna under test to calibrate the system. Usually the reference antenna will be linearly polarized. Regardless of the polarization of the reference, the relative gains of the two channels of the receiver will be set to give equal outputs proportional to $E_1$ and $E_2$ when the tilt angle of the reference is set at 45 degrees. If the reference is linearly polarized, the phase reference will be set such that the indicated value of $\delta$ is zero when the
tilt angle of the reference is between zero and 90-degrees, and it should remain at zero over this interval of \( \tau \). If the polarization of the reference is not linear, the phase reference of the receiver should be set at the value of \( \delta \) which corresponds to the known axial ratio of the reference when the tilt angle is at 45 degrees.

In the above discussion it was assumed that the sampling antenna was a single antenna with dual polarization outputs. Alternatively, two separate orthogonally polarized antennas can be placed side by side and used instead of a single antenna. This method has the advantage, especially if horns can be used, of providing more inherently linear polarizations than those typically provided by a dual polarized antenna. On the other hand, the two separate antennas do not sample the radiated field in a single direction from the antenna under test because of the physical sizes of the two antenna. The angle \( d/R \) defined by the ratio of the separation \( d \) between the antennas to the distance \( R \) to the antenna under test must be made small enough compared with the beamwidth of the antenna under test for the resulting error to be compatible with the required accuracy of the measurements.

Method (a), discussed above, is based on two sampling polarizations which are orthogonal and linear. If Method (b) is to be used, the analysis will be essentially the same. The subscripts \( L \) and \( R \) will replace 1 and 2, respectively, and \( \gamma, \delta' \) and \( \rho \) will replace \( \alpha, \delta \) and \( \rho_L \) in (10.2) through (10.8) and in the accompanying discussion. Figure 10.2 then applies. Determination of the ratio \( E_R/E_L \) for an unknown test antenna defines a circle of constant \( 2\gamma \) on the Poincare\'s sphere, while \( \delta' \) defines the meridian through \( \Lambda \).

* Even if the polarization of the reference is linear, the indicated value of \( \delta \) will not remain at zero unless the polarizations of the two sampling antennas are orthogonal and linear. See Figure 10.9 and the accompanying discussion for a means of achieving this requirement.

** It is essential here to properly define the tilt angle of 45 degrees as the \( \mathbf{u}_{3t} \) direction of Figure 10.6. Further, the indicated sign of \( \delta \) must be set such that \( \delta \) is positive for left-hand elliptical polarization. Resolution of the sense of \( \delta \) requires use of a reference antenna whose sense of polarization is known, a simple experiment to determine the sense by making a small change in line length in one of the RF signal channels, or knowledge of the indicated sense of phase from the published information on the receiver.
In Method (b), if one antenna is used, it will have a polarization network to provide orthogonal circular polarization responses. The system can be calibrated with a linearly polarized reference antenna in place of the test antenna. The gain of the receiver must be set to give equal outputs proportional to $E_A$ and $E_L$, independent of the orientation of the reference antenna. The phase angle indicating $\delta'$ should be set to zero when the reference antenna is in the $u_{1t}$ direction and it should read $+90^\circ$ when the antenna is in the $u_{3t}$ direction ($\tau = 45^\circ$).

The above procedure will not be possible if the two polarizations of the sampling antenna are not circular and orthogonal. An arrangement which can be employed to provide identically circular and orthogonal outputs is shown in Figure 10.8. This arrangement uses a single antenna with orthogonal, linear feeds A and B and a 90-degree hybrid summation network which is intended to give right- and left-circular outputs. Each

![Diagram](image)

**FIGURE 10.8** Trim networks for use in providing identically circular orthogonally polarized outputs for a receiving antenna.
channel has a trim network consisting of variable phase shifter and attenuator paralleling the hybrid.

The trim network can be adjusted to compensate for the phase and amplitude errors in the hybrid and the feeds. Adjustment is made by rotating a reference transmitting antenna about its axis and adjusting the phase shifter and attenuator in each channel to give constant outputs with rotation of the reference antenna.

A similar arrangement can be used if two separate orthogonal circularly polarized sampling antennas, a dual polarized linear antenna, or two separate orthogonal linear antennas are employed. In the linear polarization case the network is as shown in Figure 10.9.

FIGURE 10.9 Trim networks for use in providing identically linear orthogonal polarized outputs for a receiving antenna. The feeds can also represent two separate antennas, placed side by side.
Reciprocal Antennas or Non-Reciprocal Receiving Antennas --- If an antenna is non-reciprocal and is to be operated on receiving, its polarization must be measured on receiving. This prevents use of the methods described above, which are only applicable to reciprocal or non-reciprocal transmitting antennas.

Testing of this type of antenna is illustrated in Figure 10.10. The antenna under test is mounted on a test positioner which has the required degrees of freedom. The local oscillator signal from a phase amplitude receiver is directed to a mixer mounted on the antenna through one or more phase-shiftless rotary joints. At the lower frequencies, a short length of coiled, semi-rigid, solid dielectric cable (.150 OD) may provide a small enough phase shift with rotation to permit its use in place of the rotary joints. The transmitting antenna is linearly polarized and is mounted on a polarization positioner. The signal from the source is connected to the transmitting antenna, again using a rotary joint or short length of cable as described for the receiving end.

*Rotary joints are concentric with positioner axes.

FIGURE 10.10 Illustration of phase shift method of testing reciprocal antennas or non-reciprocal receiving antennas.
If the range length is short enough, as in an anechoic chamber, the phase reference can be obtained directly from the source through a directional coupler. If the range is longer, the reference can be obtained from a sampling antenna. The polarization of this antenna can be oriented at 45 degrees so that the reference amplitude will remain relatively constant for the two required orthogonal orientations of the transmitting antenna polarization. The precise amplitude of the reference signal is not important because it is only used as a phase reference and as a signal for phase locking the receiver.

Measurements are made through the desired angular sector, with the transmitting antenna first in the \( \vec{u}_1 \) direction and then in the \( \vec{u}_2 \) direction. No calibration is required if the phase reference is obtained directly from the source. In this case the ratio of the measured amplitudes \( E_2/E_1 = \rho_1(\phi, \theta) \), and the measured phase shift \( (\alpha_2 - \alpha_1) = \delta(\phi, \theta) \).

If the phase reference is obtained from a reference antenna, the phase reference may vary somewhat between horizontal and vertical polarizations of the source antenna (1) because the reference antenna may not be linearly polarized and (2) because it is not located on the axis of the beam of the source antenna. In this case a special calibration procedure is required. The antenna under test must be mounted on a polarization positioner such as that shown in Figure 5.24, and the family of cuts must include the direction defined by its axis. Calibration is made by rotation of the antenna under test about the line of sight to the source antenna, with the source antenna fixed in either the \( \vec{u}_1 \) or the \( \vec{u}_2 \) direction. The measured ratio \( E_2/E_1 \) and the measured relative phase \( (\alpha_2 - \alpha) \), will be direct measures of \( \rho_1 \) and \( \delta \), and this can be compared to the value of \( E_2/E_1 \) and \( (\alpha_2 - \alpha_1) \), obtained on axis during the course of the measurements, to provide a calibration correction.

The ratio \( E_2/E_1 \) should, of course, be identical for the two measurements if the measurements are made on an elevated range. The method is also discussed in Section 10.4 for use in making polarization measurements on ground reflection antenna ranges. In this case correction is likely to be required for both \( E_2/E_1 \) and \( (\alpha_2 - \alpha_1) \).
One disadvantage of the phase shift method is that rapid variations of phase can occur with rotation of the antenna if the phase center is located off the center of rotation of the positioner. Thus the antenna under test should be mounted so that its phase center is as near the axis of rotation as possible, especially if the measurements are made at the higher frequencies.

One must be careful in specifying the polarization of a non-reciprocal receiving antenna to indicate whether he is defining the receiving polarization of the antenna (that is the polarization of the wave to which the antenna is polarization matched) or the polarization of the wave which would be radiated by the antenna if it were reciprocal and used as a transmitting antenna. To our knowledge there is no standard defining the polarization of non-reciprocal receiving antennas.

10.4 POLARIZATION MEASUREMENTS ON GROUND REFLECTION ANTENNA TEST RANGES

It is necessary to exercise caution in making polarization measurements on ground reflection antenna test ranges because certain of the methods described in Section 10.1 cannot be applied directly.

In the ground-reflection case the wave illuminating the test antenna is an interference pattern which results from a combination of the direct path signal and the primary reflection from the range surface. Since the phase and amplitude of the reflection coefficients of the range surface are not identical for vertical and horizontal polarizations, the polarization of the field at the test aperture does not remain constant with rotation of the source antenna. Because of the difference in reflection coefficients, the tilt angle does not exactly follow that of the source antenna, and the total amplitude of the field is not the same with rotation of the polarization.

This total effect is increased also because the beam pattern of the source antenna is never rotationally symmetrical, causing the illumination of the range surface to vary with rotation of the source antenna. As a result precise polarization measurements can not be made on a ground reflection range by rotation of the source antenna without tedious calibration procedures. Polarization patterns can be made on ground reflection ranges by rotation of the antenna under test.
This is useful if the polarization is required only at the peak of the beam. It is not usually useful for polarization measurements off the peak of the beam because it is very difficult to mount antennas such that they can be rotated about an axis which is in any arbitrary direction with respect to the peak of the beam.

The phase amplitude methods which involve transmission by the antenna under test and use of fixed, orthogonally polarized sampling antennas can be employed directly on the ground reflection range. The range is calibrated by use of an antenna of known polarization as in the elevated range case.

The phase-shift method described in Section 10.2, page 10-24, has been used quite successfully on a ground reflection range to obtain polarization over a region of solid angle even though it involves rotation of the source antenna to horizontal and vertical polarizations. The measurements and calibration are made as indicated. Calibration of both the phase and amplitude channels is required to take into account change in the reflection coefficient of the range surface with polarization.

10.5 POLARIZATION STANDARDS

As indicated in Section 10.1, page 10-4, rigorous determination of polarization must be made either directly by a three antenna method or by use of a polarization standard which has been calibrated by a three antenna method. Complete calibration of a polarization standard requires determination of its axial ratio, sense and tilt angle. While the polarization of the standard can be some known elliptical polarization, linear polarization is usually the logical choice. This is because (1) nearly linearly polarized horn or dipole antennas can be fabricated relatively easily, (2) the tilt angle is precisely defined by the sharp null of the polarization pattern, and (3) the polarization is likely to be nearly independent of frequency. On the other hand the linear polarization standard cannot be used to indicate sense, and an independent method must be used for this purpose; often two orthogonal circularly polarized antennas of nearly equal gain are used.
It is possible to determine that an antenna has precisely circular polarization by rotating any non-circularly polarized antenna about its axis in the field of the antenna under test. It is further possible to produce any desired elliptical polarization by adding the fields of two orthogonal circularly polarized antennas in the proper relative phase and amplitude. This principle can be used to construct a variable polarization reference which can be calibrated in an independent polarization measurement either against a standard or by a three antenna method.

(a) A Three-Antenna Method for Evaluating the Axial Ratio of a Linear Polarization Standard.

Let three antennas A, B and C with nearly linear polarizations be operated in pairs as in Figure 10.11 to make three polarization patterns in a reflection-free test environment. It is shown in Appendix 10A that, if \( r_1 \), \( r_2 \) and \( r_3 \) are the absolute magnitudes of the apparent axial ratios, then the absolute magnitude of the true axial ratio \( r \) of the particular antenna which has the lowest axial ratio is bounded by

\[
\frac{1}{r} = \frac{1}{2} \left( \frac{1}{r_3} + \frac{1}{r_2} \pm \frac{1}{r_3} \right)
\]  

(10.19)

where \( r_3 \) is the magnitude of the highest apparent axial ratio. Thus, if any of the antennas is to be used as a standard, its axial ratio is known to be at least as high as that given by (10.19). Typical standard gain horns are likely to have axial ratios of the order of 40 decibels. An axial ratio of this order is adequate for most measurements. If a higher axial ratio is required and if tests show that the axial ratio may be too low, it may be possible to increase the axial ratio by slightly deforming the horns in an iterative manner, although the procedure is very tedious. An approach which is more orderly, but which requires two horns that are adjustable in polarization, is described below.

(b) A Three-Antenna Method for Absolute Measurement of Polarization.

Let it be required to determine the polarization of an antenna A of arbitrary polarization without the use of a polarization standard. Let antennas B and C of Figure 10.11 have adjustable polarizations (See Figure 15.6, page 15-9).
Now proceed as in (a) except that the two adjustable horns are to be adjusted to give zero nulls (within the limits of dynamic range) with antenna A. In this event $1/r_2$ and $1/r_3$ will be zero. It is shown in Appendix 10A that the axial ratio of antenna A is given by

$$\frac{r}{\rho - 1} = \frac{(r_1 - 1)\frac{\rho}{\sqrt{\rho}} + (r_1 + 1)\frac{\rho}{\sqrt{\rho}}}{(r_1 - 1)\frac{\rho}{\sqrt{\rho}} - (r_1 + 1)\frac{\rho}{\sqrt{\rho}}},$$

(10.20)

where $r_1$ is positive if B and C are right elliptical.

The polarizations of antennas B and C are identical and are orthogonal to that of horn A. The sense of polarization of antenna A can be found in the following manner. Since antennas B and C are identical, antenna C is now not necessary, and it is adjusted alternately for RHC and LHC polarizations by rotation against A or B. The ratios $\Gamma_{AB} / \Gamma_{BL}$ and $\Gamma_{AC} / \Gamma_{AL}$ are measured, where $\Gamma$ is the polarization efficiency. If $\Gamma_{AR} / \Gamma_{AL} > \Gamma_{BR} / \Gamma_{BL}$, then the sense of antenna A is right elliptical.

If all three antennas are adjustable in polarization, they can be adjusted iteratively so that $1/r_1$, $1/r_2$ and $1/r_3$ in (10.19) are zero. All three antennas are now linearly polarized.

(c) A Three-Antenna Method for Determining Tilt Angle.

Complete specification of the polarization of a polarization standard requires determination of the tilt angle of the polarization ellipse. Alternatively, it may be desired to directly determine the tilt angle of an operational antenna. The following is a three antenna method for which can be employed for this purpose.

An accurate reference line should be defined on each antenna. The major axes of the polarization ellipses lie at angles $\alpha$, $\beta$, and $\gamma$ yet to be determined with respect to the reference lines on antennas A, B and C respectively, as shown in Figure 10.11. These angles should be positive and less than 90° to avoid uncertainties in the equations which will follow.
The approximate position of the major axis of the polarization ellipse is usually known to an accuracy sufficient to allow that selection of the reference line. Antennas A and B are mounted on an antenna range facing each other. We will assume antenna A to be the transmit antenna and B the receive antenna. The reference lines of the antennas are then aligned parallel by optical means or through use of plumb lines. The major axes of the polarization ellipses are now separated by the angle $\alpha + \beta$. The receive antenna B is then rotated to the nearest null in the received power pattern and the angle through which the antennas were rotated is recorded. This angle, which we will call $\phi$, is in degrees and is positive if the rotation was in the direction of increasing $\beta$. 

FIGURE 10.11 Geometry used in determining direction of major axis of polarization ellipse.
negative if in the opposite direction. The angles $\alpha$ and $\beta$ can now be related by the equation:

$$\alpha + \beta + \phi_1 = 90^\circ.$$  \hspace{1cm} (10.21)

By repeating this procedure first with antennas A and C and then with B and C, we can relate the angles $\alpha$ and $\gamma$, and $\beta$ and $\gamma$ by the following equation:

$$\alpha + \gamma + \phi_2 = 90^\circ, \text{ and}$$  \hspace{1cm} (10.22)

$$\beta + \gamma + \phi_3 = 90^\circ.$$  \hspace{1cm} (10.23)

Solving these equations simultaneously for $\alpha$, $\beta$, and $\gamma$, we get:

$$\alpha = (90^\circ - \phi_1 - \phi_2 + \phi_3)/2,$$  \hspace{1cm} (10.24)

$$\beta = (90^\circ - \phi_1 + \phi_2 - \phi_3)/2, \text{ and}$$  \hspace{1cm} (10.25)

$$\gamma = (90^\circ + \phi_1 - \phi_2 - \phi_3)/2,$$  \hspace{1cm} (10.26)

where $\alpha$, $\beta$, $\gamma$, $\phi_1$, $\phi_2$, and $\phi_3$ are in degrees. The determination of $\alpha$, $\beta$, and $\gamma$ from the above equations uniquely determines the position of the major axes of the polarization ellipses.

10.6 ERRORS IN POLARIZATION MEASUREMENTS

Analysis of the errors which can occur in measurement of polarization is considerably more complicated than the polarization problem itself, and a complete error analysis which covers all situations is beyond the scope of this chapter. The derivations and developments of Chapter 3 should help the
reader in analyzing errors that can occur in a given measurement problem. The Poincare' sphere and the polarization box are of value as graphical aids in visualizing and solving polarization problems. The polarization matrix method of describing polarization and power transfer between antennas will prove of direct value in the analysis of errors because it can be used to calculate the relative response of non-orthogonal antennas to an incident wave.

A list which should include at least the major sources of error in making polarization measurements is given below, and a discussion and graphs relating to certain of the more important sources of error are included.

**Major Sources of Error in Polarization Measurements**

1. Error in calibration of the polarization standard.
2. Error in polarization of the sampling antennas.
3. Amplitude measurement error, including scale factor errors and non-linearity errors.
4. Phase Measurement error, including error in establishing the proper phase reference and non-linearity errors.
5. Error from extraneous signals in the test aperture.
6. Coordinate system misalignment error and other angle errors associated with the antenna positioners and angle readout instrumentation.
7. Calculation error in conversion from one set of polarization parameters to another, for example, in converting from $E_1E_2, \delta$ to $E_L, E_R, \delta'$.
8. Gross errors in calculation or interpretation which lead to blundering in the sense of polarization or in tilt angle.


Discussion

(1) **Measurement Errors Resulting From Error in Calibration of the Polarization Standard.**

Let us assume that the polarization standard has been calibrated and said to be linearly polarized. Error in polarization measurements will result from error in the measured tilt angle and axial ratio of the standard. The tilt angle error will produce a direct error in measured tilt angles. Detailed error in axial ratio will depend on the method of measurement although in general the errors are similar. If the measured axial ratio $r_s$ is determined by method (b) from the ratio

$$r_s = \frac{\rho_s + 1}{\rho_s - 1}, \quad (10.27)$$

where $\rho = E_s / E_l$, then the true axial ratio is given by

$$r = \frac{\rho_m + 1}{\rho_m - 1}, \quad (10.28)$$

where

$$\rho_s = \frac{r_s + 1}{r_s - 1}. \quad (10.29)$$

The errors in axial ratio are plotted in Figure 10.12 as a function of the absolute magnitude of the true axial ratio, with $r_s$ as a parameter. All parameters are in decibels. The error in decibels goes to infinity (1) when the axial ratio is infinite and (2) when the magnitude of the axial ratio is equal to the magnitude of that of the standard. It decreases to zero for circular polarization of either sense.

If the standard is used to make axial ratio measurements by the polarization pattern method, the source of the error is slightly different, but the results are equivalent.
The axial ratio is determined from (10.18), page 10-12. However, when the transmit antenna is not linearly polarized, $\Gamma_{\text{max}}$ and $\Gamma_{\text{min}}$ must be determined from (10.16) rather than (10.17), and the ratio in (10.18) does not represent the true axial ratio of the antenna under test. The error is again given by Figure 10.12 within the range of the figure, but consideration of the problem shows that in using method (b) the error in decibels goes to infinity when the axial ratio is the same as that of the standard while in using method (d) it goes to infinity when the axial ratio is the negative (the CPR is the reciprocal) of that of the standard.

The dashed curves of Figure 10.12 indicate the measurement error in decibels of the circular polarization ratio for the same assumed parameters. The error $\epsilon$ is constant with axial ratio of the test antenna and is given by

$$\epsilon = 20 \log (\rho - 1).$$

(10.30)

Figure 10.12 points out the problem of employing the error in axial ratio as a criterion for specifying accuracy in the measurement of a nearly linearly polarized antenna. For example, consider a standard antenna whose axial ratio is 40 decibels. If it is used to measure an antenna whose polarization is identically linear, the error in decibels will be infinite. On the other hand if the CPR $\rho$ is used as the criterion, the error is approximately 0.1 decibel. That is, the measurement indicates that the ratio of $E_R/E_L$ is 0.99 instead of unity, representing a relatively small measurement error.

There is no problem on the other hand in use of the axial ratio in specifying measurement error for axial ratios near unity. It can be seen from Appendix 3F that the axial ratio and the circular polarization ratio are equal for an axial ratio of approximately 7.6 dB. This is a convenient dividing line. For axial ratios less than 7.6 dB, the error in axial ratio is convenient; for axial ratios above 7.6 dB the error in circular polarization ratio becomes more meaningful.
FIGURE 10.12 Maximum error in measured ellipticity resulting from imperfect polarization standards. The indicated parameters are the axial ratios of the standards.
(2) **Effect of Error in the Polarizations of the Sampling Antennas.**

Assume that measurements are being made by method (b), using circular polarization components. It can be shown that the greatest measurement error can occur if for a given axial ratio of the sampling antennas they are orthogonally polarized.

Figure 10.13 is a set of graphs showing measurement errors in the polarizations of test antennas which have axial ratios of zero-decibels (circular polarization), 6-decibels, and infinity-decibels (linear) where the sampling antennas are orthogonal and have axial ratios of 0.2, 1- and 2-decibels. In the last case the error in circular polarization ratio is plotted rather than the error in axial ratio, which is infinite. See (1). Errors in δ' are also shown.

(3) **Errors in Measuring Polarization by the Multiple Component Method.**

It will be assumed that the measurements are made by measuring ρ, ρL and ρ0 directly (See page 10-7). The error in determination of the axial ratio will be the same as that indicated in (1) and (2).

Error in the tilt angle will enter through error in measurement of ρ0 and ρL in (10.15). Error in ρ0 and ρL will come from at least two sources, tilt angle and axial ratio error of the polarization standard and the same type of error of the sampling antenna or antennas. However, consideration of the Poincare' sphere geometry will show that the resulting error in tilt angle is due almost entirely to and is directly determined by errors in the tilt angle of the standard and sampling antennas.

In practice where polarization measurements are required over large regions of solid angle, measurements of $E_1$, $E_2$, $E_1$, $E_2$, $E_3$ and $E_4$ are often recorded on RDP charts or on tape to be analyzed by a computer. Readout errors due to digital recording roundoff must be considered in this type application, where amplitudes are, for example, recorded to the nearest decibel for subsequent computer analysis.
FIGURE 10.13 Measurement errors in the polarizations of test antennas which have axial ratios of (a) zero-decibels (circular polarization), (b) 6-decibels and (c) infinity decibels (linear polarization). In case (a) although the error in axial ratio is shown constant with $\delta'$ for convenience, it should be noted that for a circularly polarized antenna the phase angle $\delta'$ is not actually defined.
The roundoff error is not likely to be significant in many applications. If greater accuracy is required, considerable data smoothing can be provided in the computer to increase the accuracy, and constraints, such as the fact that \( E_1^2 + E_2^2 = E_3^2 + E_4^2 = E_5^2 + E_6^2 \), can be applied to reject error showing evidence of blunders or equipment malfunction.

REFERENCES


Let AB of Figure 10A.1 represent an axis in the equator of the Poincare' sphere. Let the polarizations of antennas A, B, and C be nearly linear so that they lie close to the equator. For example, if their axial ratios are of the order of 30 decibels, the angle $2\gamma$ will be about 87 degrees, and they will lie within 3 degrees of the equator, much closer than shown in Figure 10A.1. Let the minimum polarization efficiencies (when the tilt angles of each pair are oriented at right angles) be indicated by the squares of the

FIGURE 10A.1  Poincare' sphere, illustrating three antenna method of polarization analysis. $C'$ represents the polarization of $C$, rotated in tilt angle through 90 degrees (180-degrees in $\delta'$) so that it has minimum response with B.
radii of the three circles which are tangent to chords AB, AC and BC'. It will be evident upon consideration of Section 3.6 that under the postulated small angle conditions these radii approximate, respectively, the reciprocals of the apparent axial ratios obtained as the antennas are rotated against each other.

Figure 10A.2 represents the situation depicted in Figure 10A.1 for the small angle condition where the vertical scale has been magnified to provide resolution. Antenna A lies on a great circle near the equator. The antenna pairs are designated in order of their apparent axial ratios with AB-highest, AC-intermediate and BC-lowest.

*The particular locations of A, B and C shown in Figure 10A.1 represent one of a number of possible combinations. See note on page 10A-4.

FIGURE 10A.2  Geometry associated with the derivation of equation 10.19.
Rotation of the polarization of antenna C to the condition C' of minimum power transfer with antenna B defines the plane of the actual equator as a plane parallel with that generated by the rotation of C about the LR axis. There are two possible locations of C' which define minimum power transfer. These are defined by C₁ and C₂, where BC₁ or BC₂ may be tangent to the small circle of radius h₁ in the upper or lower hemisphere. The two possible true equators are designated A₁A₁' and A₂A₂'. Although shown exaggerated in Figure 10A.2, the planes of the two equators are tilted very little compared with plane AA. Thus h₁, h₂ and h₃ are still good approximations to 1/r₁, 1/r₂ and 1/r₃.

Radii h₄ and h₅ are those of the small circles which are tangent to chords C₁A₁ and AA₂. They are equal, respectively, to the reciprocals of the maximum and minimum possible magnitudes of the axial ratio of the antenna which has the lowest axial ratio, as indicated by (10.19). Note that A₁ and A₂ represent linearly polarized antennas, which are required to define the true axial ratio by the polarization pattern method.

In the discussion of paragraph (b), page 10-28, a special case occurs in which 1/r₂ and 1/r₃ are forced to zero. In this case the small angle approximation is not necessary. With reference to Figure 10A.3, we have

\[
\cos 2\gamma_{BC} = -\frac{1}{r_1},
\]

(10A.1)

where the sign of r₁ is determined from the sense of B and C; that is, r₁ is positive if B and C are left elliptical.

Since

\[
\rho_{BC} = \tan \gamma_{BC} = \left( \frac{1 - \cos 2\gamma_{BC}}{1 + \cos 2\gamma_{BC}} \right) = \left( \frac{r_1 + 1}{r_1 - 1} \right)^{\frac{1}{2}},
\]

(10A.2)

we can obtain

\[
r_{BC} = \frac{\rho_{BC} + 1}{\rho_{BC} - 1} = \frac{(r_1 + 1)^{\frac{1}{2}} + (r_1 - 1)^{\frac{1}{2}}}{(r_1 + 1)^{\frac{1}{2}} - (r_1 - 1)^{\frac{1}{2}}},
\]

(10A.3)
The axial ratio of antenna A is given by

\[ r_A = -r_{BC} = \frac{(r_1 - 1)^{\frac{3}{2}} + (r_1 + 1)^{\frac{3}{2}}}{(r_1 - 1)^{\frac{3}{2}} - (r_1 + 1)^{\frac{3}{2}}} \]  \hspace{1cm} (10.20) (10A.4)

**FIGURE 10A.3** Geometry associated with the derivation of equation 10A.4.

*Note:* The particular locations of A, B and C shown in Figure 10A.2 define 6 of 24 possible axial ratio combinations. Each of the three antennas can have axial ratios whose range includes the four values given by

\[ \frac{1}{r} = \frac{1}{2} \left( \frac{1}{r_1} \pm \frac{1}{r_2} \pm \frac{1}{r_3} \right) \]

and whose sign is either positive or negative. The particular set chosen defines the maximum and minimum possible values of the axial ratio of the antenna which has the lowest axial ratio.
CHAPTER 11
BORESIGHT MEASUREMENTS
T. J. Lyon and D. M. Fraley

11.1 INTRODUCTION

This chapter discusses basic concepts and techniques for boresight calibration measurements of microwave antennas, as performed on ground-based test ranges. Boresight calibration may be accomplished in one-way or two-way measurements, sensing the direction to a source antenna in one-way tests or to a target antenna in two-way tests. The angle measuring problem is the same in both cases; unless otherwise noted, we will consider the measurements to be one-way, with the understanding that the discussions apply to both types of tests.

The antenna types for which boresight calibration is required may be grouped into two broad classifications:

(a) Antennas (and associated signal-processing systems) which produce a null in the sensing function at boresight, and
(b) Antennas which produce a maximum in the sensing function at boresight.

Type (a) systems include con-scan radars, beam-switching radars, amplitude-monopulse and phase-monopulse tracking and direction-sensing systems, etc. Type (b) systems include various fixed-orientation antennas and radars of both electronic-scan and mechanical-scan types, such as height-finders and mortar-locators; important subgroups of this type of system are the family of Doppler radars such as the Janus and beam-intersection systems, and missile-borne fuse-train antennas.
Boresight calibrations are usually directed toward establishing coincidence (or in some cases simply parallelism) of the electrical boresight axis of an antenna with a mechanical reference axis. The mechanical reference is often called the optical boresight axis, and is fixed with respect to the antenna or its operational positioning structure.*

To alleviate the generally significant system-handling problem on the test range, boresight calibrations are typically performed on subsystems which include only the antenna and that portion of the associated circuitry which is necessary to generate the sensing function. The calibration is subsequently transferred to the total operational configuration by optical, mechanical or electrical measurement techniques. Also for reasons of simplicity and economy, boresight calibrations are most often performed with a CW or amplitude modulated source, as opposed to pulsed sources, regardless of the nature of the using system. Notable exceptions to these convenient approaches are sometimes made for extremely high-power systems (although the power handling capabilities of such systems are generally proven in bench tests or microwave darkroom measurements) and for such two-way systems as Doppler radars.

In any case, the test range must provide a source of radiation (or a target) whose location in the test coordinate system is known to an accuracy commensurate with the calibration tolerances. This requirement implies the necessity for calibration or proof-of-performance for the test facility itself. The major sources of error in boresight measurements include:

(a) Positioning system and readout error**
   (a. 1) Geometric error
   - Coordinate axis alignment
   - Orthogonality
   - Collimation
   (a. 2) Shaft-position error
   - Synchros
   - Resolvers
   - Encoders

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*See Chapter 5.
**See Chapter 5, section 7.
(a. 3) Deflection error
   - Load stresses
   - Solar heating
   - Wind loading
(b) Test instrumentation errors*
(c) Improper characteristics of the illuminating field **
   (c. 1) Phase curvature
   (c. 2) Amplitude taper, lateral and longitudinal
(d) Parallax
(e) Extraneous signal interference
   (e. 1) On-site reflections and diffraction
   (e. 2) Spurious radiation
(f) Improper test procedures
(g) Data processing errors.

This chapter is primarily concerned with items (c) – (f), from the viewpoint of the test engineer who must exercise final control over the range setup and test procedures in order to alleviate the effects of these and other potential error sources. For clarity, specific examples are treated in detail. The analyses, criteria and techniques discussed herein, however, have general implications and may hopefully be extended to other specific test problems as required.

11.2 BASIC RANGE CONFIGURATION CRITERIA

Most systems requiring boresight calibration are intended for use under conditions where the operating separation between transmitting and receiving antennas, or, for a radar, between the antenna and target, is of the order of miles. The fundamental considerations in establishing general purpose test ranges for such systems are discussed in detail in Chapter 14. The criteria developed there for control of the phase curvature and amplitude taper of the illuminating field generally apply to the boresighting problem.

*See Chapter 15.
**See Chapter 14 and subsequent sections of this chapter.
In fact, so long as the illuminating field is *symmetrical* in phase and amplitude about the axis of the test aperture, the boresighting problem can be less restrictive on the deviation of the field from a uniform plane wave than are the requirements for accurate far-field pattern, gain and polarization measurements. (See section 11.3.)

In some cases, boresight calibration must be performed on systems for which the operating separation between a relatively-low-directivity antenna and an associated transmitting antenna or complex reflecting target is comparatively small, as in the case of antennas used in proximity fuse vehicles. This type of specialized boresight problem is not treated in this chapter. Subsequent developments in the chapter will deal with the more prevalent semi-infinite-range simulation problem.

**11.3 THE EFFECTS OF PARALLAX IN BORESGHT MEASUREMENTS**

Error is introduced in precision antenna boresight measurements by testing at typical test separations between the antenna under calibration and the source antenna. The error components arise from three basic and interrelated sources: (1) from the physical distance which almost always exists between the antenna under test and the origin of the coordinate system in which measurements are made, (2) by the uncertainty and lack of uniqueness in the definition of a specific point which can be used as the effective location of the antenna under test, and (3) by the distortion of the radiation pattern which results because the antenna is being tested in a spherical wavefront rather than in the virtually planar phase front in which the antenna normally operates, that is for large separations between the antenna and the source of radiation. The viewpoint in this section is related to direction measurements rather than to measurements of distortion of the complete radiation pattern; the latter topic is treated in Chapters 6 and 14.

A Burroughs Datatron 5000 computer of the Rich Electronic Computer Center of the Georgia Institute of Technology was programmed to calculate antenna patterns as a function of the separation between the source antenna and an antenna under test for certain assumed asymmetrical antenna configurations.

*The investigations discussed in this section were supported jointly under Contracts NAS10-2103 and AF30(602)-3425. See references 1 and 2.*
In the following paragraphs terms are defined, a summary is given of the technique which was employed in the calculations, and the resulting data are presented.

11.3.1 Definition of Terms

Parallax - - - Parallax is defined as the difference in the apparent direction of a point or object as seen from two different station points which are not on a common straight line with the point or object under observation. In Figure 11.1 let the directions to the point $p$ be measured from station points which are the origins $o$ and $o'$ of two parallel coordinate systems. If the directions to $p$ are defined by $(\phi, \theta)$ and $(\phi', \theta')$ respectively in the two coordinate systems, the $\phi$ and $\theta$ parallax angles are defined by $(\phi' - \phi)$ and $(\theta' - \theta)$, as shown below.

![Figure 11.1 Coordinate Systems Employed for Antenna Measurements, Showing Parallax](image)

With reference to Figure 11.1, the general expression for the magnitude of the radius vector in a Cartesian coordinate system is

$$r_1^2 = x_1^2 + y_1^2 + z_1^2$$  \hspace{1cm} (11.1)
Translation from a system whose origin is \((x = 0, y = 0, z = 0)\) to a system whose origin is at the point \((x_0, y_0, z_0)\) defines
\[ r_0^2 = x_0^2 + y_0^2 + z_0^2 \] \tag{11.2}

If the point defined by (11.2) is to be the origin of a primed coordinate system \((x', y', z')\), then
\[ x' = x - x_0 \]
\[ y' = y - y_0 \]
\[ z' = z - z_0 \] \tag{11.3}

The general transformation equations for the changes of variable in going from Cartesian to spherical coordinate systems are
\[ x_i = r_i \sin \theta_i \cos \phi_i \] \tag{11.4}
\[ y_i = r_i \sin \theta_i \sin \phi_i \] \tag{11.5}
\[ z_i = r_i \cos \theta_i \] \tag{11.6}

From (11.4) and (11.5), we have \(r_i^2 \sin^2 \theta_i = x_i^2 + y_i^2\), and from (11.6), \(\cos \theta_i = z_i/r_i\).

The transcendental equations for \(\theta'\) are seen to be
\[ \sin \theta' = \frac{1}{r'} \left[ (r \sin \theta \cos \phi - r_0 \sin \theta_0 \cos \phi_0)^2 + (r \sin \theta \sin \phi - r_0 \sin \theta_0 \sin \phi_0)^2 \right]^\frac{1}{2} \] \tag{11.7}
and
\[ \cos \theta' = \frac{(r \cos \theta - r_0 \cos \theta_0)}{r'} \] \tag{11.8}

where
\[ r' = \left[ (r \sin \theta \cos \phi - r_0 \sin \theta_0 \cos \phi_0)^2 + (r \sin \theta \sin \phi - r_0 \sin \theta_0 \sin \phi_0)^2 + (r \cos \theta - r_0 \cos \theta_0)^2 \right]^\frac{1}{2} \] \tag{11.9}

Using (11.7) in (11.5) gives
\[ \sin \phi' = \frac{(r \sin \theta \sin \phi - r_0 \sin \theta_0 \sin \phi_0)}{\left[ (r \sin \theta \cos \phi - r_0 \sin \theta_0 \cos \phi_0)^2 + (r \sin \theta \sin \phi - r_0 \sin \theta_0 \sin \phi_0)^2 \right]^\frac{1}{2}} \] \tag{11.10}
Using (11.8) in (11.4) gives

$$\cos \phi' = \frac{(r \sin \theta \cos \phi - r_o \sin \theta_o \sin \phi_o)}{\sqrt{(r \sin \theta \cos \phi - r_o \sin \theta_o \cos \phi_o)^2 + (r \sin \theta \sin \phi - r_o \sin \theta_o \sin \phi_o)^2}} \quad (11.11)$$

Equations (11.7) and (11.8) yield

$$\theta' = \cot^{-1} \left\{ \frac{(r \cos \theta - r_o \cos \theta_o)^2}{(r \sin \theta \cos \phi - r_o \sin \theta_o \cos \phi_o)^2 + (r \sin \theta \sin \phi - r_o \sin \theta_o \sin \phi_o)^2} \right\} \quad (11.12)$$

and equations (11.10) and (11.11) yield

$$\phi' = \cot^{-1} \frac{r \sin \theta \cos \phi - r_o \sin \theta_o \cos \phi_o}{r \sin \theta \sin \phi - r_o \sin \theta_o \sin \phi_o} \quad (11.13)$$

for the primed angular parameters. The unprimed angular parameters are

$$\theta = \cot^{-1} \left[ \frac{z\phi}{x^2 - z\phi^2} \right] \quad (11.14)$$

and

$$\phi = \cot^{-1} \frac{x}{y} \quad (11.15)$$

The parallax angles are given by

$$\theta_p = \theta' - \theta \quad (11.16)$$

and

$$\phi_p = \phi' - \phi \quad (11.17)$$

The situation depicted in Figure 11.1 is typical of practical antenna problems, but differs markedly in severity between operational and measurement situations. Let the direction to a target at p be measured in the unprimed coordinate system defined by the shaft orientations of, say, a two-axis positioner with origin at o, the intersection of the positioner axes. Let the center of the antenna be located in o' a distance r_o from o. In the typical operating environment the distance R' to the target is of the order of miles and therefore is so large compared with r_o that \phi and \theta can almost always be considered equal to \phi' and \theta' respectively without measurable error. When measurements are being made of the radiation characteristics of such antennas, on the other hand, the range R' is often not
sufficiently great compared with \( r_0 \) that parallax can be neglected. We will be concerned with the effects of parallax at ranges which are of the order of \( D^2 / \lambda \), where \( D \) is the diameter of the antenna under test.

Center of Parallax - - - In discussions related to measurement of the radiation patterns of symmetrical antennas, parallax is sometimes discussed in terms of the center of radiation of the antennas. This term may be used synonymously with center of phase. In considering parallax in testing asymmetrical antennas it is necessary to define terms more precisely, and in this discussion we will use the term center of parallax in contrast with center of phase, and we will not employ the term center of radiation.

Parallax error can be accounted for and removed in analyzing antenna pattern data, except for a component which exists because of lack of specific information concerning the location of \( o' \), the center of parallax of the antenna under test. The center of parallax will be defined for our purpose as that location \( o' \), in or near the aperture of an antenna under test, which can be employed as an origin such that the function \( g(R', \phi', \theta') \) describing the normalized radiation pattern of an antenna will be constant with \( R' \).

Strictly speaking, of course, \( g(R', \phi', \theta') \) is not constant with \( R' \); thus a true center of parallax does not exist as defined. In pattern measurements of narrow-beam antennas, however, a major concern is that of determining the direction of the main lobe of the radiation pattern. It is of interest, therefore, to consider whether a center of parallax can be defined for practical use in locating the direction which the main beam of an asymmetrical antenna will have at large operational ranges when the measurements are made at distances that are typical in antenna test ranges.

For this study, the direction to the peak of the beam was computed in two ways:

(a) A computer search was employed at given ranges \( R \) to determine the point at which the amplitude of the beam had zero slope. The corresponding direction angles to such points are termed \( \Phi_{MR} \).
(b) The direction to the 3-decibel points on each side of the beam maximum was determined by computation at given ranges $R$, and the average of these angles was taken as the direction to the peak. These angles are termed $\Phi_R$.

The transverse displacement of $o'$ as $R$ was reduced from $R = \infty$ to some finite range $R$ was calculated in the computer program for case (b) above. The defining equation for this displacement can be obtained from Figure 11.2.

\[
\Delta Y = \frac{l}{\cos \Phi_R} \quad (11.18)
\]

\[
l = R(\Phi_R - \Phi_\infty) = R\Delta\phi \quad (11.19)
\]

\[
\Delta Y = \frac{R\Delta\phi}{\cos \Phi_R} \approx R\Delta\phi \quad \text{(for small $\Phi_R$)} \quad (11.20)
\]

Plots of $\Delta\phi$, $\Delta\phi_M$ and $\Delta Y$ are presented for several combinations of phase and amplitude asymmetries as a function of range at the conclusion of this section, where

\[
\Delta\phi_M = (\Phi_{MR} - \Phi_{M\infty}) \quad (11.21)
\]

![Figure 11.2 Geometry Defining Center of Parallax](image)
Center of Phase - - - The center of phase of an antenna can be defined as the location of the center of a sphere of radius $R'$ which is coincident with and of the same radius as the phase front produced by an antenna at a point in space $p(R, \phi, \theta)$. The location of the center of phase is of specific importance in measuring the boresight direction of phase-monopulse or amplitude-monopulse radars because such radars operate by sensing the direction of arrival of a wave as the direction of the normal to the approaching phase front.

Morita has shown that a unique center of phase does not generally exist for the practical antenna. In analogy with the location of the center of curvature of a planar curve, the radius and the location of the center of phase of an antenna are not fixed, but vary with the location of the field point in space. Although the radius of curvature of the wavefront of a beam in space is not generally constant with rotation of a plane of exploration about the axis of the beam, the axial location of the center of phase is usually not of great importance in practical applications; therefore our concentration of attention will be directed toward the transverse location of the center of phase.

In the following investigations antennas will be considered which are symmetrical in $\theta$ but asymmetrical in $\phi$. Considering only the transverse location of the center of phase, a coordinate system $X''Y''Z''$ (see Figure 11.3) will be defined which is parallel with a coordinate system $XYZ$ in which a narrow-beam antenna radiates with its beam axis nearly parallel with the $X$ axis.

Consider the sphere $S_1$ centered at $o$, and the phase front $S_2$ which intersects $S_1$ at $p_1$. Let the phase of the field at $p_1$ be $\psi_1$. Now call $\psi_2$ the phase at $p_2$ a distance $R\Delta\phi$ from $p_1$ in the $\phi$ direction.

From the geometry, for $\Delta R \ll R$,

$$\frac{\Delta R}{R\Delta\phi} \approx \alpha$$  \hspace{1cm} (11.22)

where $\alpha$ is defined as the angle at $p_1$ between $S_1$ and $S_2$ in the $XY$ plane.
Also,
\[
\alpha = \frac{\Omega \cos \phi}{R} \quad \text{for } R >> \Omega, \quad (11.23)
\]
and
\[
\Delta R = \frac{\lambda}{2\pi} (\psi_2 - \psi_1) = \frac{\lambda}{2\pi} \Delta \psi \quad (11.24)
\]

From (11.22), (11.23), and (11.24)
\[
\Omega = \frac{\lambda \Delta \psi}{2\pi \Delta \phi \cos \phi} \approx \frac{\lambda}{2\pi} \frac{\Delta \psi}{\Delta \phi} \quad \text{for small } \phi. \quad (11.25)
\]

Calculations were made in this study of \( \Omega(R) \) for certain assumed antenna asymmetries using \( p_2 \) and \( p_1 \) as the half-power points of the main lobe in the XY plane. Results of these calculations are presented at the conclusion of this section.

The values of \( \Omega \) thus calculated were checked for several cases which are considered to be extreme by defining a third point \( p' \) as the peak of the beam and solving equation (11.25) for values of \( \phi \) and \( \psi \) related to the point pairs \( p_1, p' \) and \( p_2, p' \). The average values of \( \Omega \) calculated in this manner were in close agreement with those presented here.

11-11
Boresight Deviation - - - Parallax is of importance in testing high-accuracy direction-of-arrival sensors, such as an amplitude monopulse. Many antennas of this type sense the direction of arrival of a wave as that direction for which the sum pattern signal $\Sigma$ and the difference pattern signal $\Delta$ at the terminals of each of the two channels (e.g., azimuth and elevation) of the monopulse network are in phase quadrature, assuming ideal data processing circuitry in the monopulse receiver. This is tantamount to the condition that

$$|A| = |D|, \quad |C| = |D|$$

where $A$ and $B$ are the magnitudes of the signals produced by the opposite lobes of the monopulse pattern for the crossover direction in one plane, and $C$ and $D$ are the magnitudes in the orthogonal plane. The problem of specific concern here is that of the behavior of the boresight directions with the separation between the source antenna and the antenna under test.

If we consider sensing in only one plane (say the XY plane, Figure 11.4, where we will assume the Z axis to be vertical) the monopulse antenna can be considered to consist of two asymmetrical antennas, one with its feed (A) on one side of the centerline of the reflector and the second with its feed (B) on the opposite side. The axis of the antenna is assumed to lie in the XZ plane. If the antenna possesses

![Figure 11.4 Schematic Representation of Single-Plane Monopulse Sensor](image-url)
mirror symmetry about the XZ plane, the asymmetry for channel A will be identical to that for channel B, and the boresight direction $\phi_0$ defined by $|A| = |B|$ will be in the XZ plane ($\phi=0$).

Now, still assuming absolute symmetry, if the boresight direction is measured at a source antenna separation which is sufficient for the secondary pattern to have formed (for example at a separation of $D^2/\lambda$, where $D$ is the diameter of the reflector), the measured boresight direction must lie in the XZ plane because of the assumption of symmetry. In practical cases ideal symmetry will never exist and the purpose of the investigation of this section is to provide an insight into the variation of the boresight direction as measured at different source antenna separations for assumed degrees of differential symmetry between the two antennas of the monopulse pair.

In the following section the method is described which was employed in the calculations; the input data and resulting calculations are presented in subsequent sections and Figures. It is emphasized that these calculations are made to give an insight into the problem; the design and fabrication of many high-accuracy direction sensors are directed toward achievement of as small a differential asymmetry as possible, and it is expected that the asymmetry which results in such cases will be much less than that employed in the calculations.

11.3.2 Radiation Pattern Calculations

Theoretical Development - - - Radiation patterns were calculated for a number of simulated antenna configurations by the aperture-field method, assuming the total radiated energy to be contained in a single polarization. For this case the active aperture of the antenna is assumed to be a planar surface lying in the YZ plane of Figure 11.5. The aperture-field method does not give the total field at p, but only the contribution from the aperture; however we will assume that almost all of the energy from the antenna passes through the aperture and that the contribution to the field at p of sources other than the field over the aperture can be neglected.

*The bulk of the material presented in this section was generated in connection with the previously noted USAF and NASA studies; the authors gratefully acknowledge the contributions to these efforts of Mr. J. S. Hollis of Scientific-Atlanta, Inc., and of Dr. D. T. Paris and Mr. Payne Lenoir of the Georgia Institute of Technology.
This method employs the scalar diffraction integral under the assumptions which have been made:

\[
\vec{E}_p = K \int_S F(o, y, z) e^{j\psi(o, y, z)} e^{-jkr} \frac{e^{-jkr}}{r} [(jk+\frac{1}{r})\hat{x} \cdot \hat{r} + jk\hat{x} \cdot \hat{P}] \, ds, \tag{11.26}
\]

where, with reference to Figure 11.5,

- \(K\) is a constant of proportionality,
- \(F(o, y, z)\) is the amplitude of the field distribution over \(S\),
- \(\psi(o, y, z)\) is the phase of the field distribution over \(S\),
- \(r\) is the distance from a source point \((o, y, z)\) to a field point \((x_p, y_p, z_p)\),
- \(k\) is the wave number \(2\pi/\lambda\), \(\lambda\) being the wavelength,
- \(\hat{P}, \hat{x}, \hat{R}\) and \(\hat{r}\) are unit vectors, and
- \(\hat{P}\) denotes the direction of power flow through the aperture.

For the problem at hand we can restrict \(p\) to the Fresnel and Fraunhofer regions, which are sufficiently removed from the aperture that

1. \(1/r\) is negligible compared with \(k\),
2. \(\hat{x} \cdot \hat{r} = \hat{x} \cdot \hat{R} = \sin \theta \cos \phi\),
3. \(r = R\) except in the phase term \(e^{-jkr}\)
In addition we will postulate that the direction of power flow through the aperture is nearly enough parallel with the x-axis that \( \mathbf{x} \cdot \mathbf{P} = 1 \) with negligible error.

If we designate the rectangular coordinates of \( p \) by \((x_p, y_p, z_p)\), the distance \( r \) from \( p \) to the point \((0, y, z)\) in the aperture is given by

\[
r = \left[ x_p^2 + (y_p - y)^2 + (z_p - z)^2 \right]^{\frac{1}{2}}.
\]  
(11.27)

The transformation,

\[
x_p = R \sin \theta \cos \phi,
\]
\[
y_p = R \sin \theta \sin \phi,
\]
\[
z_p = R \cos \theta
\]

allows writing

\[
r = \left[ (R \sin \theta \cos \phi)^2 + (R \sin \theta \sin \phi - y)^2 + (R \cos \theta - z)^2 \right]^{\frac{1}{2}},
\]
(11.29)

which through routine reduction gives

\[
r = \left[ R^2 - 2R(y \sin \theta \sin \phi + z \cos \theta) + (y^2 + z^2) \right]^{\frac{1}{2}}.
\]  
(11.30)

For our application symmetry will be postulated about the X-axis in \( \theta \), so we will require \( p \) to move only in \( \phi \) in the XY plane. Further the region of exploration in \( \phi \) will be near the X-axis, and \( \sin \theta \) will be equal to unity and \( \cos \theta \) equal to zero. Under these conditions equation (11.30) becomes

\[
r = \left[ R^2 - (2R y \sin \phi - y^2 - z^2) \right]^{\frac{1}{2}}.
\]
(11.31)

and \( r \) can be approximated by the first two terms of the binomial expansion, giving

\[
r = R - y \sin \phi + \frac{y^2 + z^2}{2R}.
\]  
(11.32)

These approximations allow (11.26) to be written, since we are interested only in relative phases and magnitudes at specific values of \( R \),

\[
\bar{A}_p = \int_F \int_{(o, y, z)} \exp \left[ i \left( \phi(o, y, z) + k(y \sin \phi - \frac{y^2 + z^2}{2R}) \right) \right] ds,
\]
(11.33)
where
\[ \overline{A}_p = \overline{C} \, \overline{E}_p ', \] and
\[ \overline{C} \] is an appropriate phasor.

The aperture assumed for the calculations was circular and of radius \( a \). For convenience in interpretation, the aperture coordinates are normalized to the radius as shown in Figure 11.6. The aperture was assumed to be illuminated by fields as described by the functions

\[ F(o, y, z) = \left[ K_1 + \cos \frac{\pi}{2} \rho \right] \left[ 1 + K_2 y \right] \]  
(11.34)

and

\[ \psi(o, y, z) = K_3 y + K_4 y^3 + K_5 \rho^3 \]  
(11.35)

Figure 11.6 Aperture Geometry Employed in Antenna Pattern Calculations

Equation (11.33) is accordingly written

\[ \overline{A}_p = \int_{-1}^{1} \left[ \int_{z_l}^{z_h} F(y, z) e^{j(K_5 - \frac{k}{2R})z^3} \right] e^{j[K_3 y + K_4 y^3 + (K_5 - \frac{k}{2R})y^3 + ky \sin \phi]} dy. \]  
(11.36)

Since the integral within the brackets is not a function of \( \phi \), the calculations can be made by the equivalent slit method. In programming the computer for achieving an approximation to (11.36) by a process of finite summation, \( F \) and \( \psi \) were approximated by 41 sample points along each axis.

11-16
11.3.3 Presentation of Data

In the computer calculations determining center of parallax and center-of-phase the following sets of input data were employed to represent typical asymmetries of $F$ and $\psi$ of equation (11.33).

$$K_1 = (0.462)$$
$$K_2 = (0), (0.5), (1.0)$$
$$K_3, K_4 = (0, 0), (1.4, 0), (0, 1.57), (1, 1)$$
$$K_5 = (0), (\pi/2), (-\pi/2)$$
$$\frac{R\lambda}{D^2} = (0.5), (1), (2), (4), (\infty)$$

Graphs of the functions $F(o, y, z)$ employed in the calculations are shown in Figure 11.7 showing rotational symmetry of $F$ about the $x$-axis for $K_2$ equal to zero and increasing asymmetry in the $y$ direction for increasing values of $K_2$.

The value of 0.462 was chosen for $K_1$ so that the symmetrical cosine-on-a-pedestal distribution ($K_2 = 0$) would correspond to a typical 10-decibel taper. For $K_2 = 0.5$, the $F(o, y, o)$ illumination peaks at $y/a \approx 0.26$, and the relative edge-illumination levels are approximately -17 decibels and -7 decibels. For the extremely asymmetrical function $K_2 = 1$, $F(o, y, o)$ peaks at $y/a \approx 0.4$, with relative edge levels of $-\infty$ and -5.6 decibels.

Center-of-Parallax and Center-of-Phase Calculations - - - An example of the effect of asymmetrical illumination, with linear, quadratic and cubic phase terms, is given in Figure 11.8. * Plots of

$$\Delta Y = \text{Center-of-Parallax Displacement}$$
$$\Omega = \text{Center-of-Phase Displacement}$$
$$\Delta \phi = \text{Beam-maximum Angular Displacement}$$
$$\Delta \phi_M = \text{Beam-maximum Angular Displacement}$$

are presented for

$$\psi(o, y, z) = y + y^3 - \frac{\pi}{2} \rho^2,$$
for $K_2 = 0$, 0.5 and 1.0. The particular antenna model employed in the calculations was a paraboloid of 2-foot diameter with an operating wavelength of 0.1-foot. The plotted data for $\Delta Y$ and $\Omega$ are thus universal, in that the values are normalized to the aperture radius. The angular parameters $\Delta \phi$ and $\Delta \phi_m$, on the other hand, are plotted in milliradians for the example antenna model. To convert these ordinates to universal values, one could divide the ordinate scale by the half-power beamwidth of the computation model.

Since for the example antenna the far-field patterns approximate the typical $\sin x/x$ amplitude function, the ordinate scales normalized to half-power beamwidth would be

$$\Delta \phi' = \frac{\Delta \phi}{60}$$

$$\Delta \phi_m' = \frac{\Delta \phi_m}{60}.$$
Figure 11.8 Displacement parameters for center-of-parallax and center-of-phase as a function of normalized range $R \lambda / D^2$. The lineal parameters $\Delta Y$ and $\Omega$ are normalized to aperture radius. To normalize the angular parameters $\Delta \phi$ and $\Delta \phi_*$ to half-power beamwidth, divide their ordinate scale by 60.
Boresight Deviation Calculations - Calculations were made of boresight directions $\phi$ (section 11.3.1), as functions of range for simulated amplitude-monopulse radar antennas. The aperture illumination functions employed were

$$F_L(o, y, z) = \left[ K_{1L} + \cos \frac{\pi}{2} \rho \right] \left[ 1 + K_{2L} y \right],$$  \hspace{1cm} (11.37)

$$F_R(o, y, z) = \left[ K_{1R} + \cos \frac{\pi}{2} \rho \right] \left[ 1 + K_{2R} y \right],$$  \hspace{1cm} (11.38)

$$\psi_L(o, y, z) = K_{3L} y + K_{4L} y^3 + K_{5} \rho^2,$$  \hspace{1cm} (11.39)

and

$$\psi_R(o, y, z) = K_{3R} y + K_{4R} y^3 + K_{5} \rho^2,$$  \hspace{1cm} (11.40)

where the subscripts $L$ and $R$ indicate left ($+\phi$) and right ($-\phi$) lobe illumination functions. The following parameters were employed:

$$K_{1L} = (0.462)$$

$$K_{1R} = (0.462)$$

$$K_{2L} = (0)$$

$$K_{2R} = (0), (0.5), (1)$$

$$K_{3L}, K_{4L} = (-1.4, 0), (0, -1.57), (-1, -1)$$

$$K_{3R}, K_{4R} = (1.4, 0), (0, 1.57), (1, 1)$$

$$K_{5} = (0), (\pi/2), (-\pi/2)$$

$$\frac{R\lambda}{D^2} = (0.5), (1), (2), (4), (\infty)$$

Calculated values of $\phi_o$ versus $R\lambda/D^2$ are plotted in Figures 11.9 through 11.11. The ordinates for these plots are given in terms of the parameter,

$$u = \frac{2\pi}{\lambda} \rho \sin \phi = \frac{2\pi}{\lambda} \rho \phi \quad (\text{for } \phi \ll 1).$$  \hspace{1cm} (11.41)

For the particular cases calculated, the aperture was very nearly 20 wavelengths in diameter, so that $\phi_o = u/20\pi$. To use these curves for a general aperture of diameter $n\lambda$, the corresponding crossover point is

$$\phi_{on} = \frac{20\phi_o}{n}$$  \hspace{1cm} (11.42)
Figure 11.9 Calculated Boresight Direction, $K_z=0$

1. $K_5, K_6=(4,0)$
2. $K_5, K_6=(0,1.57)$
3. $K_5, K_6=(1,1)$

(b) $K_2L=0$, $K_2R=1.0$

(a) $K_2L=0$, $K_2R=0.5$
Figure 11.10 Calculated Boresight Direction, $K_5 = \pi/2$

(b) $K_{2L} = 0$, $K_{2R} = 1.0$

(a) $K_{2L} = 0$, $K_{2R} = 0.5$
Figure 11.11 Calculated Boresight Direction. $K_5 = \pi/2$

(a) $K_{2L} = 0$, $K_{2R} = 0.5$

(b) $K_{2L} = 0$, $K_{2R} = 1.0$
11.3.4 Summary of Parallax Considerations

The above developments present theoretical studies of the effects of parallax in testing asymmetrical antennas. These studies were based on the aperture-field equivalent-slit method, and utilized digital-computer calculations of beam directions. To describe the effects of asymmetry two terms are defined, center-of-phase, and center-of-parallax, which affect the accuracy of boresight measurements in related but different manners. These effects can be seen with reference to Figure 11.12. Assume that an asymmetrical antenna $A_T$ is to be tested in a configuration where the source antenna is also asymmetrical. In the measurements it is required that the direction to the source antenna be determined from the antenna under test. Parallax error enters into such measurements basically because of the relatively small test separation which is likely to be employed compared with the operational separations which are to be simulated. The radiation from the source antenna appears to emanate from its center of phase, which is shown located a distance $\Delta$ from its geometrical center. Similarly, the operation of the antenna under test is such that it appears to be centered at a point which has been defined as the center of parallax; this point is shown located a distance $\delta$ from the geometrical center of the antenna under test. If the center of parallax and the center of phase are not in the boresight plane, $\Delta$ and $\delta$ have projections $\Delta_p$ and $\delta_p$ on this plane given respectively by

$$\Delta_p = \Delta \cos \chi_T$$ \hspace{1cm} (11.43)

and

$$\delta_p = \delta \cos \chi_R.$$ \hspace{1cm} (11.44)

At typical operational separations the parallax error angle in the sensing plane caused by the ratio $(\delta_p - \Delta_p)/R$ approaches zero. In contrast, at typical test ranges the magnitude of this angle may become significant in comparison to the boresight error specification of the antenna under test. The calculations presented in the previous paragraphs indicate magnitudes of the deviations of the center-of-phase and the center-of-parallax of antennas from their geometrical centers for assumed conditions of asymmetry.
Figure 11.12 Illustration of Parallax Error in Testing an Asymmetrical Antenna with an Asymmetrical Source Antenna.

The parallax error $\epsilon$ in the boresight plane results from assuming the source antenna's center-of-phase and the test antenna's center-of-parallax to be coincident with their geometrical centers.

With relation to specific categories of antennas involved in boresight problems, it is difficult to draw quantitative conclusions concerning parallax error because the magnitude of the errors which are produced depend on the degree of asymmetry of the antenna under test. It is possible, however, to draw some general conclusions:

1. If a source antenna is designed to be symmetrical and is made essentially symmetrical, its center-of-phase can be considered to lie at the geometrical center of the antenna with error which is so small that it can almost always be neglected unless measurements of the utmost precision are required. If the required precision is such that it is necessary to take into
account the deviation of the center-of-phase from the center of the source antenna, the antenna can be rotated about its axis through 180 degrees between measurements and an average of the measured boresights taken. If this procedure is followed, meaningful results will only be obtained if extraneous reflections are adequately suppressed.

(2) Extreme asymmetry of the source antenna is not likely to cause its center of phase to lie greater than 0.2 D from its physical center, where D is the maximum dimension of the antenna in the plane of asymmetry, but the deviation may approach this magnitude.

(3) The center of parallax of an asymmetrical antenna under test can result in a boresight error of as much as 1/20 times the half power beamwidth in tests made at separations $\leq 2D^2/\lambda$.

(4) The mirror symmetry of many types of radars tends to provide cancellation of parallax error, so that boresight measurements should be possible at ranges significantly smaller than $2D^2/\lambda$.

(5) For antennas which do not possess specific symmetry, some error from parallax is likely to be present in boresight measurements.

(6) When practicable, increases in test separation are favorable from the viewpoint of controlling parallax error, in comparison with ranges which have lengths of the order of $2D^2/\lambda$.

11.4 EFFECT OF EXTRANEOUS SIGNALS ON BORESIGHT MEASUREMENT ACCURACIES

Four specific boresight measurement problems are discussed in this section:

(a) Amplitude Monopulse Boresight Null
(b) Phase Monopulse Boresight Null
(c) One-way Single-beam Boresight Maximum
(d) Two-way Composite-pattern Boresight Maximum

The form of the analyses should apply generally to other types of systems under test.

See section 11.4.
11.4.1 Errors in Amplitude-Monopulse Systems

Consider a single-plane amplitude-monopulse system as shown schematically in Figure 11.13a. The phasors which correspond to the sum (Σ) and difference (Δ) channel signals are depicted in Figure 11.13b for the condition where an extraneous signal is present in both the A and B channels of the monopulse system. (The e^{j\omega t} time dependence of all phasors will be suppressed in this analysis.) In Figure 11.13b,

\[ \overline{A}_0 = \Lambda_0 \]

is the phasor in channel A due to the direct-path signal \( E_D \),

\[ \overline{B}_0 = \Lambda_0 \]

is the phasor in channel B due to the direct-path signal \( E_D \),

\[ \overline{a} = a e^{j\phi_1} \]

is the phasor in channel A due to the extraneous signal \( E_R \),

and

\[ \overline{b} = b e^{j\phi_2} \]

is the phasor in channel B due to the extraneous signal \( E_R \).

The phasors in channels A and B, respectively, are thus

\[ \overline{A} = (\Lambda_0 + a e^{j\phi_1}) \]  \hspace{1cm} (11.45)

and

\[ \overline{B} = (\Lambda_0 + b e^{j\phi_2}) \]  \hspace{1cm} (11.46)

Assuming the monopulse system has identical \( \sin K_\alpha/K_\alpha \) main lobes with peak amplitudes \( M \) for the A and B patterns, and has been aligned so that the lobe axes are equally displaced from the reflector axis (optical boresight axis), then for the orientation of the radar which causes the direct-path signal to arrive along the reflector axis \( \Lambda_0 \) will be equal to \( \Lambda_0 \). In this case, the sum and difference signals become

\[ \Sigma = (\overline{A} + \overline{B}) = (2\Lambda_0 + a e^{j\phi_1} + b e^{j\phi_2}) \]  \hspace{1cm} (11.47)

and

\[ \Delta = (\overline{A} - \overline{B}) = (a e^{j\phi_1} - b e^{j\phi_2}) \]  \hspace{1cm} (11.48)
The relative amplitudes and phases of the phasors $\bar{a}$ and $\bar{b}$ cannot be precisely predicted for arbitrary angles of incidence of the extraneous signal. It is of interest, therefore, to examine the effect of the extraneous energy under the worst case of phasing between $\bar{a}$ and $\bar{b}$. It is seen that, regardless of the relative amplitudes, the phase condition which will cause the greatest error is given by $\phi = \phi_1 = \phi_2 = \pi$ radians; this situation causes the phasors $\bar{a}$ and $\bar{b}$ to add in the difference channel. Applying this condition in (11.48), we have

$$\bar{\Delta}_{\text{MAX}} = (a+b)e^{j\phi} \quad \text{(11.49)}$$
Typical composite patterns of an amplitude-monopulse system are shown in Figure 11.14a. In keeping with typical radar configurations, we have assumed that the crossover level on each pattern is at −3 decibels with respect to the beam maximum. The slope $S$ of the patterns will approximate straight lines in the region of the crossover point, where

$$ S = \left. \frac{d(M \sin(\alpha/K)}{d\alpha} \right|_{\alpha = \text{crossover}} = \left. \frac{d(M \sin(\alpha)}{d\alpha} \right|_{\alpha = \text{crossover}} . \quad (11.50) $$

Since $\alpha = \theta + \gamma = 2\gamma - \alpha_B$, where $\gamma$ is one-half times the half-power beamwidth of the patterns, then

$$ \frac{d\alpha_A}{d\alpha} = -\frac{d\alpha_B}{d\alpha} = 1 . \quad (11.51) $$

and we have equal and opposite slopes for the patterns at the crossover point with the magnitude of the slopes given by

$$ |S|_{\text{crossover}} = \frac{M(\cos(\alpha/K) - \sin(\alpha))}{(\alpha/K)^2} . \quad (11.52) $$

As shown in Chapter 14, the factor $K$ is given by

$$ K = 2.28D/\lambda \quad (11.53) $$

where $D$ is the diameter of the main paraboloidal reflector and $\lambda$ is the wavelength, and the half-power points of the patterns correspond to a value of approximately 1.39 radians for $K\alpha$. Thus, the slopes of the patterns in the region of the crossover point have magnitudes of

$$ |S|_{\text{crossover}} \approx M(1.39 \cos(1.39) - \sin(1.39))/(1.39)^2 $$

or

$$ |S|_{\text{crossover}} \approx 0.38M . \quad (11.54) $$

Assuming that the amplitude-monopulse system senses boresight as the condition of orthogonality between $\Delta$ and $\Sigma$, the error in boresight direction is represented by the pattern angle $K(\delta\theta)$ shown in Figure 11.14b, at which the patterns differ in magnitude by $(a+b)$. The difference phasor $\Delta$ at the physical angle $\delta\theta$ off the
Figure 11.14. Amplitude-Monopulse Radiation Patterns About the Crossover (Boresight) Axis
optical boresight axis is given by

$$\bar{\Delta}_\theta = \bar{\Delta}_{\text{MAX}} - 2S(K\delta\theta)$$

or

$$\bar{\Delta}_\theta = (a + b) e^{i\phi} - 2S(K\delta\theta) .$$

(11.55)

This change in the difference phasor causes $\bar{\Delta}$ and $\bar{\Sigma}$ to be orthogonal, as indicated in the phasor diagram of Figure 11.15.

![Phasor diagram](image)

Figure 11.15. Orthogonality of $\bar{\Sigma}$ and $\bar{\Delta}$ Phasors at Corrected Boresight

Solution for the angle $\delta\theta$ which will produce this orthogonality for a general phase angle $\phi$ is straightforward, but tedious. Only the result will be given here, where the procedure was to obtain the arguments of the $\bar{\Delta}$ and $\bar{\Sigma}$ phasors at the point $K\delta\theta$ on the patterns, and to force ARG($\bar{\Delta}_\theta$) to equal ARG($\bar{\Sigma}_\theta$) + $\pi/2$. It can be shown that this leads to

$$\delta\theta = \frac{1}{2SK} \frac{(a^2 - b^2) \cos2\phi + (a + b) \cos\phi \bar{Z} M}{(a - b) \cos\phi + \bar{Z} M} .$$

(11.56)
The purpose here is not an exhaustive analysis based on approximations, but rather an indication of the order of error one might expect for boresight measurements. Thus we will examine (11.56) only for the case which would produce maximum boresight error. This condition can be shown from equation (11.55) to occur for $\phi$ equal to zero or $\pi$ radians. Setting $\phi$ equal to zero or $\pi$ radians in (11.56), we have

$$|\delta\theta_{\text{MAX}}| = \frac{1}{2SK} \frac{(a^2-b^2) + (a+b)\sqrt{2}}{(a-b) + \sqrt{2}} \frac{M}{M} = \frac{a+b}{2SK}.$$ (11.57)

Employing (11.53) and (11.54), this expression can be written

$$|\delta\theta_{\text{MAX}}| = \frac{0.577}{D/\lambda} \left[ \frac{a+b}{M} \right].$$ (11.58)

It is emphasized that the magnitude $(a+b)/M$ represents a signal in the monopulse circuitry. Figure (11.16) gives plots of the required suppression of extraneous energy in terms of the decibel level of $(a+b)$ referenced to the individual pattern peaks $M$ as a function of maximum allowable boresight error, with $D/\lambda$ as a parameter. For these plots, equation (11.58) is used in the form

$$20\log \frac{a+b}{M} = 20\log (\delta\theta_{\text{MAX}}) (1.73D/\lambda).$$ (11.59)

Equation (11.58) must be rewritten to determine boresight errors which can result from given magnitudes and angles of incidence of extraneous signals in terms of the ratio of the extraneous field $\bar{E}_R$ to the direct path field $\bar{E}_D$ incident on the test aperture. Also the investigator must have rather detailed knowledge of the actual pattern structures of the monopulse device. In terms of the direct-path signal $\bar{E}_D$ and the extraneous signal $\bar{E}_R$, the phasors of equations (11.45) and (11.46) can be written
Figure 11.16 Maximum Allowable Level of Extraneous Phasors in an Amplitude-Monopulse Circuit Versus Specified Maximum Boresight Errors
\begin{align}
\bar{A}_o &= C E_D d_x^\phi = \overline{B}_o , \\
\bar{a} &= C E_R d_a^\phi e^{j\phi_1} , \\
\bar{b} &= C E_R d_b^\phi e^{j\phi_2} ,
\end{align}

where C is a constant which accounts for the intrinsic impedance of free-space, the efficiency of the antenna and the waveguide impedance. The directivity terms (d) are defined as follows:

- \(d_x\) is the directivity of the A and B patterns at the crossover point, which is postulated as before to be the point of incidence of \(\overline{E}_D\),
- \(d_a\) is the directivity of the A pattern at the point of incidence of \(\overline{E}_R\),
- \(d_b\) is the directivity of the B pattern at the point of incidence of \(\overline{E}_R\).

It is noted that when both patterns intercept \(\overline{E}_R\) within their main lobes, the phases of \(\bar{a}\) and \(\bar{b}\) are constrained to be approximately equal. Since \(\overline{E}_D\) is postulated to arrive at the crossover point, then \(\bar{a}\) and \(\bar{b}\) are very nearly in phase with each other and with \(\bar{A}_o\) and \(\overline{B}_o\), and \(\bar{A}_o\) is equal to \(\overline{B}_o\). For this case the difference phasor
\[\Delta = (\bar{A}_o + \bar{a}) - (\overline{B}_o + \bar{b}) = \bar{a} - \bar{b}\]

\begin{align}
\Delta &\approx C \overline{E}_R (d_a^\phi - d_b^\phi) .
\end{align}

Thus as \(d_a\) approaches \(d_b\), \(\Delta\) approaches zero for any practical amplitudes of \(\overline{E}_R\), however large. This means that extraneous signals which arrive at the test aperture from virtually the same direction as the direct path signal have little effect on the boresight direction.

The developments which follow apply to relatively large angles of incidence for the signal \(\overline{E}_R\), which presents a high probability of the greater errors which occur
for the assumed worst-case phasing $\phi_1 = \phi_2 \pm \pi$ radians. The difference phasor at the crossover point is then written

$$\Delta_{\text{MAX}} = C \frac{E_R}{R} \left( d_a^\frac{1}{2} + d_b^\frac{1}{2} \right), \quad (11.63)$$

and the difference phasor at the angle $K\delta \theta$ is written as before as

$$\Delta_{\delta \theta} = \Delta_{\text{MAX}} - 2S K \delta \theta .$$

Since $M=\sqrt{2} A_o$ by postulation, the above expression can be written

$$\Delta_{\delta \theta} = \Delta_{\text{MAX}} - 2(0.38\sqrt{2} A_o) (2.28 D/\lambda) \delta \theta . \quad (11.64)$$

The terms of (11.64) are all in phase with the phasor $\Delta_o$, so that there is a particular value for $\delta \theta$ which will produce an absolute null for $\Delta_{\delta \theta}$. (Since we have assumed worst case phasing for the extraneous signals and ideal phasing for the direct path signals, the boresight correction will result in an absolute null for $\Delta_{\delta \theta}$.) In practice some quadrature component of $\Delta$ will always exist at boresight. The monopulse circuitry senses zero magnitude of the component of $\Delta$ which is in phase with $\Sigma$ (by sensing the condition of orthogonality between $\Delta$ and $\Sigma$). For the idealized conditions of the present analysis, this is tantamount to setting $\Delta_{\delta \theta} = 0$; the results of the analysis are thus applicable to the practical case in which a residual quadrature component of $\Delta$ exists at boresight.

Setting $\Delta_{\delta \theta}$ equal to zero in (11.64) and employing (11.60) and (11.63), we have

$$C_1 E_R \left( d_a^\frac{1}{2} + d_b^\frac{1}{2} \right) = 1.73 \sqrt{2} C_1 E_D d_x^{\frac{1}{2}} (D/\lambda) \delta \theta . \quad (11.65)$$

Since $d_o$ (the directivity at the peak of each pattern) is equal to $2d_x$, we can write (11.65) as

$$\frac{E_R}{E_D} = 1.73 (D/\lambda) \frac{d_o^{\frac{1}{2}}}{d_a^{\frac{1}{2}} + d_b^{\frac{1}{2}}} \delta \theta . \quad (11.66)$$

Equation (11.66) contains the same information as (11.58), where it is seen that

$$\frac{a+b}{M} = \frac{E_R \left( d_a^{\frac{1}{2}} + d_b^{\frac{1}{2}} \right)}{E_D d_o^{\frac{1}{2}}} . \quad (11.67)$$
Application of (11.66) to a particular problem requires that the ratio $d_o^{1/2}/(d_a^{1/2} + d_b^{1/2})$ be either postulated or approximated from experimental pattern data; if we represent this ratio by the term $\rho$, then we can write (11.66) in decibel form as

$$20 \log \frac{E_R}{E_D} = 20 \log \delta \theta + 20 \log 1.73 D/\lambda + 20 \log \rho \quad (11.68)$$

Note that for the angles of incidence of the signal $E_R$ greater than some specific value as measured from the boresight axis, $E_R$ enters one or both of the patterns through the sidelobes, and the effects of the sidelobes must be approximated in some way if a prediction is to be made of possible boresight error resulting from extraneous energy entering the aperture from angles greater than this value. For any amplitude-monopulse device whose patterns have the general character of a $\sin x/x$ amplitude variation over the main lobes, we can define an angle $\theta_R'$ at which the sidelobe approximations must be applied in terms of the aperture ratio $D/\lambda$. That is, since the first null of typical $\sin x/x$ patterns occurs at $x = 2.5 \alpha_3 - \delta$, then for $\theta_R$ to correspond to main lobe reception of $E_R$ in both patterns we must have (see Figure 11.14a)

$$K \theta_R' = 2.5 K \alpha_3 - \delta - K \alpha_3 = 1.5 K \alpha_3 - \delta,$$  

or

$$\theta_R' = 1.5 \alpha_3 - \delta. \quad (11.69)$$

Since the beam angle in degrees at the half-power points is given by $\alpha_3 = 35 \lambda/D$ for such patterns, then

$$\theta_R' = 1.5 (35 \lambda/D) = 52.5 \lambda/D \text{ degrees} \quad (11.70)$$

One approach to the approximation of the pattern effects for $\theta_R > \theta_R'$ is to obtain an envelope of the sidelobe regions of the composite patterns. This approach was used to produce the qualitative data in the sidelobe reception portion of Figure (11.17). This portion of Figure 11.17 is a plot of equation (11.68) for an assumed boresight error of one milliradian, with the ratio $\rho = d_o^{1/2}/(d_a^{1/2} + d_b^{1/2})$ approximated from the

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$^*$This example also employed a 2-foot paraboloid at X-Band. The curve can thus be interpreted as a plot for boresight error equal to $1/60$ of the half-power beamwidth.
sidelobe envelope of Figure 11.14. It is reasonable to assume that the envelope represents a useful estimate of the region of high probability for the effects of signals $E_R$ incident from angles greater than $\theta_R'$ off the boresight axis. The portion of Figure 11.17 for $\theta < \theta_R'$ is also plotted from equation (11.68), but with the ratio $\rho = \frac{d_{\theta R}^{\frac{1}{2}}}{(d_{\alpha}^{\frac{1}{2}} - d_{\beta}^{\frac{1}{2}})}$ calculated directly from sin x/x tables. (See discussion following (11.62.)

![Graph](image)

**Figure 11.17 Extraneous Signal Level Referenced to Direct-Path Signal Which Can Cause a One Milliradian Boresight Error.**

The curve is plotted versus the normalized angle of arrival of $E_R$, and represents an approximation to the quantitative effect of signals $E_R$ which enter the aperture from regions such that $\theta_R > \theta_R'$. 

To use Figure 11.17 for other values of boresight error, the ordinates would be changed by a factor of $\pm (-6 \text{ decibels for each increase (decrease)}$ in the boresight error by a factor of 2. (See Figure 11.16.) Useful qualitative conclusions can be drawn from such curves. For this example, a reflected wave which arrives at an angle of greater than $\frac{1}{7}$ of the
3-dB beamwidth from the boresight axis can cause a boresight error of about 0.004 times the 3-dB beamwidth if it is of the order of 30 to 40 decibels below the direct signal level. The most sensitive angle is between $\frac{4}{7}$ and $\frac{8}{7}$ of the 3-dB beamwidth from the boresight axis. In this region, the reflected wave has a maximum effect on one channel and may have a minimum or additive effect on the other, resulting in the possibility of maximum error in the indicated boresight direction. Reflected signals entering this region of the patterns of the order of 45 decibels below the direct signal level can cause boresight error of 0.004 times the 3-dB beamwidth.

11.4.2 Errors in Phase-Monopulse Systems

Consider a single-plane phase-monopulse system as shown schematically in Figure 11.18a. For direction sensing in the plane of the antenna pair, a typical procedure is to insert a calibrated phase shift into one signal channel of proper magnitude to cause the relative signal phases to be zero at a summation point. Assuming plane wave propagation, the angle $\theta$ to the source of radiation would then be calculable from the equation

$$\sin \theta = \frac{\lambda}{2\pi D} \phi,$$

where $D$ is the separation between the antennas, $\phi$ is the measured differential phase and $\lambda$ is the wavelength.

If an extraneous signal at the frequency of the direct-path signal is incident on the radar, the signal at the summation point of the monopulse circuitry will be a combination of phasors as depicted in Figure 11.18b. (As for the amplitude-monopulse analysis, the assumed $e^{j\omega t}$ time dependence of all phasors is suppressed.) The individual phasors are as defined below.

$$\bar{A}_D = C \bar{E}_D d_A^{\frac{1}{2}}$$ is the phasor in channel A due to the direct-path signal $\bar{E}_D$,

$$\bar{B}_D = C \bar{E}_D d_B^{\frac{1}{2}} e^{j\delta}$$ is the phasor in channel B due to the direct-path signal $\bar{E}_D$,

$$\bar{X} = C \bar{E}_R d_a^{\frac{1}{2}} e^{j\alpha_a}$$ is the phasor in channel A due to the extraneous signal $\bar{E}_R$,
and
\[ \bar{b} = C \overline{E_R} A e^{j0_b} \] is the phasor in channel B due to the extraneous signal $E_R$. 

(a) Single-Plane Phase-Monopulse Radar

(b) Alteration of the Monopulse-Circuit Phasors due to an Extraneous Signal

Figure 11.18. The Effects of Wide-Angle Extraneous Signals on the Phasors of a Phase-Monopulse Radar Circuit
The directivity terms are defined as follows:

\[ d_A \text{ and } d_B \text{ are the directivities of the } A \text{ and } B \text{ patterns, respectively, at the point of incidence of } \mathbf{E}_D, \text{ and} \]

\[ d_a \text{ and } d_b \text{ are the directivities of the } A \text{ and } B \text{ patterns, respectively, at the point of incidence of } \mathbf{E}_R. \]

The constant \( C \) accounts for the intrinsic impedance of free-space, the assumed identical antenna efficiencies, and the effects of the transmission paths of the monopulse circuitry.

To investigate the boresight error caused by the extraneous signal, we assume the monopulse to be adjusted so that \( \delta = \phi \). In this case, the monopulse would indicate true boresight in the absence of \( \mathbf{E}_R \), and the phase difference

\[ \Delta \phi = \phi' - \phi \]  

(11.73)

is seen to be proportional to the boresight error. From Figure 11.18b, with \( \delta \) set equal to \( \phi \), we have

\[ \Delta \phi = \phi' - \phi = \tan^{-1} \left( \frac{-a \sin \alpha}{A_D + a \cos \alpha} \right) + \tan^{-1} \left( \frac{b \sin (\theta - \phi)}{R_D + b \cos (\theta - \phi)} \right). \]  

(11.74)

As in the amplitude-monopulse analysis of paragraph D.2, we will examine the effects of \( \mathbf{E}_R \) for worst-case phasing of \( \mathbf{E}_a \) and \( \mathbf{E}_b \). It is seen from (D-27) that the maximum phase difference due to \( \mathbf{E}_R \) is

\[ \Delta \phi = \sin^{-1} \left( \frac{a}{A_D} \right) + \sin^{-1} \left( \frac{b}{B_D} \right). \]  

(11.75)

Typical phase-monopulse systems employ antennas of low directivity, so that a highly probable condition is one for which

\[ |\mathbf{A}_D| \ll |\mathbf{E}_D| \gg |\mathbf{a}|, \ |\mathbf{b}|. \]  

(11.76)

Assuming that these approximations apply, (11.75) becomes
\[ \Delta \phi_{\text{max}} = \frac{a + b}{A_D}, \quad (11.76) \]

or

\[ \Delta \phi_{\text{max}} = \frac{E_R (d_a^2 + d_b^2)}{E_D d_a^2} \quad (11.77) \]

From (11.72), we may write

\[ \Delta \theta = \frac{\lambda}{2\pi D} \Delta \phi \quad (11.78) \]

for small \( \Delta \theta \). If we define a factor

\[ \rho' = d_a^2 / (d_a^2 + d_b^2) \quad (11.79) \]

then from (11.77) - (11.79) we have

\[ \frac{E_R}{E_D} = \frac{2\pi D}{\lambda} \rho' \Delta \theta_{\text{max}} \quad (11.80) \]

which is written in logarithmic form as

\[ 20 \log \frac{E_R}{E_D} = 16 + 20 \log \left( \frac{D}{\lambda} \right) + 20 \log \rho' + 20 \log \Delta \theta_{\text{max}} \quad (11.81) \]

If the ratio \( \rho' \) can be determined or postulated, (11.81) will allow computation of the required suppression of extraneous energy to satisfy a specification of maximum allowable boresight error.

Figure 11.19 gives plots of \( \frac{E_R}{E_D} \) in decibels as a function of boresight error in milliradians, with the ratio \( D/\lambda \) as a parameter. The data are given for the particular case \( \rho' = 0.5 \), which corresponds to an assumption of unidirectional...
patterns for the monopulse elements in the plane of interest. For other directivity factors \( \rho' \neq 0.5 \), the ordinate scale must be changed by adding the quantity

\[
20 \log \rho' + 6 \text{ decibels}
\]

to the plotted values.

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**Figure 11.19** Required Suppression of Extraneous Energy Incident on a Phase-Monopulse Radar Versus Boresight Error for Worst-Case Phasing at the Summation Point. The curves are plotted for \( 20 \log(\rho') = -6 \) decibels. To apply these data to a general directivity ratio \( \rho' \), add \( 20 \log(\rho') + 6 \text{ dB} \) to the ordinate scale.
11.4.3 One Way Measurement of Beam Maximum Location

The electrical boresight calibration of a system which produces a maximum in the sensing function at indicated boresight consists of locating the pointing direction of the principal beam relative to some mechanical reference axis. To facilitate the measurement process, it is important to choose a coordinate system with mutually orthogonal axes with one of the axes orthogonal to the line-of-sight to the source antenna. Having established the positioner coordinate system, the relationship between the test antenna coordinates and the positioner coordinate system must be accurately determined. This is normally accomplished by optical alignment of the mechanical boresight reference of the test antenna to the direction to the source antenna and by relating the mechanical boresight direction to the indicated test antenna position as determined from the axis readouts of the test positioner. Details of this procedure require full knowledge of the particular system to be tested.

The boresight measurement technique involves, or is equivalent to, the determination of a pattern angle on each side of the peak of the main lobe where the pattern level falls to some specified relative level, typically the -3 decibel point. The boresight direction is then defined as the bisector of the two measured angles. This method provides better resolution of the boresight direction than that obtained by a single direct measurement of the direction of the beam maximum, since the slope of the radiation pattern is greater at the measurement point.

If the radiation pattern of the test antenna is not perfectly symmetrical, measurement of the boresight direction in either elevation or azimuth requires that the direction of the beam maximum be known in the orthogonal plane. The direction of the beam maximum may be approximately located by repetitive pattern measurements around the peak of the beam using a high resolution recorder. The boresight error resulting from the error in locating the beam maximum and from pattern asymmetry may further be reduced by averaging measured azimuth (or elevation) boresight directions for incremental elevation (or azimuth) angles on each side of the indicated beam maximum. Once the boresight direction is established in azimuth (or elevation), this direction may be used in the elevation (or azimuth) boresight measurement.

* See Chapter 5.
Once the boresight directions of the antenna system are determined experimentally, parallax* and misalignment** corrections must be applied to the measured angles to accurately describe the boresight direction relative to the antenna system coordinates. The presence of extraneous signals within the test environment can seriously impede high accuracy boresight measurements. The extraneous signals can take the form of on-site reflections and diffraction, or spurious radiation from off-site sources. High power radiation from spurious sources can cause measurement errors either through direct reception or saturation effects of the receiving system.

On-site reflections and diffraction are a result of the unavoidable interaction of the transmitted energy from the source antenna with the test environment. Energy which is incident on the test aperture from directions other than the line-of-sight between transmitter and receiver will result in amplitude measurement errors which directly affect the accuracy in locating the boresight direction of the antenna. The sensitivity of the measurement to amplitude errors is inversely proportional to the ratio of the desired measurement accuracy to the beamwidth of the test antenna.

For this discussion, it is assumed that the electrical boresight direction in a plane through the beam is defined as the bisector of the angle included between directions on each side of the beam axis at which the indicated pattern in that plane falls to some specified relative level, say -n decibels. It is assumed that the angles to these equal-power points are measured sequentially by a method similar to that shown in Figure 11.20.

The calculated direction of the beam maximum, with reference to Figure 11.20, is

$$\theta(0 \text{ dB}) = \frac{\theta_1(-n \text{ dB}) + \theta_2(-n \text{ dB})}{2}$$  \hspace{1cm} (11.82)

where the (1,2) subscripts imply (right, left), etc. In this case the resultant error in measuring $\theta(0 \text{ dB})$ due to the presence of an extraneous signal depends

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*See section 11.3.

**See Chapter 5.
on the pattern level selected for detection, the relative phase and amplitude of the interfering signal as referenced to the direct-path signal, and the slope of the radiation pattern's amplitude characteristic at the angle \( \theta \) (-n dB).

Consider the case of a typical directive microwave antenna such as a paraboloidal reflector. The amplitude characteristic of the radiation pattern over approximately the 6-decibel beamwidth of the main lobe may be approximated by

\[
A = A_{\text{max}} \sin(K\theta)/K\theta ,
\]

where \( \theta \) is a particular pattern angle measured from the main lobe axis in the plane of interest and \( K \) is a constant determined by the half-power beamwidth, \( \beta \), of the antenna:

\[
K = 2.78/\beta .
\]

The worst case of phasing for a given extraneous signal would be such that the relative phase at one measurement point was zero, and at the other measurement point 180 degrees, relative to the phase of the direct-path signal. Assume that the direct-path field \( E_D \) is incident at the -n dB pattern level selected for measurement, so that the corresponding detected signal amplitude may be written

\[
e_D = kE_D A(-n)/A_{\text{max}} = kE_D A_{\text{N}}(-n) ,
\]

where \( k \) is a constant of proportionality. Let the extraneous field \( E_X \) be incident at a pattern level of -m dB, at which point the amplitude characteristic is given by

\[
a = f(K\theta), \ \text{a typically \neq A} .
\]

The corresponding detected signal amplitude is given by

\[\text{The subscript N is used to denote normalization to } A_{\text{max}}.\]
\[ e_X = kE_X a(-m)/A_{max} = kE_X a_N(-m). \] (11.87)

The measurement procedure of establishing angles at which the composite detected signals are equal on each side of the beam axis may be expressed mathematically by the equality

\[ k \left\{ E_D [A_N(-n) + S_A \Delta \theta] - E_X [a_N(-m) + S_a \Delta \theta] \right\} \]
\[ = k \left\{ E_D [A_N(-n) + S_A \Delta \theta] - E_X [a_N(-m) + S_a \Delta \theta] \right\} \] (11.88)

where

\[ S_A = \left[ \frac{d}{d\theta} (A)/A_{max} \right] \] evaluated at the \(-n\)-dB point,

\[ S_a = \left[ \frac{d}{d\theta} (a)/A_{max} \right] \] evaluated at the \(-m\)-dB point,

and

\[ \Delta \theta = \text{the measurement error in establishing } \theta \text{ (0 dB)}. \]

Collecting terms in (11.88) we obtain the expression

\[ E_X/E_D = S_A \Delta \theta/a_N(-m). \] (11.89)

Substitution of the indicated derivative (which is evaluated at the \(-n\)-dB point) in this expression gives

\[ E_X/E_D = 2.78 \left| \frac{K \theta \cos (K \theta) - \sin (K \theta)}{(K \theta)^2} \right| \left| \frac{1}{a_N(-m)} \right| \left| (\Delta \theta/\beta) \right|, \] (11.90)

which may be written logarithmically as

\[ 20 \log(E_X/E_D) = 20 \log(S_N(-n \text{ dB})) + m + 20 \log \left| (\Delta \theta/\beta) \right| \] (11.91)
where
\[
S_N(-n \, \text{dB}) = 2.78 \left[ \frac{K_0 \cos (K) - \sin (K)}{(K)^2} \right] \text{ evaluated at the } -n \, \text{dB point}
\]
and
\[
m = \left| 20 \log \left[ a_N (-m) \right] \right|.
\]

Equation (11.91) allows calculation of permissible maximum relative values of \( E_X \) for specified limits of the normalized boresight error \( \Delta \theta/\beta \). A plot of \( 20 \log |S_N(-n \, \text{dB})| \) versus the selected measurement level parameter
\[
-n = 20 \log \left[ A_N(-n) \right]
\]
(11.92)
is given in Figure 11.21.

As for the monopulse cases previously discussed, some knowledge or prediction of the sidelobe structure of the pattern under test is required to establish a representative value for \( m \). In specifying minimum levels of extraneous signal suppression, the maximum relative sidelobe level usually represents a conservative choice for this parameter, so long as the peak of the beam under test is not directed toward the source of interference during the course of the measurement.

11.4.4 Two-Way Measurement of Composite-Pattern Maximum Location

In some cases it is necessary to perform a system analysis based on a two-way transmission path as opposed to one-way propagation. For instance, the measurement of radar cross sections on reflectivity ranges utilizes two-way propagation. Also, in the testing of certain systems which make use of energy scattered from a passive target, such as a doppler navigation radar, it is often desirable to use a two-way transmission path between the system and target. The following paragraphs discuss some of the major considerations pertinent to the two-way measurement problem.

Consider the measurement system which is indicated schematically in Figure 11.22. The energy incident in \( A_R \) is caused by the energy reflected from the short-circuited

\*See Chapter 13.
$-n = 20 \log \left[ A_n \left( -n \, \text{dB} \right) \right] \, \text{decibels}$

Figure 11.21 Graph of Normalized Slope Factor versus Pattern Level for a Pattern whose Normalized Amplitude varies as $\sin x/x$. 
horn, which serves as a target, and energy reflected from extraneous scatterers. The field $E_R$ at the output terminals of the receiving antenna will be

$$E_R = \left( \overline{E_D} + \sum_{n=1}^{N} E_S e^{j\phi_n} \right) e^{j\omega t}$$

(11.93)

where

- $\overline{E_D}$ is the field produced by the direct path energy from the horn,
- $E_S e^{j\phi_n}$ is the complex amplitude of a scattered field component, with its phase referred to the phase of the direct-path field,
- $\omega$ is radian frequency, and
- $t$ is time.

It is obvious that the received power for this elementary case will have error terms caused by the extraneous scatterers, which contributions cannot be separated from those due to the desired direct-path signal. On the other hand, a system which is insensitive to extraneous signals is desirable for making two-way measurements. One method of accomplishing this is to "tag" the field reflected from a target so that it can be sorted out of the total field incident on the receiving antenna.

*The signal "tagging" method does not discriminate against extraneous signals which enter the target antenna, or which on reradiation from the target enter the receiving antenna of the system under test after being reflected from extraneous objects. In this regard the same criteria which apply to one-way testing apply to two-way testing.
The direct-path signal can be separated from the signal scattered into the receiving antenna under test, but which does not enter the target antenna, if the direct-path energy is modulated in a suitable manner before being reradiated from the target antenna. Several modulation methods are possible, but direct amplitude modulation cannot be employed without use of exotic and generally impractical data processing equipment. Two typical types of phase-shift modulation techniques will be described, and amplitude-modulation will be discussed to demonstrate the problems associated with this method.

**Single-sideband (SSB) Modulation with a Cyclic Phase-shifter** - - - This technique provides an audio modulation signal and at the same time prevents scattered signals from producing error unless the reflected energy enters the target antenna or unless energy from the target antenna reaches the antenna under test by reflection. The operation of a SSB modulator which can be employed is as follows. The amplitude of the signal received by the target is proportional to $G_t^{\frac{1}{2}}$, where $G_t$ is the gain of the transmitting antenna of the system under test. The SSB modulator consists of a rotary-vane phase shifter and a short circuit, as shown in Figure 11.23. The rotary-vane phase shifter is modified for continuous rotation at a speed of, say, 2175 revolutions per minute. *

![Diagram](TARGET ANTENNA - SHORT CIRCUIT - ROTARY VANE PHASE SHIFTER)

Figure 11.23 Single-Sideband Modulator for Two-Way Boresight Measurement System

After modulation and reradiation from the target antenna, the signal received at the antenna under test is processed by the circuit shown in Figure 11.24. The received signal amplitude $E_D$ is proportional to $G_t^{\frac{1}{2}} G_r^{\frac{1}{2}}$, where $G_r$ is the gain of the receiving antenna of the system under test. Energy is also coupled

*This rate of rotation gives a modulation frequency of 145 cps, which provides low probability of interference with harmonics of 60 cps.

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from the transmitter to the receiver, at a level such that the direct-coupled signal drives the detector into its linear operating region. Under this condition the system can be analyzed with the aid of the phasor diagram of Figure 11.25. The total signal incident on the detector is given by

\[ E_T = \left( E_R + \sum_{n=1}^{N} E_S e^{j\phi_n} \right) e^{j\omega t} + E_D e^{j(\omega + \omega_m)t} \]  

(11.94)

where \( \omega_m \) is the phase modulation radian frequency, which we have set to correspond to 145 cps for convenience. Equation (11.94) can be written

\[ E_T = \left( E_F + E_D e^{j\omega_m t} \right) e^{j\omega t} \]  

(11.95)

where \( E_F \) is the amplitude of the first term of equation (11.94) consisting of the sum of the directly coupled reference signal \( E_R \) and the scattered signals, all of frequency \( \omega \). Then

\[ E_T = \left[ E_F^2 + 2E_F E_D \cos \omega_m t + E_D^2 \right]^{\frac{1}{2}} e^{j(\omega t + \alpha)}, \]  

(11.96)

\[ \alpha = \tan^{-1} \frac{E_D \sin \omega_m t}{E_F + E_D \cos \omega_m t}. \]  

(11.97)

Figure 11.24 Simplified Schematic of Two-Way Antenna Test System Using SSB Modulation for Making Boresight and Pattern Measurements.
Figure 11.25 Phasor Diagram of Signal Incident on Detector of Two-Way Measurement System Employing Single-Sideband Modulation.

If $E_D$ is small compared with $E_F$, $E_D^2 \ll E_F^2$, and can be neglected with small error.

Then

$$E_T \approx \left[ E_F^2 + 2 E_F E_D \cos \omega_m t \right]^{1/2} e^{j(\omega t + \alpha)} \quad (11.98)$$

but if $E_D$ is small compared with $E_F$, the maximum value of $E_F E_D \cos \omega_m t$ will be small compared with $E_F^2$, and $E_T$ can be written, again with small error, as
The linear detector of the system under test operates on $E_T$ to give an alternating term of frequency $\omega_m$ and amplitude $E_D$, which is independent of $E_F$. Thus the output signal $E_D$ is independent of the level of the coupled signal $E_R$ and the scattered signal, $\Sigma E_S$.

**Digital Phase-Shift Modulation** - - - The digital phase-shift modulation method is in essence the same as the cyclic phase-shift method described above, except that the phase of the signal received by the target antenna is delayed in discrete steps rather than linearly with time. Consider the circuit of Figure 11.26. The signal received by the target antenna passes through a 2-bit digital phase shifter and is reflected at a short circuit. The phase shifter consists of two elements, having 0-45 degree and 0-90 degree steps respectively. Since the signal passes through the phase shifter twice, the total phase shift steps are 0-90 degrees and 0-180 degrees. By cyclic programming of the phase shifters, the phase of the reflected signal can be retarded to give equal length steps of 0, 90, 180, and 270 degrees. The phasor diagram associated with this technique is as shown in Figure 11.27. The total signal at the detector is given by

$$
\bar{E}_T = \Sigma (\bar{E}_S) + \bar{E}_R + E_D e^{j(\alpha + n\pi/2)}
$$

(11.100)

where

- $\Sigma (\bar{E}_S)$ = summation of scattered signals
- $\bar{E}_R = E_R e^{j\alpha}$ (phase reference)
- $n = 0, 1, 2, 3 \cdots$
- $\alpha$ = phase angle of direct-path signal as a function of path length between system under test and target antenna.

If $\Sigma (\bar{E}_S)$ and $\alpha$ are constant with time, $\bar{E}_T$ experiences discrete steps in level.

*This analysis implies that $\Sigma \bar{E}_S$ is constant in magnitude, that is, it has only the frequency $\omega$. Actually $\Sigma \bar{E}_S$ will fluctuate in magnitude and phase. However, the only component of this fluctuation which will add error is that which is within the pass band of the filter of Figure 11.24, centered at $\omega_m$. This component will normally be small compared with $E_D$. 

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as \( n \) increases with time, and these steps are reproduced at the output of the detector.

![Signal Tagging Circuit Employing Digital Phase Shift Modulation](image)

**Figure 11.26** Signal Tagging Circuit Employing Digital Phase Shift Modulation.

When the system under test is changed in orientation by the antenna test positioner during the course of measurements, the path length to the target antenna changes, causing \( \alpha \) to change. * Inspection of Figure 11.27 shows that as \( \alpha \) changes, the shape of the output signal will change. For example, Figures 11.28a and 11.28b indicate the detected output wave shapes for \( \alpha = 0 \) degrees and 45 degrees respectively, for \( \Sigma \left( \overline{E}_S \right) \) and \( \overline{E}_D \) small compared with \( \overline{E}_R \).

Although the output wave shape corresponding to \( E_D \) changes with \( \alpha \), it can be shown by Fourier analysis that the fundamental component of \( E_D \) is independent of \( \alpha \). The fundamental component can be extracted from \( E_T \) by use of a bandpass filter. Thus the 90-degree digital phase shift method operates in a manner which is equivalent to that of the cyclic phase shifter method. It has the advantage that the modulation frequency is not limited by the requirement for mechanical rotation as is the cyclic phase shifter.

*While a similar change occurs for the cyclic phase modulation case previously discussed, the result in that case is no more than a slight phase modulation of the detected output signal.*
Figure 11.27 Phasor Diagram Corresponding to Discrete Phase Steps of 90 Degrees.

Figure 11.28 Output Wave Shapes for Digital Phase Shift Modulation Employing 90-Degree Phase Steps. (a) $\alpha = 0$, (b) $\alpha = 45$ degrees.
Amplitude Modulation - - - The problem of tagging the signal of a two-way measurement system by employing amplitude modulation is illustrated by Figure 11.29. In the phasor diagram shown, $E_R$ represents either the sum of the scattered signal or a direct coupled signal or both. It is shown as a single phasor for convenience. The signal $E_D$ is represented by the phasor summation

$$E_D = E_c e^{j(\omega t + \alpha)} + E_m e^{j(\omega - \omega_m) t + \alpha} + E_m e^{j(\omega + \omega_m) t + \alpha} \quad (11.101)$$

where $\alpha$ is the relative phase between the signal carrier $E_c$ and $E_R$, and $\omega_m$ is the modulation frequency.

Figure 11.29 Sinusoidally Amplitude-Modulated Wave With Carrier Frequency $\omega$ Added to Reference Wave of Same Frequency.
$\vec{E}_D$ is always in phase with $\vec{E}_C$ and its magnitude varies over the interval $(E_C - E_m)$ to $(E_C + E_m)$. The sum of the modulated signal and the unmodulated carrier is $E_T$, as in the previous discussions. $E_T$ can be seen to vary in magnitude by a value

$$\Delta E_T = 2E_m \cos \alpha,$$

(11.102)

if $E_R$ is large compared with $E_D$, by consideration of the analysis of the paragraphs on the cyclic phase shifter (equation (11.99) and accompanying discussion) and inspection of Figure 11.29. The variation is more complex for smaller ratios of $E_R$ to $E_m$, but can be seen to depend radically on $\alpha$.

The crux of the above discussion is that the output signal $\Delta E_T$ varies with $\alpha$ in some manner which cannot be readily determined as $\alpha$ changes during the course of a measurement program. Amplitude modulation represents a poor choice as a signal tagging method and should be avoided.

11.5 TEST RANGE CALIBRATION

The boresight-facility calibration problem for systems which are to operate over semi-infinite ranges can be divided into the following categories:

(a) Establishment of an incident electromagnetic field of the proper frequency and polarization over the test aperture which adequately simulates an incident plane wave of constant amplitude (see Chapters 10 and 14 for discussions of polarization-measurement and aperture-field-measurement techniques),

(b) Provision for supporting, positioning and indicating the orientation of calibration devices or systems under test, and establishment of a frame of reference in which the measurements can be made (see Chapter 5),

(c) Determination of the location of the source of radiation, and

(d) Establishment of an optical line of sight from the source of radiation to the device under test to permit its comparison with the electrical line of sight indicated by the test device.

11.5.1 Alignment of Coordinate Systems - - - Prior to any radiation tests, optical and mechanical alignment of the test-range coordinate system and the
test device coordinate system must be accomplished. Details of alignment
techniques, and formulae for error calculations, were presented in Chapter 5; parallax-correction terms were discussed in section 11.3. In both cases, knowledge of the center-of-phase of the source of radiation and the center-of-parallax of the device under test was assumed in order to generate correction terms.

As shown in section 11.3, the ambiguity in parallax corrections is to some extent predictable if detailed knowledge of the mechanical and electrical properties of the test device is available, and the ambiguity for a specific device can be reduced by increasing the test separation. Once the basic electromechanical and electromagnetic criteria for the facility are satisfied, the primary experimental problem for boresight-facility calibrations is thus the determination of the location of the source of radiation, and establishment of a boresight-comparison reference to this location.

11.5.2 Location of the Source of Radiation -- To determine the boresight measurement capability of a test range, it is necessary to measure the direction of arrival of the phase front of the incident field. The measurements must be made under conditions that simulate the specific boresight measurement problem at hand; for example, if the device to be tested has low directivity and correspondingly high susceptibility to wide-angle reflections, the range calibration should be accomplished using low-directivity sensors. From the same viewpoint, if the system to be tested will be mounted on a support structure (such as a spacecraft mockup) which simultaneously screens the test positioner from illumination and introduces unique multiple-path reflection effects between the support structure and fixed objects, the same support structure or a facsimile should be employed in the calibration.

Since any practical measurement device will perturb the incident field, the measurement technique should be such that the effects of reflections from the measurement device itself are taken into account.* A method for determining

*Alternatively, direct measurement of the phase and amplitude of the incident field over the test aperture may be employed, but only if practicable mechanical tolerances and suppression of probe-structure reflections are commensurate with the required accuracy. See Chapters 9 and 14.
the direction of arrival of the incident field and of evaluating the levels of extraneous signals by means of the scatter in the measured direction of arrival as the perturbing fields interfere in varying phases with the direct path field is described in detail in references 2 and 6.*

This method is based on the fact that the direction OT from an antenna under test to a source antenna can be effected in two ways in a vertical \( \theta \)-axis positioner configuration such as that shown in Figure 5.24.**

In Figure 11.30 the line OT is defined under two sets of conditions: (1) with clockwise rotation about the \( \theta \) axis to a direction \( \phi, \theta \), and (2) with counterclockwise rotation about the \( \theta \) axis and 180 degree rotation in \( \phi \) to the same position \( \phi, \theta \). The direction in case (2) is identified with the underscored coordinates \( \phi, \theta \).

In this manner the direction OT can be maintained fixed relative to a direction sensor which is mounted on the upper azimuth axis while the sensor is inverted and is changed in position relative to earth fixed objects because of the natural parallax (See page 11-5) which exists. This permits measurement of the direction OT as \( (\phi + \phi)/2 \) and \( (\theta + \theta)/2 \), using the angle readout system of the positioner as the direction indicator. Further, if the sensor is an interferometer or monopulse antenna whose null direction can be varied, a scatter in the measured direction of OT will result as \( \phi \) and \( \theta \) are varied because of the variations in phase which occur between the direct path and extraneous energy. The magnitude of this scatter permits evaluation of the extraneous energy level.

The calibration device used in the Gemini range evaluation was a dual-phase interferometer (1428 MHz), similar to that employed on the mission, whose boresight direction was adjustable remotely. Thus, its electromagnetic

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*Reference 6 is included as an Appendix in reference 2.

**In Figure 5.24 the upper azimuth and the lower azimuth axes can be used as the \( \phi \) and \( \theta \) axes of Figure 5.30. The elevation axis then permits adjustment of the \( \theta \) axis normal to the Z axis.
Figure 11.30 Illustration of the Method Employed for Positioning the Calibration Device to Obtain Two Identical Orientations Relative to OT. The four crosses represent antennas of a dual-plane interferometer or monopulse sensor.
characteristics and its sensitivity to the environment were similar to those of the operational interferometer. A similar method was used in connection with evaluation of the same range for tests of the X-band (10 GHz) LM rendezvous radar of the Apollo program except that the antenna employed amplitude monopulse sensing and had electrical characteristics similar to those of the LM radar.

REFERENCES


6. J. S. Hollis, R. E. Pidgeon, Jr., and R. M. Shutz, A Precision Ground-Reflection Antenna Boresight Test Range, presented at the 14th Annual Symposium on USAF Antenna Research and Development, University of Illinois; October 1964. This reference is also included in reference 2 as an Appendix.
This chapter discusses techniques for electrical tests of operational radomes. The particular measurements covered include:

- Boresight Shift
- Beam Deflection
- Transmission Efficiency
- Power Reflection
- Pattern Distortion
- Depolarization.

The material presented here is largely practical, as opposed to theoretical. For those interested in a survey of the general topics of radome design theory and quality control type measurements, the several articles and bibliographical sections of Techniques for Airborne Radome Design are recommended.

12.1 Boresight Shift and Beam Deflection Measurements

This section discusses the measurement of the effect of radomes on the direction of propagation of electromagnetic waves.

The antenna over which the radome under test is placed will be termed the test antenna. When the test antenna has a boresight null, we are concerned with apparent changes in the direction of arrival of illuminating wavefronts as sensed by the test antenna. Measurement of such changes will be termed boresight shift measurements. When the test antenna has no boresight null, we are concerned with changes in the orientation of the mainlobe axis of a radiation pattern of the test antenna. Measurement of these changes will be
termed beam deflection measurements.

The objective of such radome tests is usually to prove that boresight shift or beam deflection does not exceed specified limits over a given sector of the spherical coordinate system defined by a specific test system's $\phi$ and $\theta$ axes (see Figure 1). For vehicle-mounted antennas, the $\phi$ and $\theta$ coordinates are often related to the vehicle's pitch, roll and yaw axes as shown in Figure 2. In some cases, complete quantitative data for boresight shift must be obtained over a sector of coverage in order to program computer-controlled servo systems for guidance or tracking antennas.

The various measurement systems which are used to assess boresight shift and beam deflection may be grouped into two broad categories, closed-loop (servo) systems and electronically calibrated systems.

In closed-loop boresight shift systems, the transmitting antenna is continuously oriented along the apparent electrical boresight axis of the test antenna. The positioner employed for this purpose is termed a null seeker, and typically consists of a precision servo-controlled X-Y mechanism.

A similar positioner, termed a beam straddler, is used in closed-loop beam deflection systems. The beam straddler continuously orients a set of receiving antenna pairs at equal-power points about the mainlobe axis of the test antenna pattern. Each pair of beam straddling antennas has a fixed separation along a baseline between antennas, and the geometric centers of the baselines for all pairs are made coincident.

It is common practice to use the terms "null seeker antenna" and "beam straddling antennas" in both closed-loop and electronically calibrated systems, even though these antennas are fixed in place for the latter.

Figure 3 compares the basic elements of null seeker systems and electronically calibrated boresight shift systems; the subsystems shown shaded in this figure are similar for either approach to boresight shift measurement. Beam straddler systems are compared with electronically calibrated beam deflection systems in Figure 4; the shaded subsystems are similar for either approach to beam deflection measurement.
FIGURE 12.1 Standard spherical coordinate system.

FIGURE 12.2 Consolidated vehicle and antenna coordinate system.
FIGURE 12.3  Block diagrams of boresight shift measurement systems.
FIGURE 12.4 Block diagrams of beam deflection measurement systems.
The items discussed in the following paragraphs include:

(1) Radome Positioners
(2) Receiving and Detection Networks
(3) RF Source Characteristics
(4) Closed-Loop Positioning System
(5) Electronic Calibration Network

12.1.1 Radome Positioners

The radome positioner and its control system are designed to simulate the operational antenna-radome orientations, and are in general identical for either closed-loop or electronically calibrated measurement systems. For practical reasons, such positioners typically hold the test antenna fixed in space while moving the radome through the required sectors of coverage.

Details of the positioner design should be based on at least the following criteria:

(a) Axis order — the relative motion of the radome and test antenna should be identical to the motion experienced in the operational system.
(b) Scan angle — the mechanical limitations imposed by support members should make allowance for complete coverage of required sectors.
(c) Electromagnetic interference — the configuration of the positioner should minimize interference with the electromagnetic field.

Where possible, consideration should also be given to factors of human engineering such as convenience, simplicity of operation, and the elimination of critical adjustments. An example of positioner design based on the major criteria is discussed below.

Axis Order — In a fire-control radar system, a common order of axes from antenna to radome is azimuth-on-elevation-on-roll. Tests of radomes for such systems require a radome positioner of the roll-on-elevation-on-azimuth type; an example of this type of positioner is shown in Figure 5. In many operational coordinate systems, these three axes do not intersect at a common point. For such systems, the radome positioner must be equipped with sliding adjustable offsets in order to duplicate the operational configuration.
FIGURE 12.5 Three-axis (roll-on-elevation-on-azimuth) radome positioner for fire-control type radar systems.
Note that if the test antenna and radome were rotated through 90 degrees from their normal aircraft mounting, the illustrated positioner would be suitable for radars with elevation-on-azimuth-on-roll axis orders. This apparent versatility must however be considered in the light of the total test requirement. In the case of a fan-beam test antenna, for example, orientation of the wide beam-width in the vertical plane might result in unacceptable interference due to enhancement of reflections from the range surface. (See Chapters 11 and 14.)

**Scan Angle** - - The positioner must orient the radome through all radar operational scan angles. Utilizing a fixed antenna mounting post as in Figure 5, the available scan angles are limited by the radome base diameter and antenna insertion-depth requirements.

**Electromagnetic Interference** - - Reflection and diffraction effects caused by physical structures in the test facility are often major sources of measurement error. The radome positioner can be a significant contributor to the error budget assigned for such environmental effects.

The positioner shown in Figure 5 is designed to minimize the amount of structure forward of the radome base. The elevation axis is supported on one side only, in preference to a yoke arrangement. The major portion of the roll ring support is placed behind the roll ring. The roll ring itself is kept as small as practical in order to minimize blockage of the test-antenna aperture at wide scan angles without limiting the size of radomes which may be tested.

Often the positioner structure must be covered with RF absorbing material to reduce reradiated electromagnetic interference beyond the levels which can be achieved in the mechanical design. The configuration of the positioner, essentially independent of material, establishes the interference level due to diffraction of the electromagnetic wavefront.

12.1.2 Receiving and Detection Networks

A wide variety of reception and detection techniques is possible for any of the four fundamental measurement systems depicted in Figures 3 and 4. In the following paragraphs, detailed discussions of several example networks are presented. The intent here is not to survey the topic, but rather to illustrate some of the more pertinent concepts and tradeoffs with specific examples.
For any detection technique, the basic requirement is to process RF signals from the receiving antenna or antennas (the test antenna in boresight shift systems or the beam straddling antennas in beam deflection systems) and supply to the measurement system signals whose amplitude and phase or sense represent angular magnitude and direction, respectively, of the boresight shift or beam deflection.

The three most common types of test antennas which are used in radome boresight shift measurements include conical-scanning, amplitude-sensing monopulse and phase-sensing monopulse.

(a) Conical-Scanning - - In these systems, the magnitude of displacement of a target from the boresight axis is indicated by the degree of amplitude-modulation of the signal received through a single beam whose mainlobe axis is mechanically rotated in a cone about the boresight axis. The scanning drive unit also provides a reference signal for synchronous demodulation of the received signal in order to recover the sense of the boresight displacement in two orthogonal directions. A typical detection network for conical-scanning systems is discussed later.

(b) Amplitude-Sensing Monopulse - - In these systems, boresight sensing in a given plane is accomplished by instantaneous comparison of the magnitudes of signals received in a pair of patterns which are symmetrically squinted off the boresight axis.

(c) Phase-Sensing Monopulse - - In these systems, boresight sensing in a given plane is accomplished by instantaneous comparison of the phase of signals received in a pair of patterns. The mainlobe axes of the patterns typically lie in a plane containing the boresight axis.

For either of the basic monopulse sensors, RF conversion networks may be employed to accommodate one of three types of angle detection:

1. Sum-and-Difference
2. Amplitude
3. Phase
Although some operational monopulse radar systems employ phase detection networks in combination with phase-sensing antennas, the boresight stability in such cases is sensitive to instabilities of the signal channels in both amplitude and phase. For this reason, most radome boresight-shift measurement systems involving phase-sensing test antennas incorporate conversion networks which permit either amplitude or sum-and-difference detection techniques.

Example circuits for realizing these latter two techniques are discussed following the conical-scan discussion. The principles involved in the various boresight-shift detection networks presented here are applicable to other types of direction-sensing antennas, such as those for beam-switching radar systems.

Detection networks for beam deflection measurements are always associated with the beam straddling antennas, so that the choice of a detection technique is essentially independent of the particular type of test antenna employed. Of the possible beam deflection detection techniques, two are most widely used:

(a) Post-detection amplitude comparison at audio frequencies, using direct detection of source-modulated signals. This technique is described following the monopulse discussions.

(b) Post-rectification amplitude comparison of intermediate-frequency signals. This technique is analogous to the amplitude comparison technique discussed in connection with the monopulse sensors.

Other beam-deflection detection techniques, such as pre-detection amplitude comparison at RF or IF, are feasible in theory but seldom used due to the sensitivity of the error output to phase instabilities.

Conical-Scanning Boresight-Shift Detection - - Figure 6 presents a simplified block diagram of a detection network which may be used in boresight-shift measurement systems employing con-scan test antennas. Amplitude modulation at a convenient audio frequency (typically 1 KHz) is usually provided at the source, but may be incorporated into the detection network.

For any displacement of the source antenna from the electrical boresight axis of the test antenna, additional amplitude modulation is imposed at the scan
rate of the rotating feed. The first detection stage and band-pass amplifier suppresses the RF carrier. The amplifier output is detected and passed through a low-pass filter network centered at the scan rate. Synchronous demodulation at the scan frequency then provides signals which yield both magnitude and sense of the boresight shift in two orthogonal directions.

These signals may serve as inputs to a closed servo loop or alternatively may be routed through calibration circuits for direct recording. In many systems the vector sum of the orthogonal boresight shift components is derived electronically either at the nullseeker or in the detection network, and recorded simultaneously with the error components.

**Amplitude Detection of Boresight Shift for Monopulse Antennas** - A single-plane amplitude-sensing monopulse antenna is depicted schematically in Figure 7 (a). Phasor diagrams of the signals in each feed channel are shown in Figure 7 (b) as the source moves from the A-pattern (B-feed) side of
boresight to the B-pattern (A-feed) side of boresight. These diagrams assume fixed phase error $\phi$ of the B channel relative to the A channel, and are shown for $\omega t = 2N\pi$.

Note that error in the phase function has no effect on the ratio $A/B$ either at boresight or at any angle $\phi$ within the range of unambiguous directions of arrival of the incident wavefront. This result is valid only in the absence of coherent interfering signals (See Chapter 11).

A single-plane phase-sensing monopulse antenna with a conversion network to provide for amplitude detection is depicted schematically in Figure 8 (a). Phasor diagrams of the output signals from the sensing elements and from each converter channel are shown in Figure 8 (b) as the source moves from the B-pattern side to the A-pattern side of boresight. These diagrams assume a fixed differential amplitude error in the sensor channels, and are shown for $\omega t = 2N\pi$.

Note that differential error in the amplitude functions has no effect on the ratio $E_1/E_2$ at boresight. However, this ratio is sensitive to amplitude error for all other angles of arrival of the incident wavefront; this result is of
particular significance in electronically calibrated measurement systems. As for the previously discussed amplitude-sensing/amplitude-detecting technique, coherent extraneous signals can introduce both boresight and angle-tracking error.

A heterodyne amplitude-comparison detection network as diagrammed in Figure 9 is insensitive to the relative phase of the input signals insofar as boresight stability is concerned. This detection scheme is, however, sensitive to instabilities of the signal channels in amplitude and to differential mixer-conversion, amplifier-gain or rectifier-conversion effects due to simultaneous (common-mode) changes in input power level. The common-mode problem is of most concern between calibration of the boresight reference with no radome in place and the dynamic measurements of boresight shift through the radome.

FIGURE 12.8 Single-plane schematic and associated phasor diagrams for phase-sensing/amplitude-detecting monopulse antenna.

A heterodyne amplitude-comparison detection network as diagrammed in Figure 9 is insensitive to the relative phase of the input signals insofar as boresight stability is concerned. This detection scheme is, however, sensitive to instabilities of the signal channels in amplitude and to differential mixer-conversion, amplifier-gain or rectifier-conversion effects due to simultaneous (common-mode) changes in input power level. The common-mode problem is of most concern between calibration of the boresight reference with no radome in place and the dynamic measurements of boresight shift through the radome.
As indicated in the expression for the output signal $c$ in the block diagram, mismatch or drift effects in the detection network produce both additive (offset) and multiplicative (change in slope) errors in the desired angle output $M_0 \log (A/B)$. Accordingly, these effects are of concern in both closed-loop and electronically calibrated test systems.

![Block diagram of a heterodyne amplitude detection network for amplitude sensors or converted phase sensor outputs.](image)

**FIGURE 12.9** Block diagram of a heterodyne amplitude detection network for amplitude sensors or converted phase sensor outputs.

**Sum and Difference Detection of Boresight Shift for Monopulse Antennas** — A single-plane amplitude-sensing monopulse antenna with a conversion network to provide for sum-and-difference angle detection is depicted schematically in Figure 10 (a). Phasor diagrams of the sensor outputs and the converter outputs are shown in Figure 10 (b) as the source moves from the B-feed side to the A-feed side of boresight. These diagrams assume a fixed phase error $\phi$ of the B channel relative to the A channel, and are shown for $\omega t = 2N\pi$.

A single-plane phase-sensing monopulse antenna with a conversion network to provide for sum-and-difference angle detection is depicted schematically in Figure 11 (a). Phasor diagrams of the sensor outputs and the converter outputs are shown in Figure 11 (b) as the source moves from the B-pattern side to the A-pattern side of the boresight. These diagrams assume a fixed differential amplitude error in the sensor channels, and are shown for $\omega t = 2N\pi$. 

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FIGURE 12.10 Single-plane schematic and associated phasor diagrams for amplitude-sensing/sum-and-difference detecting monopulse antenna.
For both the amplitude-sensing and phase-sensing antennas, errors in the non-sensing functions produce a non-zero magnitude of the difference signal $\Delta$ at the desired boresight ($\theta = 0$). This situation can produce boresight error if means are not provided for suppression of sensor errors.

The detection circuit diagrammed in Figure 12 is inherently insensitive to the components of the modulated difference signal $\Delta_m$ which are in phase quadrature with the attenuated sum signal $k\Sigma$ at the input ports of the hybrid tee. Isolators are employed to flatten the input VSWR of the detection channels, and the sum-channel attenuator allows adjustment of the sensitivity of the circuit about boresight.

The phase shifter in the difference channel is set to suppress the sensing-system error. Since $\Delta$ is in phase quadrature with $\Sigma$ at boresight for amplitude sensors (Figure 10), and $\Delta$ is in phase with $\Sigma$ at boresight for phase sensors (Figure 11), the phase shifter is set at $N\pi$ and $(2N-1)\pi/2$, respectively, for use with these test antennas. The choice of $N = 1, 2, \ldots$ determines the sense of the audio-difference-circuit output $\delta$ corresponding to boresight shift for $\theta \neq 0$.

![FIGURE 12. 12 Simplified block diagram of an RF-bridge sum-and-difference angle detection network.](image)

The output signal $\delta$ is directly proportional to the difference input $\Delta$, and is an odd function of the angle of arrival $\theta$. This signal may be employed to derive an error signal in closed-loop systems, or may be routed through calibration circuitry for direct recording.

12-17
In some cases, it is desirable to normalize the difference signal so that the angle detection output is proportional to the ratio $\Delta/\Sigma$. This can be accomplished in RF-bridge systems by amplitude modulation of the transmitted wave at an audio frequency separated from the modulating frequency in the difference channel. The center-tap output, $\sigma$, from the audio-difference-circuit primary then becomes directly proportional to $\Sigma$, and can be used for normalization in either closed-loop or electronically calibrated systems.

In a closed-loop nullseeker system, bridge performance is important chiefly at or near electrical boresight, since the nullseeker "tracks" electrical boresight. The input signal-to-noise ratio at boresight for such systems should be 20 decibels or greater. (See section 12.1.3.)

For electronically calibrated systems, it is desirable to maintain linearity of the detected error signal versus boresight shift. Although this capability is inherent in monopulse sensors and RF bridge detection circuits for small angles off boresight, the recording system should incorporate conformity adjustment networks when large boresight shifts are anticipated.

The major effect of common-mode amplitude variations in electronically calibrated systems employing a sum-and-difference RF bridge network is a change in scale of error signal amplitude versus boresight shift. The common-mode effect can be offset in practice by use of the sum signal as a normalization factor in the recording system.

The RF bridge sum-and-difference detection technique is in wide use at microwave frequencies. Each detection network is in general restricted to a particular waveguide band. The bridge technique can be employed at IF frequencies in combination with a heterodyne receiver if broad-band capability is desired, although this approach reintroduces the possibility of common-mode errors due to the various stages of harmonic mixing and filtering.

**Amplitude Detection of Beam Deflection** - The block diagram of Figure 13 indicates the basic elements in a single plane of a widely used type of beam-deflection detection network. This circuit is applicable in closed-loop systems and in electronically calibrated systems.
A reference beam direction in the plane defined by the line of sight and the receiving antennas is established by the condition of equal power outputs from each receiving antenna, with no radome in place over the test antenna. With the radome mounted and exercised through its required scan angles, deflection of the transmitted wavefront will cause the output of one receiving antenna to increase and the second antenna output to decrease.

The direction of the beam deflection is determined by amplitude modulating the transmitted signal, detecting the modulation at the receiving antennas, and adding the two audio signals out of phase in a difference network. The resultant audio-frequency output signal is normalized to the sum of the output signals in a synchronous demodulator, producing a signal which has magnitude proportional to the angular beam deflection and sense corresponding to the direction of the deflection. The heterodyne amplitude comparison circuit previously discussed (Figure 9) can also be employed in beam deflection tests.

The primary requirements for accurate measurements are stability of the detectors or harmonic mixers with temperature, and matched amplitude tracking versus common-mode variations in signal level. It should be noted that the common-mode amplitude variations (those experienced simultaneously in each channel) caused by radome wall losses can be 30 percent or greater over a sector scan of some radomes. The stringent requirement such variations impose on electronically calibrated beam straddling systems of either the heterodyne or audio post-detection type can be illustrated as follows:

Assume a pattern under test whose half-power beam width is β radians.
and a beam straddling array designed to subtend some fraction
of the angle $\beta$, say $a\beta$, at the test separation. Let the maximum
allowable error in measurement of beam deflection which can be
budgeted to common-mode effects be some fraction of the angle $\beta$, say
$\rho\beta$. The maximum allowable amplitude tracking error, $\epsilon$, of the two
measurement channels is then given by

$$\epsilon \text{ (decibels)} = 20 \log \left[ \frac{f_1(\rho\beta + a\beta/2)}{f_2(\rho\beta - a\beta/2)} \right]$$

(12.1)

where

$$f_1(\rho\beta + a\beta/2) = \text{the apparent amplitude of the undeflected beam at receiving antenna 1, and}$$

$$f_2(\rho\beta - a\beta/2) = \text{the apparent amplitude of the undeflected beam at receiving antenna 2.}$$

When the pattern function of the beam under test is known or postulated,
equation (1) allows direct computation of allowable amplitude tracking
error as a function of array spacing and specified beam deflection
measurement accuracy. Figure 14 presents graphs of solutions to
equation (1) for a typical $\sin x/x$ normalized amplitude characteristic
of the beam under test. Allowable amplitude tracking error in decibels
is plotted versus beam deflection measurement error normalized to
the half power beamwidth of the test pattern, with array spacing as a
parameter.

As an example of the use of these data, consider the case in which a
beam having a one-tenth-radian half-power beamwidth is to be tested
with straddling antennas spaced to subtend the half-power beamwidth of
the pattern ($a=1$). Let the allowable beam deflection measurement error
assigned to amplitude variation be 0.05 milliradian ($\rho = 5 \times 10^{-4}$). From
Figure 14, we find that the allowable amplitude tracking error between
the two measurement channels is 0.013 decibel. Should this require-
ment be imposed for common-mode signal level variations of up to 3
decibels, the amplitude tracking specification of the detection circuit
would be of the order of 0.005 decibel/decibel.
FIGURE 12. Allowable amplitude tracking error with normalized beam deflection measurement error with beam-straddler array spacing as a parameter.

$D =$ BEAM DEFLECTION ERROR NORMALIZED TO TEST PATTERN HALF-POWER BEAMWIDTH (DIMENSIONLESS)

$E =$ MAXIMUM ALLOWABLE AMPLITUDE TRACKING ERROR OF DETECTORS (DB)
For a given test-antenna beamwidth, the narrower the spacing employed on the receiving antennas, the more critical is the amplitude tracking specification. For the same test antenna as above but with a 1-decibel-beamwidth straddling spacing, the amplitude tracking specification which would accommodate a 50 percent power variation would be approximately 0.002 decibel/decibel.

The beam straddling technique is widely used for beam deflection measurements, and is a straightforward and easily instrumented approach to the measurement problem. It is emphasized, however, that this two-point sampling of the test beam in a given plane of interest makes no allowances for simultaneous asymmetric beam distortion due to the radome (see section 5 of this chapter).

The sensing function of stationary straddling antennas is an inherently non-linear function of angular beam deflection. Thus, electronically calibrated systems must incorporate conformity adjustment circuitry to provide differential gain control to the recording system if linear recordings of beam deflection versus radome orientation are desired.

12.1.3 RF Source Characteristics

For precision measurement systems the transmitter must be extremely stable. In the monopulse antenna and RF bridge case, for example, frequency variations will result in phase changes in the microwave networks which necessitate re-balancing the bridge. Also, the phase of any reflections received at the antenna with reference to the direct signal depends on the reflection path distance in wave lengths.

Conventional signal sources with 0.1% stability are inadequate to assure that phase changes due to frequency variations will not cause significant errors in boresight and beam deflection measurements. For this reason a phase-lock frequency control system is required. Such systems typically provide a signal frequency stability of better than one part per million.

The minimum power output required for RF sources in beam deflection and boresight shift test systems is a function of the desired detection sensitivity, the noise figure and bandwidth of the detection circuitry, the conversion law and efficiency of detectors or mixers, and the gains and separation of transmitting
and receiving antennas. Typical specifications require that the error signal for a 0.1-milliradian beam deflection or boresight shift be at least a factor of 10 greater than the error signal which results from system noise with no RF power transmitted.

Source Level Requirements in Boresight Shift Systems — For a closed-loop or electronically calibrated boresight shift system utilizing an RF bridge circuit (Figure 12), the above specification can be related to the various test parameters shown in Figure 15. Assume an amplitude-sensing monopulse test antenna with pattern functions characterized by the typical $\sin x/x$ variation. Let the crossover level of the sensing patterns be -3 decibels relative to their maxima, and assume 100% amplitude modulation of the difference signal by means of a variable attenuator in the RF bridge. We may then write

$$E_\epsilon = 10 \log \left( \frac{2E_N}{\text{P}_{\text{in}}} \right) = \frac{G_2}{4} \text{P}_\text{o} \text{G}_\text{t} \text{G}_{\text{ro}} (\lambda/4\pi R)^2 \left[ \text{F}_1 - \text{F}_2 \right]$$  \hspace{1cm} (12.2)

where

- $E_\epsilon$ = the amplitude of the error signal voltage produced across the audio transformer primary by a 0.1 milliradian boresight offset,
- $E_N$ = the amplitude of the voltage at each detector output due to system noise with no RF energy incident on the detectors,
- $G_2$ = the conversion ratio (volts/watt) of the detectors operating in the square-law region,
- $P_o$ = the input power at the transmitting antenna terminals,
- $G_t$ = the gain of the transmitting antenna in the direction of the source antenna,
- $G_{ro}$ = the peak gain of the sum pattern of the test antenna,
\[ \lambda = \text{the wavelength of the RF signal}, \]
\[ R = \text{the separation between the transmitting and receiving antennas}, \]
and
\[ F_1, F_2 = \text{the magnitudes of the individual test antenna patterns relative to their peaks at a 0.1 milliradian offset from boresight}. \]

**FIGURE 12.15** Major parameters which affect the specification of RF source level for boresight shift systems.

If significant waveguide or cable runs are required in the test system, allowance must be made for additional losses and for noise-equivalent pickup directly in the passive circuitry.

As an example of the use of (12.2), consider a test situation for which

\[ \lambda = 0.1 \text{ foot} \]
\[ R = 1000 \text{ feet} \]
\[ G_t = G_{to} = 0.5 (20\pi)^2 \text{ 2 foot paraboloids; 50% efficiency} \]
\[ E_N = 0.012 \text{ microvolts 30HzBW closed-loop system} \]
\( C_2 = 350 \text{ millivolts/milliwatt.} \)

From \( \sin x/x \) tables we find that for the assumed pattern configuration

\[
(F_1^2 - F_2^2) \approx 5(10)^{-3}
\]

Substituting the assumed and calculated parameters in (12.2), the required basic value of \( P_0 \) is seen to be approximately 2 milliwatts. A 200-milliwatt signal source would thus provide a margin of approximately 20 decibels to account for attenuation and pick-up in the closed-loop system, and approximately 23 decibels of safety factor for electronically calibrated systems with 6 Hz bandwidths.

If modulation of the transmitted signal is employed for purposes of normalization or for simultaneous transmission-loss measurements, these safety factors are reduced by approximately 3 decibels.

**Source Level Requirements in Beam Deflection Systems** -- For a beam straddling system employing audio detection circuits (Figure 13), the basic error-signal/noise specification can be related to the various test parameters shown in Figure 16.

In this case we may write

\[
e_{\Delta} = 10 e_N = C_1 C_2 P_0 G_r (G_{t1} - G_{t2})(\lambda/4\pi R)^2
\]

(12.3)

where

\( e_{\Delta} \) = the difference signal voltage produced by a 0.1 milliradian beam deflection,

\( e_N \) = the difference signal voltage due to system noise with no RF power incident on the detectors,

\( C_1 \) = a proportionality constant whose value depends upon the modulation technique,
\[ G_r = \text{the gain of the beam straddling antennas in the direction of the test antenna,} \]

\[ G_{t1}, G_{t2} = \text{the gain of the test antenna in the direction of the straddling array antenna} \]

\[ \text{1, 2 for a 0.1 milliradian beam deflection,} \]

and where \( C_2, P_0, \lambda \) and \( R \) are as defined in (12.2).

\[ \text{FIGURE 12.16 Major parameters which affect the specification of RF source level for beam deflection systems.} \]

For circuits employing square-wave modulation, tunnel diode envelope detectors and solid-state crystal amplifiers, the following parameters are typical:

\[ C_1 = \frac{2}{\pi} \approx 0.64, \]

\[ C_2 = 350 \text{ millivolts/milliwatt at X-band into a matched load,} \]

\[ e_N = 0.006 \text{ microvolts for a 6-Hz bandwidth direct-detection electronically calibrated system,} \]

\[ e_N = 0.012 \text{ microvolts for a 30-Hz bandwidth closed-loop servo system.} \]

As an example of the use of equation (12.3), consider a direct-detection test at
10 GHz employing a radar antenna with a symmetrical \( \text{sin } x/x \) pattern in the plane of investigation which has a half-power beamwidth of 0.1 radian and a peak gain of 30 decibels \( (G_{t, \text{max}} = 1000) \). Assume receiving antennas at a range of 200 feet having peak gains of 15 decibels \( (G_r \approx 31.6) \). Let these antennas be spaced at the half-power points of the undeflected test beam. From \( \text{sin } x/x \) tables we find that a beam deflection of 0.1 milliradian corresponds to a test antenna gain differential given by

\[
G_{t1} - G_{t2} = 0.003 G_{t, \text{max}} = 3.
\]

Substitution of the calculated and assumed parameters in (12.3) yields a required value for \( P_0 \) of approximately 2 milliwatts. For this example case, a 200-milliwatt source would provide a margin of 20 decibels to account for circuit attenuation and noise-equivalent pickup in cable or waveguide runs. Under the same test conditions but with a closed-loop servo system, a 200-milliwatt source would provide a margin of 17 decibels above a basic requirement of 4 milliwatts.

12.1.4 Closed-Loop Positioning Systems

In closed-loop measurement systems, the nullseeker or beam-straddler carriage position represents boresight shift or beam deflection due to the radome. In monopulse or other boresight-null type radome testing, the nullseeker carriage positions the RF source antenna into the test antenna's boresight direction in response to the error signals from the test antenna's receiving network. In beam deflection tests, the beam-straddler carriage positions the straddling antennas in response to the error signals derived from their outputs to force them into the balanced power position.

The antenna positioner is a component in a closed RF servo loop as indicated in Figure 17. It must be properly designed for operation in a dynamic measurement situation. Boresight shift or beam deflection constantly changes during the data-recording interval and dynamic response will directly affect overall system accuracy. Factors influencing the antenna positioner design are range length, expected boresight shift or beam deflection, data rate, and required accuracy.
Range length in a boresight test facility is established by criteria formulated to assure an adequate simulation of the operational electromagnetic environment. In combination with the maximum expected boresight shift or beam deflection induced by the radome, the range length establishes the required carriage travel. The carriage travel and expected error rate determine the required carriage velocity. These factors and the dynamic accuracy requirements determine the design specifications for the closed-loop positioning system.

Figure 18 illustrates a two-axis X-Y nullseeker designed for use in a monopulse test. The required range length was 25 meters and the expected boresight error was ±10 mr. The specifications for this unit are given below. Note in Figure 18 that the RF oscillator is mounted directly on the nullseeker carriage to eliminate flexing in the RF cable during dynamic tests.

### Specifications

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Height</td>
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</tr>
<tr>
<td>Width</td>
<td>7 feet</td>
</tr>
<tr>
<td>Depth</td>
<td>3 feet</td>
</tr>
<tr>
<td>Weight</td>
<td>2000 lbs</td>
</tr>
<tr>
<td>Axes</td>
<td>Two orthogonal axes</td>
</tr>
<tr>
<td>Travel</td>
<td>±13 mr at 25 meters</td>
</tr>
<tr>
<td>Drive</td>
<td>Bidirectional ac servomotors</td>
</tr>
<tr>
<td>Speed</td>
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<tr>
<td>Acceleration</td>
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</tr>
<tr>
<td>Static Servo Error</td>
<td>0.025 mr</td>
</tr>
<tr>
<td>Input</td>
<td>dc signals, one volt/milliradian slope</td>
</tr>
</tbody>
</table>

---

**FIGURE 12.17**  
X-Y antenna positioner and servo system used in boresight shift and beam deflection test systems.
FIGURE 12.18  Servo-driven X-Y nullseeker with direct-mounted oscillator unit.
An additional design requirement may result if transmission loss and angular error measurements are to be made simultaneously. As the nullseeker moves linearly up or down, the peak of the transmitting beam moves above or below the monopulse antenna causing a drop in received signal. This loss in signal occurs simultaneously with transmission loss due to the radome, and degrades the transmission data. Increasing the transmitting antenna beamwidth to significantly reduce the signal-loss effect unfortunately magnifies reflection and diffraction problems through illumination of larger areas of the test facility. Accordingly, curved track nullseekers have been designed that move on a constant radius about the receiving antenna to eliminate the problem.

12.1.5 **Electronically Calibrated Systems**

The electronically calibrated system eliminates the need for the mechanical antenna carriage positioner and servo system by direct measurement of boresight error or beam-deflection at the receiving antenna outputs. Special electronic circuitry is utilized in processing the error signals to compensate for level variations and sensing-function non-linearities.

The block diagram of an electronically calibrated recording system is shown in Figure 19. The detected sum signal from the monopulse bridge or beam straddling antennas is used as a normalizing signal in the boresight recorder to suppress scale-factor changes resulting from RF signal level variation.

![Block diagram of an electronically calibrated recording system.](image)
The modulated difference output from the bridge or straddling network is synchronously detected and fed directly to the input of the calibration circuit for the recorder. A conformity adjustment system may be utilized to compensate for non-linearities in the sensing functions.

A precision calibrated gimbal for the test antenna is required for periodic calibration of the recorder scale.

12.1.6 Summary

Two basic types of systems for measuring radome-induced antenna boresight shift or beam deflection have been described. One utilizes a mechanical null-seeker or beam-straddler operating in a closed servo loop to track boresight or beam direction. The second electronically detects angular error signals, calibrates these signals through electronic compensation techniques, and records the calibrated signals directly.

The calibrated system eliminates the moving nullseeker or beam-straddler antenna systems and servo systems and replaces them with a fixed transmitting antenna or fixed receiving antennas. Significant advantages result, both from simplification of instrumentation and reduction of rigidity requirements demanded of the antenna support structures.

A particularly important advantage results in systems which require long range lengths. The requirements on velocity of travel and structural stability of a nullseeker with, say, ±10 feet of travel (±10 mr at a 1000-foot range) become technically and economically impractical. The calibrated system requires only a single stationary antenna regardless of range.

The ability to maintain a constant orientation of the line of sight relative to the range surface represents one of the most technically significant differences between the two systems, in that the effects of variable-phase extraneous signals can significantly degrade the overall test system accuracy. For either type of system, closed loop or electronically calibrated, the limiting accuracy is invariably set by the electromagnetic environment provided by the facility. As was shown in the previous chapter, measurement accuracies of the order of 0.1 milliradian demand state-of-the-art suppression of extraneous signal effects. The electromagnetic factors of the facility design problem are
treated in detail in Chapter 14.

12.2 TRANSMISSION EFFICIENCY MEASUREMENTS

The objective of radome transmission tests is usually to prove that transmission efficiency remains equal to or greater than some specified minimum value over a sector of the radome. In some cases, quantitative assessment of the composite gain or effective area of the test antenna-radome combination is required.

Most transmission efficiency tests are based on a free-space reference at ambient temperatures. A reference power level is established with no radome in place. The radome is then inserted and exercised through its required scan angles. Assuming the signal source to be stabilized by leveling, or the data to be normalized to the source level, deviations of the received power level from the reference level may be attributed to a combination of reflection, diffraction, absorption, refraction and depolarization of the electromagnetic wave by the radome, and to multiple-path interference phenomena between the test antenna, radome and radome positioner.

In order to discriminate against the multipath effects, provision is usually made for calibrated displacement of the test antenna along the line-of-sight. Comparative recordings made before and after a quarter-wavelength displacement of the test antenna would exhibit the effects of reflections between the test antenna and radome, since these reflections would essentially reverse their relative phasing with the direct-path signal. Similarly, half-wavelength displacements would exhibit the effects of reflections occurring near 90 degrees from the line-of-sight, such as those from positioning equipment, test cubicle apertures, etc.

The technique chosen for reception and detection of transmission efficiency data is largely dependent on (1) the type of test antenna to be employed, and (2) any requirement for simultaneous recording of boresight shift or beam deflection data.

12.2.1 Transmission Tests for Boresight Antennas

As for boresight-shift tests, the antenna types most frequently used in transmission tests of radomes which are to be employed with boresighting or tracking radars are con-scan, amplitude-monopulse and phase-monopulse.
With minor modifications, con-scan systems such as that previously discussed (Figure 6) may be utilized in simultaneous boresight and transmission tests. The required modifications to the angle-detection circuit are indicated in Figure 20. The circuit includes a power coupler, a precision variable attenuator for use in linearity checks, a detector and a band-pass amplifier. Usually, square-law detection is employed, so that the detector output voltage is directly proportional to the power incident at the detector. With a linear band-pass amplifier and recording system, the recorded signal is directly proportional to received power.

Transmission efficiency tests employing monopulse antennas make use of the sum signal as the transmission indicator. One approach which accommodates simultaneous boresight and transmission tests was shown in Figure 12, where the input to the nullseeker antenna was modulated and the detected sum signal was transformer-coupled into a servo or recording system. If this approach is taken, the sum output must be divided between transmission recorders and normalization circuits. The sum-channel attenuator should be a precision, variable type to permit linearity checks.

The circuit diagrammed in Figure 21 is often used in transmission tests with monopulse antennas. In this case, the signal source is operated CW, and sum-signal modulation is effected in the sampling circuit. The detected signal may
again be divided between transmission data channels and normalization circuits for boresight shift recordings.

![Simplified block diagram of a monopulse transmission-efficiency measurement system.](image)

**FIGURE 12.21** Simplified block diagram of a monopulse transmission-efficiency measurement system.

### 12.2.2 Transmission Tests for Single Beam Antennas

Transmission measurement with a single-beam radar antenna system is usually accomplished by sum-signal detection from receiving antenna pairs when the system is also used to measure beam deflection. In some cases, a center-mounted receiving antenna is used solely for transmission tests. A typical system is shown in Figure 22.

The energy received by each antenna is square-law detected, and the detected signals are combined in a summing unit. The audio sum signal is amplified and recorded in the same way as for monopulse radar systems. A precision attenuator may be inserted in one of the RF lines to check the overall system linearity.

If beam deflection measurements are not required, the remote antenna system can be replaced with a single antenna located at the peak of the beam of the
radar antenna, and the detected signal provided directly to the audio amplifier. The location of the detector and RF source may be interchanged if transmission tests only are being performed.

**FIGURE 12.22**
Simplified block diagram of a transmission-efficiency measurement system for conventional (single-beam) radars.

### 12.3 POWER REFLECTION MEASUREMENTS

For many test radomes, it is necessary to determine the relative level of energy reflected from the radome back into a transmitting test antenna. A directional coupler type reflectometer bridge is frequently used for such measurements when continuous recording of reflection level versus radome scan angle is required. Slotted line VSWR techniques are sometimes used when only a small number of discrete angular orientations are required.

Figure 23 is a simplified block diagram of a typical reflectometer type measuring system. The system provides constant RF power to the test antenna through the directional coupler, RF switch and tuner. The tuner allows line matching between the test antenna and feed lines to establish minimum reflected power with the radome removed.

The RF switch is used during calibration to provide a reflected reference signal which, in combination with the precision variable attenuator, allows a known reference level to be set at the detector. The recorder gain is then adjusted to agree with this reflected power level. Since the signal source
power output is made constant by leveling, or has provision for normalization, during a fixed-frequency measurement it is not necessary to have a forward-signal reference level.

FIGURE 12.23
Simplified block diagram of a reflectometer bridge circuit for measurement of internal radome reflections.

Once the system is calibrated, the radome is placed over the test antenna and the relative reflected power level is recorded directly. The radome can then be rotated and a plot of reflected power versus radome position recorded. These tests may be performed simultaneously with beam deflection and transmission tests for single-beam test antennas. For boresight-null type test antennas, the reflection tests may be performed simultaneously with transmission tests, but typically must be independent of boresight shift tests.

12.4 ANTENNA PATTERN DISTORTION MEASUREMENTS
In many applications, the peak gain and relative sidelobe levels of radiation patterns are of concern. The effects of a radome on these parameters, as well as halfpower beamwidth, angle between first nulls, and effective polarization must often be measured in conjunction with the previously described radome characteristics.
12.4.1 Relative Power Pattern Measurements

Recordings of the relative-power patterns of test antennas are typically restricted to E- and H-plane cuts for symmetrical on-axis patterns, or orthogonal cuts through the beam maximum for asymmetrical patterns.

The orientation of the radome relative to the test antenna remains fixed for each pattern measurement; i.e. the radome and antenna are scanned as a unit. This capability could be provided in the positioner of Figure 5, for example, by a removable antenna mounting bracket which would mate with the radome roll ring. The fixed-post antenna mount would be removed during pattern tests.

Pattern measurements are generally repeated for a number of scans sufficient to sample the operational sector of relative radome-antenna orientations. All such patterns are compared with a reference pattern recorded with no radome in place. In cases where rather detailed pattern distortion specifications must be complied with, reference patterns are usually recorded for each selected relative orientation of the test antenna and the radome support (no radome in place).

Pattern distortion measurements are typically performed with the test antenna operated on receiving, particularly when video detection is employed. In many cases, the required combination of accuracy and dynamic range demands a heterodyne receiving technique to provide the necessary linearity and sensitivity.

12.4.2 Polarization Measurements

Many test specifications require assessment of the depolarizing effects of radomes. The most common technique for such measurements is to operate the test antenna on receiving, and to provide for rotation of a linearly polarized remote antenna about the line of sight.

For test antennas which are nearly circularly polarized, axial ratio data may be obtained simultaneously with pattern data by spinning the remote antenna at a high rate in comparison to the scan rate of the radome-antenna cut. In this case, the axial ratio of the test antenna-radome combination is
superimposed on the relative power pattern.

For linearly polarized test antennas, point-by-point axial ratio measurements are usually specified. For each required $\phi$, $\theta$ orientation of the test antenna, the remote antenna is rotated through 360 degrees. Assuming reciprocal circuit components, these measurements may be performed with the test antenna transmitting or receiving.

When synchro outputs for the angular rotation of the remote antenna are available, the tests described above may also provide tilt-angle data for the effective radome-antenna polarization ellipse. Techniques for performing complete polarization measurements, including axial-ratio, tilt-angle and sense-of-rotation are described in Chapter 10.

REFERENCES

2. "Maintenance Repair and Electrical Requirements of Fiber Laminate and Sandwich Constructed Radomes All Aircraft", T.O. 1-1-24; Dec. 1962, Fig. 7-19.

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INTRODUCTION

This chapter will be concerned with reflectivity measurements fundamentals. Although there has been much theoretical work toward calculating the cross section of targets, exact solutions are practicable for only a few geometrical shapes. The need for experimental data occurs in the investigation of complex targets or when verification of a theoretical solution is desired. Experimental data are of particular value when polarization properties of the target are of concern or when cross section modification (enhancement or reduction) techniques are employed.

13.1 BACKGROUND MATERIAL

The basic problem in reflectivity measurements is the determination of an object's scattering cross section. The scattering cross section \( \sigma \) may be defined as \(^1\) "the area intercepting that amount of power which, when scattered isotropically, produces an echo equal to that observed from the target". In general, \( \sigma \) is a function of the orientation of the target with respect to the incident wave and the position chosen for sampling the scattered energy. A useful analytical definition of \( \sigma \) is

\[
\sigma(\phi_1, \theta_1, \psi_1, \theta_s) = 4\pi R^2 \frac{S_s(\phi_s, \theta_s)}{S_i(\phi_i, \theta_i)}.
\]  

(13.1)

where \( \phi, \theta, \) and \( R \) are spherical coordinates referenced to the target, \( S_i(\phi_i, \theta_i) \) is the power density of the energy incident upon the target from the \( \phi_i, \theta_i \) direction, \( S_s(\phi_s, \theta_s) \) is the power density of the energy scattered
by the target in the $\phi_s, \theta_s$ direction, and $R$ is the distance to the sampling point. The geometry is indicated in Figure 13.1.

At this point, the investigation will be limited to those cases where only linearly polarized waves are incident on the target and only the component of scattered energy with similar polarization is considered in determining $\sigma$. Later in this section the concept of the scattering matrix, which retires this restriction, will be introduced.

A target's scattering cross section depends on the direction of arrival of the incident energy and on the direction to the sampling point. When these two directions are the same; that is, $\phi_i = \phi_s$ and $\theta_i = \theta_s$, the scattering cross section is called the monostatic, or radar cross section. When the directions are not the same, the term bistatic cross section is used.

The need for investigating an object's scattering cross section is due mainly to its involvement with radar, where $\sigma$ enters directly in the monostatic radar equation. Let us assume that a radar and target are situated in free space so that both the radar and target are isolated from extraneous signals. This hypothetical case is closely approximated in practice by a highly directional radar tracking an airborne object directly overhead. Let us
further assume that the radar antenna has a gain $G$ in the direction of the target and upon transmitting has a power $P_o$ watts across its input terminals. The power density on a target which is at some distance $R$ from the antenna is

$$S_1 = \frac{P_o G}{4\pi R^2} \quad (13.2)$$

if the target is in the far-field of the antenna. The power density of the scattered energy at the antenna is

$$S_s = \frac{S_o \sigma}{4\pi R^2} \quad (13.3)$$

where $\sigma$ is the scattering cross section of the target. The power received by the antenna is $P_r = S_s A_e$, where $A_e$ is the effective area of the antenna. The relationship between $A_e$ and $G$ is given by $A_e = G \lambda^2 / 4\pi$, and the equation for the power received becomes

$$P_r = \frac{P_o G \lambda^2 \sigma}{(4\pi)^3 R^4} \quad (13.4)$$

which is the familiar radar equation.²

It is conceivable that with a calibrated radar, $\sigma$ can be measured directly from (13.4) if the range $R$ to the target is known. In practice, the desired accuracy in determining $\sigma$ places such a severe requirement on the calibration of a radar that this method is impractical and other techniques are used for measuring $\sigma$.

The most common method of determining $\sigma$ for an object is the comparison method in which the power scattered from the object is compared with the power scattered from a standard located in the same position as the object. This method eliminates the problem of calibrating the radar since the scattering cross section of the standard is known. Therefore, it is only necessary to measure the difference in power scattered from the two targets to calculate $\sigma$.

*See Appendix 2A.
The metallic sphere is one of the most common cross section standards. The sphere is very popular as a standard in the frequency region where the radius of the sphere is much larger than the wavelength because in this region it behaves as an isotropic scatterer which eliminates the need for critical alignment of the sphere on the mounting pedestal. In addition, in this frequency region, its cross section is easily determined.

For the frequency region in which the radius of the sphere is much larger than the wavelength, the geometrical optics technique of analytically determining $\sigma$ is applicable with small error. This method, as its name implies, is a ray tracing technique. If there is a smooth curved surface upon which energy is incident, then in some direction (determined by the direction of the incident energy and the geometry of the surface) there will be specular reflection of energy, and from a consideration of the power reduction in the ray bundle produced by the divergence of the scattered beam, the cross section is found to be

$$\sigma = \pi R_1 R_2, \quad (13.5)$$

where $R_1$ and $R_2$ are the principal radii of curvature of the surface. For the sphere, $R_1 = R_2$ and $\sigma$ becomes

$$\sigma = \pi r^2, \quad (13.6)$$

where $r$ is the radius of the sphere.

There are three frequency regions of interest for the monostatic scattering cross section of a metallic sphere: (1) Rayleigh region, (2) resonance region, and (3) optical region. These regions are shown in Figure 13.2.*

In the Rayleigh region the scattering cross section is relatively independent of the precise shape of the object, but $\sigma$ is directly proportional to the square

---

* The lines indicating the transitions between regions are not intended to define specific ratios of $a/\lambda$, but rather to indicate general regions. This figure was adapted from reference 1, page 453.
FIGURE 13.2 Calculated value of $\sigma$ for a metallic sphere as a function of its radius in wavelengths.

of the volume of the object. This is easily recognized from the equation for the scattering cross section of an object in the Rayleigh region,

$$
\sigma = k^4 \frac{V^2}{\pi C} F^2 ,
$$

where $k = (2\pi/\lambda)$, $V$ is the volume, $C$ is a numerical constant, and $F$ is a shape factor. For a wide variety of shapes

$$
F = 1 + \frac{1}{\pi y} e^{-y} ,
$$

This discussion of the Rayleigh region is thus not limited to the sphere, but applies to any object whose dimensions are small compared to the wavelength.
where $y$ is a measure of the length-to-width ratio of the body. For more detailed information about the Rayleigh region, refer to references 4, 5, and 6.

It is evident from Figure 13.2 that if the sphere is to be used as a standard for monostatic measurements in the Rayleigh region, the radar cross section for specific ratios of $a/\lambda$ must be employed. The problem is further complicated for bistatic measurements because in the Rayleigh region the sphere does not behave as an isotropic scatterer. This is evident from Figure 13.3 where the bistatic cross section of a sphere with $a/\lambda = 0.175$ is plotted as a function of the angular difference between directions to the transmitter and receiver.

![Graph](image-url)

**FIGURE 13.3** Bistatic scattering cross section for a metallic sphere with $a/\lambda = 0.175$ as a function of the bistatic angle.

*The data for these curves were taken from reference 7.*
Since, in this region, the sphere is not an isotropic scatterer, one must make certain that operation is not in the Rayleigh region if it is desired to use the sphere as a standard for bistatic measurements, unless correction factors are used.

The resonance region* is in general rather difficult to define. It cannot be defined as the region where the cross section exhibits rapid fluctuations with frequency because for a sphere the cross section varies ± 1 per cent about its optical value when a/λ = 6.85. However, for all practical purposes, the resonance region can be thought of as the region intermediate to those for which high frequency and low frequency techniques are applicable. In this region, the cross section of a body may vary widely with frequency, and the oscillations are, in some cases, critical functions of the shape and aspect of the body.

From the above discussions on the Rayleigh and resonance regions, it is apparent that an object, particularly a sphere, should not be used in these regions as a reflectivity standard unless: (1) there are accurate correction factors available that will allow extrapolation for frequency, aspect angle, or bistatic angle variances, or (2) unless the object has been calibrated for the particular frequency and orientation by some absolute calibration technique.

For a better understanding of the cross section of objects, it is worthwhile to write the radar equation in the form

\[
Pr = \left( \frac{P_o G_t}{4\pi R_1^2} \right) \left( \frac{A_T \eta D_T}{4\pi R_2^2} \right) \left( \frac{G_r \lambda^2}{4\pi} \right)
\]

(13.9)

where:

\[ P_r = \text{power received from scattering by the target} \]
\[ P_o = \text{power at the antenna terminals upon transmitting} \]
\[ G_t = \text{gain of transmitting antenna in the direction of the target} \]
\[ D_T = \text{directivity of target in the direction of the receiving antenna.} \]

*Again, the discussion is not limited to the sphere.
\( A_\tau \) = projected area of target normal to line of sight from the transmitter

\( \eta \) = scattering efficiency of the target (The ratio of the total scattered power to the total incident power in the projected area of the target.)

\( G_r \) = gain of receiving antenna in the direction of the target

\( R_1 \) = distance to target from transmitter

\( R_2 \) = distance to target from receiver.

For the monostatic case, which is the most common, \( G_\tau = G_r = G \) and \( R_1 = R_2 = R \). In this case, (13.9) becomes

\[
P_r = \frac{P_0 G^2 \lambda^2}{(4\pi)^3 R^4} (A_\tau \eta D_\tau) .
\] (13.10)

Since equations (13.10) and (13.4) should predict the same amount of received power, it follows that

\[
\sigma = A_\tau \eta D_\tau .
\] (13.11)

To expedite matters later, we will absorb the \( \eta \) term into a gain expression that includes directivity and dissipation. Therefore, equation (13.11) becomes

\[
\sigma = A_\tau G_\tau ,
\] (13.12)

where \( G_\tau \) is the gain of the target. Equation (13.12) allows one to consider a target as an antenna with gain \( G_\tau \), transmitting an amount of power equal to the power intercepted by the projected area \( A_\tau \). It is interesting to note that if the scattering pattern of a target is directional rather than isotropic, the maximum value of its radar cross section must be larger than its projected area unless it has a low scattering efficiency.

For illustrating the concept of (13.12), consider the metallic sphere again. If it is assumed that the metal is a perfect conductor, then there will be no losses. As previously stated, the sphere is an isotropic scatterer in the optical region and therefore has a gain \( G_\tau = 1 \). The power intercepted by the sphere of radius \( r \) is the incident power density multiplied by the projected area \( A_\tau = \pi r^2 \). The
cross section of the sphere is

\[ \sigma = \pi r^2 \quad , \tag{13.13} \]

which is the same as previously derived.

The flat plate is another object that is often used as a reflectivity standard. In the optical region the flat plate represents the largest obtainable monostatic cross section for a given projected area. This is at times very desirable, since it would take a very large sphere to reflect power equivalent to that reflected from a small flat plate. The flat plate's large cross section may be advantageous when the object under investigation has a very large \( \sigma \). Since spheres with large cross sections are impractical, there will be a large difference between the power received from the large target and the power received from the sphere. Because of the increasing probability of calibration and non-linearity errors of the measurement equipment as its dynamic range is increased, significant errors in determining the cross section of the object under test could be introduced.

From the theory of diffraction from apertures, \(^9\) it is well known that the illumination which provides maximum gain from a plane aperture whose dimensions are large compared with the wavelength is that of uniform amplitude and phase. Also, for this illumination the effective area is equal to the physical area, and the gain and physical area are related by*:

\[ G = \frac{4\pi A}{\lambda^2} \quad . \tag{13.14} \]

The illumination of a flat plate oriented normal to a plane electromagnetic wave is uniform in amplitude and phase and therefore if the scattering from the plate is considered as radiation from an antenna, the gain of the plate will be given by equation (13.14), where \( A \) is the area of the plate. From equation (13.12), equation (13.15) gives the maximum monostatic scattering cross section for a plate. As the aspect of the plate changes, its monostatic scattering cross section

\[ \sigma = \frac{4\pi A^2}{\lambda^2} \quad . \tag{13.15} \]

*See Appendix 2A.
varies with the aspect angle. For a rectangular flat plate oriented as in Figure 13.4, \( \sigma \) as a function of \( \phi \) near normal incidence and for \( \theta = 90^\circ \), is given by

\[
\sigma = \frac{4\pi a^2 b^2}{\lambda^2} \cos^2 \phi \left[ \frac{\sin(ka \sin \phi)}{(ka \sin \phi)} \right].
\]

(13.16)

FIGURE 13.4 Coordinate system for determination of the cross section of a flat plate that is in the yz-plane.

The dihedral reflector is another interesting geometrical shape that could be used as a reflectivity standard. This reflector is illustrated in Figure 13.5. It consists of a pair of planes which intersect in a right angle. The dihedral reflector has the characteristics that any ray which impinges upon the reflector in a plane which is normal to the intersection of the planes comprising the reflector and from a direction less than \( 45^\circ \) from the bisector plane is reflected twice and returns in the direction of incidence.

If a ray entering the reflector is reflected twice, the path length for such a
ray, if traced from a plane that is normal to the incident ray, having traversed the reflector and returned to the normal plane, is the same for any path chosen. Therefore, over the portion of the plane normal to the incident rays where the above conditions exist, this area can be considered an aperture with uniform amplitude and phase illumination. The effective area of this aperture for $a/\lambda >> 1$ is given by

$$A_e = 2ab \sin (45^\circ - \theta),$$

(13.17)

where $\theta$ is referenced to the bisector of the dihedral and is always less than 45 degrees. The radar cross section can be determined from (13.15).

An interesting feature of the dihedral reflector is its influence on the polarization of the incident wave. When a linearly polarized wave enters the reflector in a plane perpendicular to the intersection of planes comprising the reflector and is returned in the direction of incidence, the plane of polarization of the reflected wave is rotated through an angle $2\phi$, where $\phi$
is the angle between the plane of polarization of the incident wave and the intersection of the planes comprising the reflector. The rotation is in the opposite direction from the angle between the incident plane of polarization and the intersection of planes comprising the reflector.

Figure 13.6 illustrates the relative sizes for a given radar cross section of the three targets discussed. The frequency chosen for this example has a wavelength of 10 centimeters. The maximum cross section for each target is 117 square feet.

FIGURE 13.6 Comparison of three targets for equal radar cross sections.

The decision of which target to use as a reflectivity standard should be based upon such factors as: (1) the desired accuracy of measurements, (2) the expense of fabricating the standard, (3) the size of the object whose cross section is to be compared with the standard, (4) the support structure for the target, (5) the reflectivity range configuration, and (6) the polarization.
characteristics of the standard. The previous discussions on typical reflectivity standards are intended merely as a guideline for making the decision of which object to choose. In practice, there have been other objects that have been used as standards. Some of these are: Luneberg lenses, cylinders, corner reflectors, dipoles, cone-sphere and circular disks. It is beyond the scope of this text to discuss all of these. The reflectors discussed were chosen because they have characteristics that illustrate points that will be discussed later.

Scattering Matrix - - Thus far in the discussion on the scattering cross section, the investigation has been limited to cases where only linearly polarized waves are incident upon a target and only the component of scattered energy with similar polarization has been considered in determining $\sigma$. This section will consider the polarization properties of targets.

By considering the previously discussed polarization properties of the dihedral reflector, it can be seen that if the monostatic scattering cross section of the dihedral reflector was recorded, as a linearly polarized transmit-receive antenna was rotated about the line of sight, the cross section of the reflector would vary with the polarization. Therefore, even for this very simple target there is a need for knowledge about the polarization properties of the target in order that its cross section can be determined for any polarization. This need is even more pronounced when complex targets are considered.

The scattering matrix $S$ provides a method that removes the aforementioned limitation and allows the cross section to be determined for any polarization. Knowledge of $S$ completely defines the target polarization characteristics for the particular frequency, radar bistatic angle, and target aspect angle involved: the cross section of the target may be calculated for any desired transmitting and receiving antenna combination.

Before discussing the scattering matrix it will be helpful to consider the polarization properties of antennas. ** The polarization characteristics of

*A bar beneath a letter will denote a matrix.

**For a more detailed coverage, refer to Chapter 3.
an antenna are defined by the polarization characteristics of the wave transmitted by the antenna. For all cases, it is assumed that the far-field characteristics of the antennas (or scatterers) are of interest, where the field consists of only transverse components and varies as $1/r$. Therefore, describing the polarization characteristics of the wave can be accomplished in two dimensional space that is transverse to the direction of propagation, and any polarization can be described by two orthogonal components in the transverse plane.

The orthogonal components of a wave $\mathbf{W}$ may be represented in matrix form by

$$\mathbf{W} = \begin{bmatrix} \mathbf{E}_M \\ \mathbf{E}_N \end{bmatrix},$$

(13.18)

where $\mathbf{E}_M$ and $\mathbf{E}_N$ are the orthogonal components of the wave. * As established in Chapter 3, these components may be linear, circular, or more generally elliptical, so long as they are orthogonal in average power. ** In the following discussion the orthogonal components of the wave will be represented by linear components.

For a right-handed Cartesian coordinate system, as illustrated in Figure 13.8, any elliptically polarized field in a plane which is normal to the direction of propagation of the wave producing the field can be expressed in terms of $\mathbf{u}_1$ and $\mathbf{u}_2$ components as

$$\mathbf{W} = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha e^{j\omega t} \\ \sin \alpha e^{j(\omega t + \delta)} \end{bmatrix},$$

(13.19)

where $\delta$ is the relative phase of the $\mathbf{u}_2$ component with respect to the $\mathbf{u}_1$ component and $\alpha$ is the angle whose tangent is the ratio of the magnitude of the

---

* A bar above a letter denotes a phasor quantity.
** The subject of orthogonality is discussed in Appendix 3B.
The components are normalized such that the effective value of the total field is unity.

\[ \vec{u}_2 \text{ component to the } \vec{u}_1 \text{ component.} \]

An antenna that transmits a general elliptically polarized wave can be represented by the matrix \( A \), where

\[ A = \begin{bmatrix} \cos \alpha & e^{i\delta} \\ \sin \alpha \end{bmatrix}. \quad (13.20) \]

To define the receiving polarization \( A_r \), the rules of Appendix 3D must be used. Also, if a wave \( W \), where \( W \) is given by

---

* A dot beneath a letter denotes normalization.

** See section 3.7.
is incident upon the antenna $\Delta_r$, as illustrated in Figure 13.9, the polarization efficiency can be determined as in Chapter 3 from the inner product

$$\vec{V} = (\Delta_r, \vec{W}) \quad . \quad (13.22)$$

\[
\vec{W} = \begin{bmatrix} \cos \alpha_w \\ \sin \alpha_w e^{j\delta w} \end{bmatrix}
\quad (13.21)
\]

The inner product of two vectors using matrix multiplication is defined as

$$ (\vec{A}, \vec{B}) = \vec{A}^{\dagger} \vec{B} = \vec{a}_1 \vec{b}_1 + \vec{a}_2 \vec{b}_2 \quad , \quad (13.23)$$

where $\vec{a}_1$, $\vec{a}_2$ and $\vec{b}_1$, $\vec{b}_2$ are components of $\vec{A}$ and $\vec{B}$ respectively.

Using $\vec{A}_r$ with (13.21) in (13.22), the inner product $\vec{V}$ is given by
Let us return now to the consideration of scattering from targets. The polarization of the energy scattered by an object is determined by (1) the polarization of the incident energy, and (2) the polarization properties of the scattering object. If the polarization effects of the medium on the wave are neglected, then the polarization of the incident energy at the scattering object is the same as the polarization of the transmitting antenna.

The scattering properties of an object are defined by a two by two matrix \( S \) which relates the incident and scattered fields, and if these fields are resolved into linear components, the relationship is given by

\[
\begin{bmatrix}
E_{1s} \\
E_{2s}
\end{bmatrix} = S \begin{bmatrix}
E_{1i} \\
E_{2i}
\end{bmatrix},
\]

(13.25)

where the subscripts \( s \) and \( i \) designate the scattered and incident fields, and the subscripts 1 and 2 denote components in the \( \vec{u}_1 \) and \( \vec{u}_2 \) directions respectively. The scattering matrix \( S \) for the general orthogonal polarizations \( M \) and \( N \) is

\[
S = \begin{bmatrix}
S_{MM} & S_{MN} \\
S_{NM} & S_{NN}
\end{bmatrix},
\]

(13.26)

where the first subscript designates the polarization transmitted, and the second subscript designates the polarization scattered or received. The components of \( S \) are defined as

* Under most laboratory environments, this assumption is valid.
** See page 3.31 for typical polarizations \( M \) and \( N \) can represent.
For each of the elements in (13.27), the polarization of the incident energy consists of only the component indicated by the subscript of the respective denominators.

From (13.25), it is seen that the matrix $S$ transforms the components of the incident energy into components of the scattered field. As stated previously, the polarization of the incident wave is the same as the polarization of the transmitting antenna. The scattering cross section $\sigma$ will be seen to be determined by the polarization matrix of the transmitting antenna, the scattering matrix and the polarization matrix of the receiving antenna. From equation (13.4) it is seen that if all the other variables are held constant, the power received by the radar antenna is directly proportional to the scattering cross section of the object under test.

Since the range dependence of the incident and reflected fields is suppressed in the treatment of polarization given herein, the magnitudes of the elements of the scattering matrix are equal to the square root of the scattering cross sections of the target for the respective transmitting and receiving polarizations. Also, it is sufficient to know only the relative phases of the matrix elements.

For the monostatic case, there is no unique way for specifying the relative phases of the elements in the matrix, and care must be taken when interpreting these phase angles. This problem arises because the polarization of the incident wave and the reflected wave are described in different coordinate systems. The problem is illustrated by considering the scattering from a flat plate.

Assume that a flat plate has dimensions that are large compared to the
wavelength and there is no depolarization due to the edges of the plate. Let us further assume that the monostatic cross section of the plate, which is oriented normal to the line of sight from the transmitter-receiver location, is $k$ square meters when the $\vec{u}_1$ polarization is both transmitted and received. Since there is no depolarization, $\tilde{S}_{21} = \tilde{S}_{12} = 0$.

If we choose to define the scattering matrix such that it transforms the incident wave as defined in its own coordinate system into a scattered wave that is defined in its coordinate system, then the scattering matrix for the plate will be

$$ S = \begin{bmatrix} \sqrt{k} & 0 \\ 0 & -\sqrt{k} \end{bmatrix}. \tag{13.28} $$

As a specific example, let a wave whose polarization is defined upon transmitting in its own coordinate system by $\alpha = 45^\circ$ and $\delta = 0$ be incident upon the target. This is a linearly polarized wave rotated $45^\circ$ from the $\vec{u}_1$ axis toward the $\vec{u}_2$ axis, and is described by the matrix

$$ E_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}. \tag{13.29} $$

The coordinate system is shown in Figure 13.10, and the scattered field is

$$ E_2 = \begin{bmatrix} \sqrt{k} & 0 \\ 0 & -\sqrt{k} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} \sqrt{k}/2 \\ -\sqrt{k}/2 \end{bmatrix}, \tag{13.30} $$

which, as is intuitively known, must be polarized in the same plane of polarization as the incident wave. If we had chosen the relative phase of $\tilde{S}_{11}$ and $\tilde{S}_{22}$ to be zero, then the scattered wave would be orthogonal to the incident wave, which we know from physical considerations is impossible.
FIGURE 13.10 Coordinate system showing relative phase of $s_{11}$ and $s_{22}$ of the scattering matrix for a flat plate where the incident and scattered waves are described in their respective coordinate systems.

If the transmitting antenna is used also for receiving, then the receiving polarization is

$$A_r = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$  \hspace{1cm} (13.31)$$

and by substituting (13.30) and (13.31) in (13.22) the inner product $\bar{V}$ becomes

*See page 13-15.

*Note: The inner product $\bar{V}$ of (13.32) for the scattering case is not equal to the inner product $\bar{V}$ of (13.22) for the direct transmission case, because (13.32) contains information relating to the magnitude of the scattering cross section in addition to its polarization properties while (13.22) is normalized to give only information related to the polarization efficiency between the receiving antenna and the incident wave.
The scattering cross section $\sigma$ for any target is given by

$$\sigma = \overline{V} \overline{V}^*.$$

(13.33)

For this example $\sigma$ is equal to $k$.

The scattering matrix can also be defined such that it transforms an incident wave, whose polarization has already been transformed to the coordinate system of the scattered wave, into a scattered wave whose polarization is described in its own coordinate system. For this case the scattering matrix for the flat plate is

$$S = \begin{bmatrix} \sqrt{k} & 0 \\ 0 & \sqrt{k} \end{bmatrix}.$$  

(13.34)

The reason the relative phase is zero can be seen from Figure 13.11.

If the same transmit-receive antenna as used in the previous example is again used, then the polarization of the incident wave, after being transformed to the coordinate system of the scattered wave is

$$\overline{E}_i = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}.$$  

(13.35)
FIGURE 13.11 Coordinate system showing relative phase of $\tilde{s}_{11}$ and $\tilde{s}_{22}$ of the scattering matrix of a flat plate when the incident and scattered waves are described in the coordinate system of the scattered wave. In (13.34) the phase reference for the scattering matrix has been rotated through $\pi$ radians since only the relative phase of the matrix elements is of consequence.

The scattered wave becomes

$$E_s = \begin{bmatrix} \sqrt{k} & 0 \\ 0 & \sqrt{k} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} \sqrt{k}/2 \\ -\sqrt{k}/2 \end{bmatrix},$$

which is polarized in the same plane of polarization as the incident wave and is that predicted by (13.30).

The receiving polarization is again given by (13.31), and $\sigma$ is found to be equal to $k$ as before.
For bistatic problems the transformation of the incident wave to the coordinate system of the scattered wave is quite complicated and is beyond the scope of this text. The reader is referred to reference 11 for a discussion of the problem.

A procedure identical to that used in the examples can be used for finding the monostatic cross section for any general transmitting and receiving polarizations. For instance, if the scattering matrix elements are measured in the $\vec{u}_1$ and $\vec{u}_2$ polarization basis, then the general equation for $\bar{V}$ will be

$$\bar{V} = \begin{bmatrix} \cos \alpha_R \sin \alpha_R e^{-j(\pi - \delta_R)} \\ \sin \alpha_R e^{j\delta_R} \end{bmatrix} \begin{bmatrix} \vec{S}_{11} & \vec{S}_{21} \\ \vec{S}_{12} & \vec{S}_{22} \end{bmatrix} \begin{bmatrix} \cos \alpha_T \\ \sin \alpha_T e^{j\delta_T} \end{bmatrix}, \quad (13.37)$$

where the subscripts $R$ and $T$ denote the receiving and transmitting antennas respectively, which may or may not be the same antenna. Also, $\delta_R$ is the phase angle used for defining the polarization of the receiving antenna on transmitting.* To find the cross section for any polarization combinations (13.37) can be used in (13.33).

13.2 REFLECTIVITY MEASUREMENT PROBLEMS

A problem of primary importance in reflectivity measurements is that caused by the unwanted signal produced by energy scattered by the background. The term background implies any obstacle such as mounting pedestal, turntable, the natural environment, and similar objects which are sources of radar returns other than the target being measured. Unless some means of discrimination or cancellation is used, an error in the determination of the cross section of the target will result.

One of the major objectives of the measurement techniques that will be described in the following section is to reduce the effects of background interference so that cross sections can be measured accurately. The magnitude

*Note that $(\pi - \delta_R)$ in (13.37) is the phase $\delta_r$ in radians of the receiving polarization defined in Chapter 3. See page 13-15.
of the possible error due to the background is illustrated by the following example. For simplicity, it will be assumed that the fields at the receiving antenna due to both the target and background are of the same polarization as the receiving antenna. The measured field $E_M$ at the receiving antenna is

$$E_M = E_t + E_b e^{j\phi}$$

(13.38)

where $E_t$ is the field due to the target, $E_b$ is the field due to the background, and $\phi$ is the difference in phase of the fields of the target and background.

Since the power received is proportional to the scattering cross section, the fields are proportional to the square root of the respective cross sections. The factor of proportionality will appear on both sides of (13.38), and therefore in terms of cross sections, (13.38) becomes

$$\sqrt{\sigma_M} = \sqrt{\sigma_t} + \sqrt{\sigma_b} e^{j\phi}$$

(13.39)

where $\sigma_M$ is the measured cross section, $\sigma_t$ is the cross section of the target, and $\sigma_b$ is the cross section of the background.

Using Euler's identity and the law of cosines in (13.39) gives

$$\sigma_M = \sigma_t + \sigma_b + 2\sqrt{\sigma_t \sigma_b} \cos \phi$$

(13.40)

From equation (13.40) it is seen that the measured cross section can vary between the limits $\sigma_t + \sigma_b + 2\sqrt{\sigma_t \sigma_b}$ and $\sigma_t + \sigma_b - 2\sqrt{\sigma_t \sigma_b}$. Therefore the error in $\sigma_M$ can be appreciable if $\sigma_b$ is not negligible. The upper and lower limits on the error are plotted in Figure 13.12 as a function of $\sigma_t/\sigma_b$. It is interesting to note that for a measured accuracy of ±1 dB, $\sigma_t$ must be 20 dB greater than $\sigma_b$.

The phase of $\sigma_t$ can be varied with respect to that of $\sigma_b$ by moving the target in the direction of incidence. If the relative phase change is such that $\sigma_M$ passes through a maximum and a minimum, the magnitude of the ripple can be identified as the error due to the background.
In the operational environment there is almost always a very large separation between the source and the target and between the target and the receiving antenna. The fields which are incident on the target and on the receiving antenna have spherical wave fronts of such large radii that they can usually be considered plane waves. The measurements must be made at ranges which adequately simulate plane wave conditions.

Errors due to inadequate separation are caused by phase and amplitude variations of the incident fields at the target and receiving antenna. The distance requirements are analogous to the requirements for antenna ranges and the reader is referred to section 14.2 for a detailed treatment of the subject.

FIGURE 13.12  Upper and lower limits of the possible error in cross section measurement due to the background.
13.3 Reflectivity Measurements Techniques

In this section certain of the more commonly used measurement techniques will be discussed. The discussion will be kept brief, and references will be given to enable the reader to pursue particular techniques in greater detail.

Standing-Wave Method -- The standing-wave method is an absolute calibration technique in that no standard is required for cross section determination. This technique is used in conjunction with a ground plane, and only objects that have a plane of symmetry can be measured by this method.

The basic setup for the method is shown in Figure 13.13. By moving the probe in a direction along the line of sight between the transmitter and target, there will be maximum and minimum signal at specific points. If the distance from the target to a particular maximum is $\omega_1$ and to the adjacent minimum is $\omega_2$, then by measuring the SWR in the region between $\omega_1$ and $\omega_2$, the cross section can be calculated with the aid of equations 13.41 and 13.42.

![Figure 13.13 Basic block diagram for measuring cross sections by the standing-wave method.](image-url)
\[ \Gamma = \frac{\omega_1 (\text{SWR} - 1)}{(\omega_1/\omega_0) \text{SWR} + 1}. \]  

(13.41)

\[ \sigma = 4\pi |\Gamma|^2 \]  

(13.42)

In (13.41) \( \Gamma \) is the reflection coefficient of the target for the particular aspect involved. \(^{12,13}\)

Cross sections have been measured with the standing-wave technique with an accuracy between 5 and 10 percent.

**CW Method** -- In the CW method the received signal due to scattering from the background is cancelled by a nulling technique. The basic block diagram for this system is shown in Figure 13.14.

![Block Diagram](image)

**FIGURE 13.14** Basic block diagram for the CW method of measuring radar cross section.

With the target removed from the mounting pedestal, the slide-screw tuner is adjusted until the receiver is nulled. Then the object is placed on the pedestal and the power received recorded. The power received from a cross section standard can then be compared with that received from the test target, and from this comparison, the cross section of the target can be determined.
The dynamic range and accuracy which can be achieved depends to a large extent on the degree of background cancellation that can be obtained. One of the prerequisites for a high degree of cancellation is a frequency stable transmitter. Phase locked oscillators (described in Chapter 15) provide a high degree of stability and are often employed. It is also important that the magic tee and other waveguide components have very low temperature coefficients or be placed in a temperature controlled environment.

Further information can be found in the articles of references 12, 13, 14, and 15.

**FM/CW Radar Method** -- This method discriminates against reflections from the background by frequency selectivity. The basic block diagram for the method is illustrated in Figure 13.15. The transmitted frequency varies linearly as a function of time, and some of the transmitted signal is introduced into the mixer for use as a local oscillator power by reflection from the branch of the magic tee that contains the tuner. The L.O. signal is
mixed with the signal received from scattering by the target, and gives an appropriate IF signal. The frequency of the IF signal is a function of the distance to the target; and unwanted signals, which are outside a range interval, or cell, defined by the constants of the system, can be discriminated against by selective filtering.

When physically large targets are being investigated, care must be taken to insure that the frequency does not change significantly during the transit time of the slow wave around the target. If this occurs, degradation of the measured cross section will result because large differential phase shifts between incremental scatterers over the target can result from a small frequency change.

Detailed information on the FM/CW method may be found in references 12, 15, 16, and 17.

Pulse Radar Method - - The pulse technique provides discrimination against the background by using a short RF pulse and a range gate. There are two basic types of pulse measurement methods. In one method the pulse is sufficiently larger than the target to provide essentially continuous wave illumination of the target during the pulse interval. In the second method the transmitted pulses are short (with widths into the sub-nanosecond range) to permit investigation of individual scatterers (flare spots) within the target.

While the pulse radar tends to discriminate against scatterers which are outside of a cell defined in angle by the beamwidth of the antenna, or antennas, and in range by the pulse width and the range gate of the receiver, it does not discriminate against unwanted scatterers within this cell. The problem of mounting and orienting the target without introducing excessive error in the measurements thus constitutes one of the major areas of concern associated with the pulse technique.

When measurements are required at short ranges, especially with true-monostatic systems, multiple reflections within the waveguide system can cause spurious targets which occur within the range gate. Spurious signals can also result if reflections from large distant targets occur within the range gate. Additional spurious signals can occur from large targets at
the range of the target under test which are in the side-lobes of the antenna or antennas.

For additional information on the pulse technique, see references 15, 18, 19, and 20.
REFERENCES

CHAPTER 13


10. From personal communication with Dr. Maurice W. Long, Director, Engineering Experiment Station, Georgia Institute of Technology.


*Reference 21 appears on page 13-10 of the text.
CHAPTER 14
ANTENNA RANGE DESIGN AND EVALUATION
T. J. Lyon, J. S. Hollis, and T. G. Hickman

14.1 INTRODUCTION

The major objectives of antenna test facilities have been described in previous chapters of this text. Subsequent chapters will present concepts and details of major items of test instrumentation, and of the integration of such instrumentation into measurement systems.

This chapter discusses principles and techniques for the establishment and proof of adequate electromagnetic characteristics of the test environment. In particular, the discussions here will relate to test facilities which are intended for use in measuring the free-space far-zone performance of antennas.

For purposes of consistency in notation, it will be convenient to assume that all antennas are tested on receiving. The criteria and techniques discussed herein are also valid for the design and evaluation of ranges used to test antennas operated on transmission.

The ideal test environment for determining far-zone antenna performance would provide for a plane wave of uniform amplitude to illuminate the test aperture. Various approaches to simulation of this ideal electromagnetic environment have led to the evolution of two basic types of antenna test ranges,

1. Free-space Ranges
2. Reflection Ranges.

Free-space ranges are those in which an attempt is made to suppress or remove the effects of all surroundings, including the range surface or
surfaces, on the wavefront which illuminates the test antenna. This suppression is sought through one or more of such factors as (a) directivity and sidelobe suppression of the source antenna and the test antenna, (b) clearance of the line of sight from the range surface, (c) redirection or absorption of energy reaching the range surface, and (d) special signal processing techniques such as tagging by modulation of the desired signal or by use of short pulses.

The typical geometries associated with the free-space approach include the ELEVATED RANGE, the SLANT RANGE, the RECTANGULAR ANECHOIC CHAMBER, and, above certain limiting frequencies, the TAPERED ANECHOIC CHAMBER. A recent development in this area is the COMPACT RANGE, in which the test antenna is illuminated by collimated energy in the aperture of a larger point- or line-focus antenna.

Reflection ranges are designed to make use of energy which is reradiated from the range surface(s) to create constructive interference with the direct path signal in the region about the test aperture. The geometry is controlled so that a small, essentially symmetric amplitude taper is produced in the illuminating field. The two major types of reflection ranges in use are the GROUND REFLECTION RANGE and, for low frequencies, the TAPERED ANECHOIC CHAMBER.

14.2 ELECTROMAGNETIC DESIGN CONSIDERATIONS AND CRITERIA

For either basic type of range, the fundamental electromagnetic design criteria deal with control of

1. Inductive or radiation coupling between antennas,
2. Phase curvature of the illuminating wavefront,
3. Amplitude taper of the illuminating wavefront,
4. Spatially periodic variations in the illuminating wavefront caused by reflections,
5. Interference from spurious radiating sources.

Items (1) through (4) primarily establish the dimensional requirements on the range design, and limiting values of source-antenna directivity. Item (5) must be considered in the overall design.

14.2.1 Effects of Coupling between Antennas - - - At the lower microwave frequencies, the effects of inductive coupling between the source antenna and
the test antenna must be considered. Such effects are usually considered negligible when the criterion

$$R \geq 10\lambda$$

(14.1)

is satisfied, where \(R\) is the separation between antennas and \(\lambda\) is the wavelength. This criterion is based on the field equations for an elemental electric dipole, from which the ratio of the amplitude of the induction field to that of the radiation field is seen to be

$$\rho_e = \frac{\lambda}{2\pi R} .$$

(14.2)

At \(R \geq 10\lambda\), \(\rho_e \leq 1/20\pi\), and the criterion is seen to be equivalent to the requirement that

$$20 \log (\rho_e) \leq -36 \text{ decibels} .$$

(14.3)

**Mutual coupling** due to scattering and reradiation of energy by the test and source antennas is also of concern. If the source antenna produces a significant illumination taper between the center and edges of the test aperture, interaction between the antennas can cause a measurable error in the signal levels observed near the peak of the test antenna's main lobe. The effect of mutual coupling on sidelobe accuracies is usually negligible.

As an example, consider a measurement involving paraboloidal test and source antennas with diameters \(D\) and \(d\), respectively, and \(\sin x/x\) amplitude characteristics. The 3-dB beamwidth of the source antenna is very nearly

$$\theta_s = \frac{1.22\lambda}{d} \text{ radians} .$$

(14.4)

For a range \(R\), which is large with respect to \(D\), the plane angle subtended at the source antenna by a diameter of the test aperture will be

$$\alpha_0 = 2 \tan^{-1} (D/2R) = \frac{D}{R} \text{ radian} .$$

(14.5)

If the range is given by
then from (14.4)-(14.6) we may write

\[ d = 1.22 R \frac{D \phi}{\theta_s} \quad \text{(14.7)} \]

The power \( P_r \) received by the test antenna at the peak of the beam is, for the polarization-matched case,

\[ P_r = \frac{P_o G_s}{4\pi R^2} \epsilon_r A_r \quad \text{(14.8)} \]

where

- \( P_o \) = power input to source antenna,
- \( G_s \) = peak gain of source antenna,
- \( \epsilon_r \) = aperture efficiency of test antenna, and
- \( A_r \) = physical area of the test aperture.

Assuming a reciprocal source antenna, we may write *

\[ G_s = 4\pi \epsilon_s A_s / \lambda^2 \quad \text{(14.9)} \]

where \( \epsilon_s \) and \( A_s \) are defined as for the receiving use. From (14.6)-(14.9), we have

\[ P_r = P_o \left[ 0.92 \epsilon_s \epsilon_r \left( \alpha_g / \theta_s \right)^2 \right] \quad \text{(14.10)} \]

The energy within the solid angle subtended by the test antenna is transmitted to the load, dissipated or scattered. The paraboloid (or any antenna with a line or point focus) directs a large fraction of the scattered energy back toward the source. If the load is not ideally matched, a fraction of the received signal will also be reradiated. The net result is that the combined back-scattered signal due to scattering and reradiation may be only a few decibels below the received signal level.

*See Chapter 2.
Let the back-scattered power be $k_r P_r$, and similarly let the power available for retransmission at the source antenna be $k_s P_s$, where

$$P_s = k_s P_r \left[ 0.92 \epsilon_s \epsilon_r (\alpha_0 / \theta_s)^2 \right]. \quad (14.11)$$

The power received by the test antenna due to reradiation becomes

$$P'_t = P_0 \left[ k_s k_r (0.92 \epsilon_s \epsilon_r)^2 (\alpha_0 / \theta_s)^4 \right], \quad (14.12)$$

so that

$$\frac{P'_t}{P_r} = k_s k_r (0.92 \epsilon_s \epsilon_r)^2 (\alpha_0 / \theta_s)^4. \quad (14.13)$$

Typical values for the reflection and efficiency terms are $k_s = k_r = 0.25$ and $\epsilon_s = \epsilon_r = 0.5$; thus we may write

$$10 \log (P'_t / P_r) = -24.7 + 40 \log (\alpha_0 / \theta_s) \text{ decibels}. \quad (14.14)$$

The level of the signal arriving at the test antenna due to retransmission from the source antenna will be at least 45 decibels below the original received signal if the ratio $\alpha_0 / \theta_s$ is made equal or less than $1/3.2$. This ratio corresponds to a subtended angle approximately equal to the $1/4$-decibel beamwidth of the source antenna; the reradiated signal in this case could cause an error in measured gain of about $\pm 0.05$ decibel.

In addition to the error produced by retransmission from the source antenna, error can be caused if the signal source is not isolated from the source antenna. The power level and frequency of the source may change because of the variation in loading caused by the mutual coupling effects. Error from this source can be virtually eliminated by isolation and stabilization of the signal source.*

14.2.2 Effect of Curvature of the Incident Phase Front -- In the absence of reflections, the phase variation of the field over the aperture of a

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*See Chapter 15.
receiving antenna of a given size and operating at a given frequency depends almost entirely on the separation between the source antenna and the antenna under test and not upon the beamwidth of the source antenna. If the receiving antenna is in the far zone of the transmitting antenna, the phase front of the approaching wave deviates very little from a section of a sphere centered on the transmitting antenna over the major portion of the main lobe. This is shown in Figure 14.1 which is a graph of the calculated phase deviation in degrees over a spherical surface through the main lobe of the beam produced by a circular transmitting aperture. A 30-dB Taylor distribution is assumed, and four different distances are assumed from the transmitting antenna to the spherical surface: (a) \( d^2/\lambda \), (b) \( 2d^2/\lambda \), (c) \( 4d^2/\lambda \), and (d) infinite, where \( d \) is the aperture width of the transmitting antenna.

![Figure 14.1 Deviation of transmitted phase fronts from spheres centered on transmitting antenna. R is radius. A 30-dB Taylor aperture distribution is assumed](image)

The main lobe extends from approximately \( U = -1.6\pi \) to \( U = 1.6\pi \), where \( U = (\pi/\lambda)d\sin\theta \). Even at a range as small as \( d^2/\lambda \) the phase front is spherical.
to within 2 degrees between the 1-decibel points of the pattern; this condition is typical of reasonably focused symmetrical antennas. If the transmitting antenna is focused at the test range, the phase front will be essentially that for $R = \infty$. If the transmitting antenna is significantly defocused, slightly greater phase variation will be experienced. In any event, the deviation of the phase front from spherical between the 1/4-decibel points of the beam will be small.

It is seen that over a planar receiving aperture the variation of the phase of the incident field is caused almost entirely by deviation of the test aperture from the sphere centered on the transmitting antenna if the receiving antenna subtends less than a half-power beamwidth of the transmitting antenna's wave front. In practice, the antenna under test will subtend considerably less than a half-power beamwidth in order to reduce error from mutual coupling and from amplitude taper of the incident field over the test aperture.

An expression for the phase deviation over a planar test aperture can be determined from Figure 14.2(a). Since

$$R^2 + \frac{D^2}{4} = (R + \Delta R)^2,$$

(14.15)

$$\Delta R \approx \frac{D^2}{8R}.$$

(14.16)

if $\Delta R^2$ is neglected. The corresponding phase deviation is given by

$$\Delta \phi = \frac{2\pi \Delta R}{\lambda} \approx \frac{\pi D^2}{4\lambda R} \text{ radians}.$$

(14.17)

Figure 14.2(b) is a family of graphs of the ranges which will produce different spherical phase variations over antenna apertures up to 100 feet in diameter. The data are presented in terms of range in feet divided by frequency in GHz versus aperture width in feet. The phase variations are from $\pi/256$ to $2\pi$ radians.

A commonly employed criterion for determining the minimum allowable separation between the source antenna and the antenna under test is to restrict $\Delta \phi$ to a maximum of $\pi/8$ radian, or 22.5 degrees. Under this
FIGURE 14.2 Deviation of spherical phase-fronts from planar at extremes of planar apertures of width D. The corresponding phase deviations are \((2\pi/\lambda) \Delta R\) radians at ranges \(R = K(D^2/\lambda)\).
condition, \( R \geq 2D^2/\lambda \). If antenna measurements are made at a range of \( 2D^2/\lambda \), there will be a significant departure of the nulls of the radiation pattern and the location and levels of the minor lobes from their infinite-range values. The amount of the deviation depends on the original side-lobe level and structure. D. R. Rhodes calculated that at a range of \( 2D^2/\lambda \) the first null of the pattern produced by a rectangular aperture with uniform illumination has a relative level of about -23 decibels instead of -23 decibels. This theoretical deviation is due solely to phase-error effects; the incident-wave amplitude over the test aperture was assumed constant. The infinite range pattern in the above case has a \( \frac{\sin x}{x} \) configuration, with a relative first-lobe level of about -13 decibels.

Figure 14.3 is a graph showing calculated patterns of a circular aperture with a 30-decibel Taylor distribution at separations of \( 2D^2/\lambda \), \( 4D^2/\lambda \), and infinity. If an antenna such as this is adjusted for optimum focus at a range of \( 2D^2/\lambda \) or \( 4D^2/\lambda \), for example, the antenna will be slightly defocused for operation at extreme ranges. It is evident that, if extreme accuracy of the infinite-range side-lobe structure is desired, measurements must be made at a range which is appreciably greater than \( 4D^2/\lambda \).

If the antenna under test is of a type whose focus can be changed, the feed can be focused for optimum patterns at the range employed and subsequently readjusted axially by a calculated amount to focus the antenna at infinity. Certain aberrations are introduced into the measurements by focusing the antenna at finite range. If the antenna should have an extremely large \( f/D \) ratio, 2 or greater, such as that found in typical optical devices, the phase error caused by axial focusing would be almost entirely proportional to the square of the normalized radial position in the aperture. This is the same type of phase error caused by operation of the antenna at less than infinite range, and patterns made by focusing the antenna at the test range will be virtually identical with the infinite-range patterns of the antenna when focused at infinity.

However, for most antennas the \( f/D \) ratios are less than 0.5, and the phase errors caused by axial focusing contain significant even terms of order higher than 2. The shorter the range at which the antenna is focused, the greater will be the amplitude of the high order aberration terms.
FIGURE 14.3 Calculated radiation patterns of a paraboloid with quadratic phase errors encountered in measuring at three ranges as indicated.
Figure 14.4(a) is a graph of the phase error over the apertures of paraboloids with four f/D ratios focused at finite range R and operated at infinite range. The phase error is normalized to $\pi D^2/4\lambda R$ radians and plotted versus radial aperture position normalized to the aperture radius. Figure 14.4(b) shows the approximate resulting phase error over the aperture when the same paraboloids are operated at ranges that produce zero phase error at the peripheries of the paraboloids.

In practice, focusing will be accomplished by adjustment of the feed for deepest nulls. This will not generally result in zero phase difference between contributions from the periphery and the center of the aperture; rather the phase at the periphery will be slightly negative. This will result in an average phase error which is somewhat less than that indicated in (b). Cheng gives a graph which defines the axial feed motion required to refocus an antenna at infinity as a function of the test separation measured in antenna focal lengths.

14.2.3 Effect of Amplitude Taper Over the Test Aperture -- For accuracy in simulated far zone measurements, the illuminating field must be sufficiently constant in amplitude both along the line-of-sight and in planes normal to the line-of-sight.

Consider an antenna under test on receiving, which has a maximum dimension, L, of its active region along the line-of-sight. If the separation between the source antenna and the center of the active region is $R_o$, then the ratio $\rho_p$ of the power density at the forward extreme of the active region to that at the rear is given by

$$10 \log (\rho_p) = 20 \log \left( \frac{R_o + L/2}{R_o - L/2} \right) \text{ decibels} \quad \ldots \quad (14.18)$$

Severe axial variations of the illuminating field can cause measurement error, particularly in the minor lobe structure of radiation patterns. For most antenna types which have significant depth to their active regions, such error is usually negligible when the power density over the region is constant to within one decibel. This condition corresponds to an approximate restraint
FIGURE 14.4 Normalized Phase Errors of Paraboloids; (a) Focused at finite range and operated at infinity, and (b) operated at the range for which focusing was accomplished. Unity corresponds to a phase error of $\pi D^2/4AR$ radians. In these graphs, focusing is defined by zero phase difference of the contributions from the center and periphery of the antenna.
The criterion, for such structures as high-gain disc-on-rod antennas, often is more restrictive than the greater of the previously discussed range-length criteria which were based on suppression of inductive coupling and phase curvature.

The effect of amplitude taper of the incident field over a plane normal to the line of sight and adjacent to the test aperture can be considered from the viewpoint of reciprocity. Variation of the amplitude of the field over the aperture on receiving is analogous—within the accuracy of the aperture field approach—to modification of the aperture illumination by the primary feed on transmitting. For example, consider the pattern of an antenna whose feed would produce an aperture illumination \( f(\theta, r) \) on transmitting, where \( (\theta, r) \) indicates position in the aperture. If illuminated on receiving by a source antenna which produces over the test aperture an amplitude taper \( g(\theta, r) \), the measured pattern would be analogous to that of a transmitting antenna illuminated by a feed which produces an illumination of \( f(\theta, r)g(\theta, r) \) over the aperture. If \( g(\theta, r) \) is constant in amplitude and phase over the aperture, the measured pattern will be the same as the infinite-range pattern for the illuminations \( f(\theta, r) \). The greater \( g(\theta, r) \) deviates from constant, the greater will be the deviation of the measured pattern from the infinite-range pattern.

The quantitative effect of nearly constant functions \( g(\theta, r) \) cannot be determined, however, without assumption of \( f(\theta, r) \).

Figure 14.5 is a calculated infinite-range pattern of a circular aperture with a 10-decibel cosine distribution as tested with a source antenna which produces a circularly symmetric aperture taper to -0.5 decibel at the periphery. The taper is assumed to have a \( \sin x/x \) form. The calculated patterns show nearly identical close-in side lobes and a reduction in gain of about 0.15 decibel for the 0.5-decibel taper.

The decrease in measured gain caused by aperture taper is determined by the amount of taper and by the aperture-illumination function of the antenna under test. A criterion of 0.25 decibel is commonly employed for the limit of the
FIGURE 14.5 Calculated radiation patterns of a paraboloid with a 10-decibel aperture illumination taper; (1) measured with a 0.5-decibel \((\sin x)/x\) taper of the source antenna pattern, and (2) with no taper.
amplitude taper over the test aperture. For typical illumination functions, the approximate decrease in measured gain due to a 0.25-decibel taper would be 0.1 decibel. If a taper not exceeding 0.25 decibel is employed, the error in pattern structure from this source is likely to be less on many antenna test sites than that which would be produced by reflections if wider source beam-widths were employed. Such errors are discussed in paragraph 14.2.4.

If a source antenna is employed which is calculated to produce a taper of the field over the test aperture, it is essential that the transmitting antenna be directed such that the peak of its beam is centered on the antenna under test to prevent excessive and asymmetrical illumination taper with a resultant increase in the measuring error. It is important to note that error from symmetrical amplitude taper within the accepted criterion of 0.25 decibel does not produce a defocusing type of error but a small modification of the measured side-lobe levels and an error in measured gain.

For elevated ranges, assuming the reflections from the range surface to be negligible, the 0.25-decibel amplitude taper criterion may be related directly to the source directivity, the test aperture width and the range length. Consider a range geometry as sketched in Figure 14.6, where the range length R is much greater than the test aperture width D. The plane angle \( \alpha \) subtended at the source antenna by the aperture width D is

\[
\alpha = 2 \tan^{-1} \left( \frac{D}{2R} \right) = \frac{D}{R} \text{ radian } . \quad (14.20)
\]

Assuming a typical \( \sin \frac{x}{x} \) amplitude characteristic for the main lobe of the source antenna pattern, its 0.25-decibel beamwidth is

\[
\theta_{(0.25)} = \frac{0.37 \lambda}{d} \text{ radian } , \quad (14.21)
\]

where \( d \) is the source diameter. The 0.25-decibel restriction on amplitude taper is seen to correspond to the criterion

\[
\frac{\theta_{(0.25)}}{\alpha} = \frac{0.37 \lambda R}{D d} \leq 1 \quad , \quad (14.22)
\]

or equivalently

14-15
FIGURE 14.6 Elevated range geometry.

\[ d \leq \frac{0.37\lambda R}{D} \]  \hspace{1cm} (14.23)

For a range length \( R = KD^2/\lambda \), the source diameter would be restricted to

\[ d \leq 0.37KD \]  \hspace{1cm} (14.24)

Often the antenna under test is of such size or configuration that it must be mounted with its phase center considerably displaced from the center of rotation of the test positioner. In these cases, the aperture width \( D \) should be interpreted as the width of a section of a plane which subtends the maximum projected excursion of the test antenna.

For ground-reflection ranges, the amplitude taper of the illuminating field along a horizontal line through the test aperture, normal to the line of sight, will be determined almost entirely by the source directivity, the test
aperture width and the range length, just as for elevated ranges. The taper along a vertical line through the test aperture, however, is virtually independent of the directivity of the source antenna, and depends almost entirely on the height of the center of the test aperture above the range surface. This may be shown as follows.

Consider a test-range geometry as sketched in Figure 14.7. Let us set the phase reference at zero for the direct-path wave at the source antenna, and neglect the slight difference in loss due to dispersion along the path lengths for the direct and reflected waves. The phasor representing the amplitude of the field at points along a vertical through the test aperture can then be written as

\[ E = E_0 e^{-j\beta R_n} + kE_0 e^{-j(\phi + \beta R_n)} \]  

(14.25)
where the time dependence is suppressed. In this expression,

$$\beta = \frac{2\pi}{\lambda},$$

(14.25a)

$$R_0 = \left[R^2 + (h-h_t)^2\right]^{\frac{1}{2}},$$

(14.25b)

$$R_r = \left[R^2 + (h+h_t)^2\right]^{\frac{1}{2}},$$

(14.25c)

$E_0$ - the direct-path field amplitude, and

$$ke^{-j\phi} = \text{an effective reflection coefficient for the range surface.}$$

For typical ground-reflection geometries, $\phi$ will be very nearly equal to $\pi$ radians for a given polarization of the transmitted wave. Thus, (14.25) may be written

$$E = E_0 e^{-j\beta R_0} \left[1 - ke^{-j\beta (R_r - R_0)}\right].$$

(14.26)

The field is seen to have a periodic interference pattern as a function of $h$, with peaks at those values of $h$ for which

$$R_r - R_0 = (2N - 1)\frac{\lambda}{2}$$

(14.27)

where $N = 1, 2, 3, \ldots$.

For a test point at $h = h_r$, the least amplitude taper over a test aperture results when the first interference lobe is centered at the test point. Assuming $R$ to be large with respect to $h_r$, we can approximate $R_0$ and $R_r$ by

$$R_0 = R + \frac{(h_r - h_t)^2}{2R},$$

(14.28)

and

$$R_r = R + \frac{(h_r + h_t)^2}{2R},$$

(14.29)
so that for $N = 1$ we have

$$R_R - R_0 = \frac{2 h_t h_r}{R} = \frac{\lambda}{2} \quad \text{(14.30)}$$

Typically, the range length $R$ and test height $h_r$ are fixed, and (14.30) is applied in defining the proper source height at a given frequency:

$$h_t = \frac{\lambda R}{4 h_r} \quad \text{(14.31)}$$

Over the first interference lobe of the illuminating field, the normalized field magnitude is seen from (14.26) to vary with $h$ as

$$E_N(h) = \left[1 + k^2 - 2k \cos\beta(R_R - R_0)\right]^{1/2}$$

and for the conditions of (14.30) becomes

$$E_N(h) = \left[1 + k^2 - 2k \cos\left(\frac{\pi h}{2 h_r}\right)\right]^{1/2} \quad \text{(14.33)}$$

Ground-reflection ranges are designed such that $k$ is as near unity as is practical.* Setting $k$ equal to unity in (14.33) yields a useful approximation to the field amplitude about the test point, when the first interference lobe is peaked at that point:

$$E_N(h) = \sin\left(\frac{\pi h}{2 h_r}\right) \quad \text{(14.34)}$$

We can now apply an appropriate criterion on the amplitude taper of the illuminating field, and relate the criterion specifically to the height of the center of the test aperture. For an aperture of diameter $D$ in the vertical

*See section 12.2.4
dimension, the field magnitude at the extremes of the aperture as referenced to that at the center will be found by setting

\[ h = h_r - \frac{D}{2} \]  \hspace{1cm} (14.35)

in (14.34), for which

\[ E_n = \cos \left( \pi \frac{D}{4h_r} \right) \]  \hspace{1cm} (14.36)

If the typical 0.25-decibel restriction on amplitude taper is applied, we require

\[ \left( \frac{\pi D}{4h_r} \right) \leq \cos^{-1}(0.9716) \]  \hspace{1cm} (14.37)

or equivalently

\[ h_r \geq 3.3D \]  \hspace{1cm} (14.38)

In view of the simplifying assumptions employed in the above development, a suggested criterion for such ranges is \( h_r \geq 4D \).

The phase of the illuminating field within the first interference lobe is dependent upon the amplitude \( k \) of the effective complex field-reflection coefficient. However, for any range length greater than approximately \( 2D^2/\lambda \), the phase front differs negligibly from that of an apparent phase center located directly beneath the source antenna at a height above the range surface given by

\[ h_t' = \left( \frac{1 - k}{1 + k} \right) h_t \]  \hspace{1cm} (14.39)

The plane in which the phase and amplitude vary essentially symmetrically about the center of the test aperture is seen to be inclined from the vertical by an angle \( \alpha \) given by

\[ \alpha = \tan^{-1} \left[ (h_r - h_t')/R \right] \]  \hspace{1cm} (14.40)
Ground reflection ranges are in wide use as general-purpose test facilities at frequencies through Ku-band. With proper calibration techniques, such ranges can also be employed in precision boresighting and gain-standard applications.

14.2.4 Effects of Extraneous Reflections - - - Practical earth-based antenna test ranges are always subject to possible measurement error due to extraneous reflection and diffraction of the transmitted wavefront. The field at a point in the aperture under test is the phasor sum of the desired and extraneous radiation fields. The relative phases and amplitudes of the desired and extraneous field components vary, in general, with position over the aperture. Measured radiation patterns will depart significantly from the free-space patterns if the variation of the resulting spatially periodic field over the aperture is excessive.

In the elevated mode of operation, the primary source of extraneous energy is usually the range surface in the region near the range axis. Effective suppression of reflected energy must be provided on such facilities by a combination of line-of-sight clearance, source- and test-antenna directivity and sidelobe suppression, and in some cases range surface screening. Ground-reflection ranges, on the other hand, depend on near-specular reflection of the transmitted energy which strikes the range surface, so that the relative smoothness of the primary range surface at the operating frequency is of great importance.

The subject of reflections from lossy dielectric surfaces has been treated from several viewpoints in the literature. Depending on the theoretical model chosen for analysis, predicted reflection coefficients for microwave-frequency radiation

* See Chapter 8.

** The term "specular reflection" applies literally only in the case of geometrical optics where the wavelength approaches zero. Specular reflection at wavelengths in the microwave frequency region actually implies a particular predictable super-imposed effect of elemental field contributions reradiated from all illuminated regions of the surface. These elemental surface contributions are usually studied in terms of "zones of constant phase," or Fresnel zones, as discussed briefly in this section.
vary typically from 0.3 to unity as a function of surface roughness, electrical properties of the range surface, angle of incidence, polarization and frequency.

For the purposes of establishing range design criteria, the reflection of electromagnetic waves which illuminate typical range surfaces is conveniently studied in terms of "zones of constant phase," or Fresnel zones, on the surface. For source and test heights of \( h_s \) and \( h_t \), with a separation \( R \) between the bases of the antenna support structures, the shortest path between the source and test points via the range surface is

\[
R_{R0} = \left[ R^2 + (h_t + h_s)^2 \right]^{\frac{1}{2}} .
\]  

For vanishingly small wavelengths, this path would define the point of specular reflection (the center of the region of constant phase) at which the grazing angle \( \psi \) is given by

\[
\psi = \tan^{-1} \left[ (h_t + h_s)/R \right] .
\]

![Fresnel Zone Boundary](image.png)

FIGURE 14.8 Sketch for Fresnel zone boundary on a planar range surface.

If we establish a coordinate reference as shown in Figure 14.8, the path length via any other point \((o, y, z)\) on the surface is written

\[
R_s = \left[ h_t^2 + y^2 + z^2 \right]^{\frac{1}{2}} + \left[ h_s^2 + y^2 + (R - z)^2 \right]^{\frac{1}{2}} .
\]  

(14.43)
Since $R_R > R_{R0}$, the phase of a ray traveling along $R_R$ will lag that of the ray along $R_{R0}$ by $\Delta \phi$ radians, where

$$\Delta \phi = \frac{2\pi}{\lambda} (R_R - R_{R0}) \quad (14.44)$$

By definition, the locus of points $(0, y_1, z_1)$ for which

$$\Delta \phi_1 = N\pi \quad ; \quad N = 1, 2, 3, \ldots \quad (14.45)$$

or correspondingly

$$R_{R1} - R_{R0} = \frac{N\lambda}{2} \quad (14.46)$$

determines the outer boundary of the $N$th Fresnel zone. The inner boundary of the $N$th zone is given by

$$R_{R1} - R_{R0} = (N - 1) \frac{\lambda}{2} \quad (14.47)$$

The definitions are seen to be such that energy arriving at the test point from the outer bound of the Fresnel zone lags in phase by $\pi$ radians that energy arriving from the inner bound of the zone.

In range design problems, the pertinent parameters of a given Fresnel zone are the center, length and width of its outer bound. These parameters can be calculated from (14.46), which is rewritten in terms of the range dimensions and coordinates as

$$\left[ h_x^2 + y^2 + z^2 \right]^\frac{1}{2} + \left[ h_x^2 + y^2 + (R - z)^2 \right]^\frac{1}{2} - \left[ R^2 + (h_x + h_y)^2 \right]^\frac{1}{2} = \frac{N\lambda}{2} \quad (14.48)$$

This expression shows that the successive outer bounds of the Fresnel zones on a planar range surface describe a set of expanding ellipses whose major axes lie along the longitudinal range axis. The algebraic manipulations for solution of the desired parameters are simplified by definition of the following
functions:

\[ F_1 = \left( \frac{N\lambda}{2R} + \sec \psi \right) \]  

(14.49)

\[ F_2 = \frac{(h_r^2 - h_t^2)}{(F_1^2 - 1)R^2} \]  

(14.50)

\[ F_3 = \frac{(h_r^2 + h_t^2)}{(F_1^2 - 1)R^2} \]  

(14.51)

It can be shown that the following expressions result for the parameters of the outer-bound of the Nth zone, where the center is measured from the base of the source tower:

Center:

\[ C_N = R \left( \frac{1 - F_3}{2} \right) \]  

(14.52)

Length:

\[ L_N = RF_1 (1 + F_3^2 - ZF_3)^{\frac{1}{2}} \]  

(14.53)

Width:

\[ W_N = R \left[ (F_1^2 - 1)(1 + F_3^2 - ZF_3)^{\frac{1}{2}} \right] \]  

(14.54)

Since practical antenna range surfaces are not true planes, and the wavelengths at microwave frequencies do not satisfy the conditions of geometrical optics, the above expressions are inexact in practice. This inexactness does not, however, destroy the utility of these expressions in formulating basic range design parameters, as shown in subsequent developments in this section.

An approximate indication of the effects of a composite coherent extraneous signal on pattern-measurement accuracies is also useful. Assume that at the
terminals of the test antenna the direct-path field produces a voltage $e_0$, and the extraneous field produces a voltage $e_x$. For typical logarithmic pattern recordings, the error in measured relative level can be

$$\Delta L = 20 \log \left( \frac{e_R + e_x}{e_0} \right) \text{ decibels} \quad (14.55)$$

for $e_0 > e_x$. In the less likely event that $e_0 < e_x$, we write

$$\Delta L = 20 \log \left( \frac{e_x - e_0}{e_0} \right) \text{ decibels} \quad (14.56)$$

Graphs of equations (14.55) and (14.56) are given in Figure 14.9.

Suppression of Surface Reflections in the Elevated Mode - - - As a basic design principle, when practicable it is suggested that for any elevated range configuration the illumination of the range surface be restricted to the side-lobe region of the source-antenna radiation pattern. This general criterion then leads to quantitative criteria on controllable parameters such as range length, source and test heights, and source antenna size.

Consider again the elevated range geometry of Figure 14.6. The plane angle subtended at the source antenna by the test height $h_r$ is

$$\alpha_h = \tan^{-1} \left( \frac{h_r}{R} \right) \approx \frac{h_r}{R} \text{ radian} \quad (14.57)$$

for $R > h_r$. If the source-antenna pattern has a mainlobe width of $\theta_M$, our general surface-illumination criterion becomes

$$\theta_M \leq \frac{2h_r}{R} \quad (14.58)$$

Most elevated ranges are designed for tests of antennas for which the control of phase curvature in the illuminating wavefront establishes the minimum range length at

$$R = \frac{KD^2}{\lambda} \quad (14.59)$$
FIGURE 14.9 Possible error in measured relative pattern level due to coherent extraneous signals. Linear scales are employed for signal ratios of +20 to -30 decibels; the plus-or-minus errors are essentially equal for ratios of -25 decibels or less, as indicated in the logarithmic plot for ratios down to -75 decibels.
with \( K \) usually specified at 2 or greater. Source antennas for such ranges typically have mainlobe widths \( \theta_M \) of 2.3 to 2.7 times their 3-dB beamwidths; a useful nominal value based on wavelength and source antenna diameter is

\[
\theta_M = \frac{3\lambda}{d} \quad (14.60)
\]

Substitution of (14.59) and (14.60) into (14.58) yields the following rule-of-thumb:

\[
h_r d \geq 1.5 K D^2 \quad (14.61)
\]

If we simultaneously impose the previously developed 0.25-decibel amplitude-taper criterion of (14.24),

\[
d \leq 0.37 K D \quad (14.24)
\]

we have the relation

\[
0.37 h_r K D \geq h_r d \geq 1.5 K D^2 \quad (14.62)
\]

between the parameters. It is seen, for example, that a test height

\[
h_r \geq 4 D \quad (14.63)
\]

would be required for simultaneous satisfaction of the amplitude-taper, and surface-illumination criteria.

Frequently, the practicality and economics of the test situation dictate some compromise of the combined criteria of (14.62). From the opposite viewpoint, some combinations of test antenna characteristics and specified measurement accuracies demand criteria more stringent than those developed here.

In any case, the test procedures should be outlined with care so that full advantage is taken of the possible sidelobe suppression of the test antenna. For example, if the nominal sidelobe levels of the source and test antennas are -25 decibels, and the range surface produces a 10-decibel attenuation of
the reflected wave, the extraneous signal level will be approximately 60 decibels below the direct-path signal level. Reference to Figure 14.9 shows that measurement of the peak of the beam will be in error by less than 0.01 decibel due to this source. However, if the peak of the pattern under test is directed toward the range surface and a -25-decibel sidelobe is directed toward the source antenna, the relative level of the extraneous signal will be only some 10 decibels below the level of the direct-path signal, and the measurement error can approach ±3 decibels.

Since many reflecting situations can be worse than the above, it is advisable not to direct the peak of the beam of the antenna under test toward the ground in exploring the side-lobe region above the main lobe. A polarization positioner should be employed (see Chapter 5) to rotate the antenna under test through 180 degrees about a direction through or near the peak of the beam if concern exists that interference can occur from this source.

Diffraction fences on the range surface can reduce the level of extraneous reflections beyond that achievable through satisfaction of the basic design criteria. Such fences can be of simple wooden frame construction, covered with ordinary conducting screen having a mesh of approximately $\lambda/10$. The desired effect of such fences is to redirect away from the test aperture a portion of the reflected energy which would, in the absence of the fences, be incident from points within the region of the range surface including those Fresnel zones which are illuminated by relatively high-level regions of the source pattern.

Since the fences necessarily intercept a portion of the transmitted wavefront, they simultaneously introduce perturbations in the incident field due to diffraction effects along their edges. A practical fence design is thus a compromise between suppression of reflections and residual diffraction. Some reduction in edge diffraction can be achieved by installation of tuned slots or serrations along the edge. Either approach requires structures which extend several wavelengths above the edge; the former is perhaps more effective at a single frequency, but is accordingly frequency sensitive. The most obvious approach to reduction of diffraction interference is to maintain low-level illumination of the fence edge, if possible. This approach sometimes leads to the use of multiple-fence configurations using relatively low fence heights, with proper
tilting and skewing of the individual fences to suppress multiple-bounce effects. For most range geometries, however, little additional suppression is realized for more than one to three fences.

Based on experimental data collected on antenna ranges whose surfaces vary from randomly rough to specularly smooth, a suggested criterion for fence designs is to screen from direct view of both the source antenna and the test antenna the first twenty or more Fresnel zones on the range surface, while maintaining the fence-edge illumination at as low a relative level as is practical.

For purposes of establishing specific fence designs and error budgets, the Cornu spiral derived from classical straight-edge diffraction theory can be applied to the diffraction fence problem to obtain an estimate of residual diffraction effects. The typical range geometry for which fence installations are under consideration is as sketched in Figure 14.10. It is assumed that

\[ h_r = h_t = h > h_f \]

and

\[ R \gg h. \]

FIGURE 14.10 Relation of diffraction parameters to range geometry for a single diffraction fence at the center of the range.
The Cornu spiral is a plot of \( C(s) \) versus \( S(s) \), where \( C \) and \( S \) are the Fresnel integrals, and where the spiral factor, \( s \), is arc length on the spiral. The variable \( s \) is usually taken as positive in the illumination region of a source, and is normalized such that

\[
s = \sqrt{2} \text{ for } d^2 = \left( x + \frac{\lambda}{2} \right)^2 - x^2 \pm \lambda x
\]

where \( x \) and \( d \) are as defined in Figure (14.10). Normalized field magnitudes are computed as the amplitude of the vector from the asymptote at \( s = -\infty \) to the point on the spiral corresponding to the field point.

For the present assumptions, the spiral factor \( s_1 \) for the point \( P_1 \) in the shadow region of the image antenna is

\[
s_1 = -\left( \frac{2}{\lambda x_1} \right)^{\frac{1}{2}} d_1 \quad . \tag{14.64}
\]

and the spiral factor \( s_2 \) for the point \( P_2 \) in the illumination region of the source antenna is

\[
s_2 = \left( \frac{2}{\lambda x_2} \right)^{\frac{1}{2}} d_2 \quad . \tag{14.65}
\]

The receiving point is in the shadow region of the fence for radiation from the image antenna, and the corresponding Cornu spiral factor is given approximately by

\[
s_1 = -\left( \frac{16}{\lambda R} \right)^{\frac{1}{2}} h_r \quad . \tag{14.64a}
\]

The receiving point is in the illumination region of the source antenna, with a Cornu spiral factor of approximately

\[
s_2 = \left( \frac{16}{\lambda R} \right)^{\frac{1}{2}} (h - h_r) \quad . \tag{14.65a}
\]
The probable maximum bias and ripple components due to diffraction of the image-and source-antenna wavefronts, respectively, can be estimated by entering the graphs of Figure 14. 11 or 14. 12 at the values computed from (14. 64a) and (14. 65a).

Figure 14. 11 is a plot of the diffraction-field amplitude versus the spiral factor for a point-source radiator, where the amplitudes are normalized to that of the unobstructed free-space wave. A graph of the envelope of the normalized illumination-region diffraction-field amplitude about the free-space asymptote is given in Figure 14. 12 for s = 1 to 25.

For example, consider an X-band test problem in which

\[ D = 7.5 \text{ feet} \]
\[ h = h_r - h_t = 30 \text{ feet} = 4D \]
\[ \lambda = 0.1 \text{ foot} \]
\[ R = 1000 \text{ feet} \leq 2 \frac{D^2}{\lambda}, \text{ and} \]
\[ h_f = 6 \text{ feet} = h/5. \]

The spiral factors are \[ s_1 \approx -2.4 \] and \[ s_2 \approx 9.6. \] Entering Figure 14. 11 at \( s = -2.4 \), we find that a point-source image antenna of amplitude equal to the source-antenna amplitude would contribute a field component of approximately 0.1 times the direct-path field which would exist in the absence of the reflecting range surface. This is a worst-case estimate of the contribution of the image antenna, since in practice the image antenna for an elevated range would be attenuated in amplitude both by the directivity of the source antenna and the non-specular nature of the range surface.

To estimate the periodic nature of the incident field in the illumination region of the source antenna, we enter Figure 14. 12 at \( s = 9.6 \). The maximum peak-to-peak variation of the field which would result from diffraction of a direct-path wavefront radiated by a point-source antenna is seen to be approximately 0.05 times the unobstructed field amplitude. For typical logarithmic field
FIGURE 14.11 Normalized field amplitude for diffraction over a straight edge (point-source illumination).
FIGURE 14.12 Envelope and free-space asymptote for the straight-edge diffraction field.
amplitude plots, such a variation could produce a ripple component of about ± 0.2 decibel. This value is also a worst-case estimate. Since the fence edge is preferably at a level of illumination significantly below the direct-path level, the successive maxima and minima of the perturbed field will approach the free-space asymptote somewhat more rapidly than indicated in Figure 14.12. Also, the asymptote will exhibit some concave curvature for relatively small displacements from the shadow-illumination boundary, so that both the relative amplitude and the peak-to-peak variations of the resultant field will be less than indicated by the theoretical curves.

Longitudinal ramps along a range axis can provide moderate suppression of surface reflections. An approximate indication of the relative suppression of a ramp whose apex extends the length of the range axis, with an included angle at the apex of 2α, can also be obtained from a basic diffraction model. 14

Consider a ramped-range geometry in which

\[ h_r = h_t = h << R, \]

as sketched in Figure 14.13. The imaginary extension of each ramp face beyond the apex is considered to form a virtual opaque screen between a corresponding image of the source antenna and the receiving antenna. Each image antenna is a distance \( h \sin \alpha \) below its virtual screen, along the normal through the source antenna.

For oblique incidence of a wave on the edge of a half-plane, the emerging wave is conical. The corresponding diffracted rays make the same angle with the edge as do the incident rays. 15 For the assumed geometry, the ray diffracted at the center of the apex thus bends through an angle \( \psi_1 \), to reach the test point, where

\[ \sin^2 \left( \frac{\psi_1}{2} \right) = \frac{4h^2 \cos^2 (\alpha)}{R^2 + 4h^2}, \]

or approximately

\[ \psi_1 \approx \frac{4h \cos (\alpha)}{R}. \]  \hspace{1cm} (14.66)

The corresponding Cornu spiral factor is approximately

\[ \sigma_1 \approx - \psi_1 \left( \frac{R}{2\lambda} \right)^{1/2}, \] \hspace{1cm} (14.67)
which is seen to be

\[ S_1 = \cos(\alpha) \left[ \frac{8h^2}{\lambda R} \right]^{\frac{1}{2}} \]  \hspace{1cm} (14.68)

**FIGURE 14.13 Elevated range with longitudinal ramp.**

In keeping with previously defined criteria, let

\[ h = K_1 D, \quad R = K_2 D^2 / \lambda, \]  \hspace{1cm} (14.69)

where \( D \) is the diameter of the test aperture.

We then have

\[ s_1 = \cos(\alpha) \left[ \frac{8K_1^2}{K_2} \right]^{\frac{1}{2}}. \]  \hspace{1cm} (14.70)
From Figure 14.11, it is seen that a value of \( s_1 = -4.5 \) is required to produce a relative field suppression factor of 0.05 for a single image. Since the apex of the ramp is a line of symmetry for a vertical through the test point, the shadow-zone contributions from each side of the apex add in phase along the vertical. Thus, \( s_1 = -4.5 \) corresponds to a total relative suppression of approximately 0.1, or -20 decibels.

The previously described diffraction-fence example showed that a single fence of height \( h/5 \) at the center of a range would produce approximately 20 decibels of suppression, when the parameters \( K_1 \) and \( K_2 \) were approximately 4 and 2, respectively. It is of interest to compare the ramped configuration which predicts the same suppression for the same parameters \( K_1 = 4, K_2 = 2 \). In this case we require

\[
s_1 = -4.5 = -\cos(\alpha) \left[ \frac{8K_1^2}{K_2} \right]^{\frac{1}{2}} = -8 \cos(\alpha)
\]

or equivalently \( \alpha = 56 \) degrees.

To accomplish suppressions of this order, the ramp faces should subtend the first twenty or more Fresnel zones which would exist on a horizontal plane at the elevation of the apex. For the present assumptions of equal tower heights of \( 4D \) and a range length of \( 2D^2/\lambda \), the width of the 20th Fresnel zone would be very nearly

\[
W_{20} \leq (N\lambda R)^{\frac{1}{2}} = [(20\lambda)(2D^2/\lambda)]^{\frac{1}{2}}
\]

or

\[
W_{20} \approx 6.3 D
\]

Each ramp face should thus have a width of about

\[
\frac{W_{20}}{2 \sin(56^\circ)} \approx 3.8D \pm h
\]

and the ramp height would be about

\[
\frac{W_{20}}{2 \tan(56^\circ)} \approx 2.2D \pm \frac{h}{2}
\]

Lower-level interference in both the horizontal and vertical planes of the incident field will result from diffraction effects at the interfaces of the range surface with the ramp. Further, the phasing of the higher-level ramp diffraction with the direct-path signal, and the asymmetry of the ramp as viewed from points along a horizontal line through the test aperture, will result in periodic
interference in both the vertical and the horizontal plane, whereas the simpler transverse-fence configuration is essentially insensitive to practical transverse displacements of the test point. In light of the above results, and the significantly greater ground-preparation and maintenance costs of equivalent ramps, it appears preferable to employ the diffraction-fence approach to obtain additional surface-reflection suppression in the elevated mode.

Control of Surface Reflections in the Ground-Reflection Mode - - - Reflections from a range surface may be classified in two general categories, diffuse scattering and specular reflection. The transition from conditions causing diffuse scattering to those producing essentially specular reflection is gradual. Accordingly, criteria for specifying the boundaries and tolerances for the reflecting surfaces and other controlled areas in the design of ground-reflection ranges are largely empirical.

Many of the recent theoretical treatments of reflection of electromagnetic waves make extensive use of diffracted rays in combination with the usual rays of geometrical optics; Luneburg and others have shown that in expressions relating reflected and incident fields of large radii of curvature, the reflection process is essentially the same as for a local plane wave incident upon a plane interface. The Fresnel reflection and transmission coefficients thus apply locally over large surfaces, and satisfaction of the Rayleigh criterion for smoothness over a sufficiently large area ensures an efficient use of the surface as a reflector of microwave energy. The derivation of the criterion is illustrated by Figure 14.

Consider two rays, $r_a$ and $r_b$, from a plane wavefront $F_1$, which approach a surface on which a pedestal of height $\Delta h$ is located. Let $r_a$ strike the top of the pedestal and let $r_b$ strike the plane surface near the pedestal. The two rays are reflected such that the grazing angle $\dot{\psi}$ for the reflected rays is equal to the grazing angle for the incident rays, and proceed to form part of a second wavefront $F_2$. The Rayleigh criterion is based on the geometrical difference in path length traversed by $r_a$ and $r_b$ in travelling from $F_1$ to $F_2$. Let the distance traversed by $r_a$ be $d_1 + d_2$. From inspection it is seen that $r_b$ will travel the distance $d_1 + d_2 - c + c + 2 \Delta h \sin \dot{\psi}$, and that the difference in path length
is given by

$$\Delta d = 2\Delta h \sin \psi \quad .$$  \hspace{1cm} (14.71)

The corresponding phase difference of the rays at a distant receive point is

$$\Delta \phi = \frac{2\pi}{\lambda} \Delta d = \frac{4\pi}{\lambda} \Delta h \sin \psi \quad .$$  \hspace{1cm} (14.72)

Restriction of \( \Delta \phi \) to \( \pi/m \) radians and defining a smoothness factor, \( M = 4m \), yields the following form of the Rayleigh criterion:

$$\Delta h \leq \frac{\lambda}{M \sin \psi} \quad .$$  \hspace{1cm} (14.73)

Various authors have suggested smoothness factors ranging from 8 to 32 or greater as defining a "smooth" surface, corresponding to phase differences from \( \pi/2 \) to \( \pi/8 \) or less. Empirically, this range of values for \( M \) is considered to represent surfaces which range from tolerable to very smooth.

For purposes of establishing range design criteria, the grazing angle is usually computed only at the "point of specular reflection," where

$$\tan \psi_o = \frac{h_r + h_t}{R} \quad .$$  \hspace{1cm} (14.74)

Most range geometries are such that \( R >> (h_r + h_t) \), for which

$$\sin \psi_o \approx \tan \psi_o \quad .$$  \hspace{1cm} (14.75)

Also, for fixed \( R \) and \( h_r \), the \( \sin \psi \) term is relatively insensitive to frequency over two or more octaves in frequency range; thus the controlling factor in (14.73) is typically the wavelength.

Once one has chosen a practicable value for \( \Delta h \) based on the highest frequency of interest, he must specify the boundaries of the area over which this surface tolerance is to be maintained. Based largely on experimental results, but

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in agreement with current theory, the most stringent surface-roughness criterion should be met over a nominally rectangular region of a width equal to that of the outer-bound ellipses of the first 20 to 30 Fresnel zones on the range surface, and extending the full length of the range.

This primary surface should fair into a secondary region for which the surface tolerance is nominally two or three times that of the primary surface, and which in any case is free of steep local slopes. Again based on experimental results, the recommended minimum width of a rectangle defining the secondary region boundary is twice that of the primary zone. The secondary surface should also start at the source site, and should extend considerably beyond the receive site, as far as 0.2 to 0.25 times the range length if

*See equations (14.49) — (14.54).
possible, particularly for removal of steep local slopes.

Finally, a third cleared area should be maintained under the control of the range personnel, and kept free of all major reflecting and diffracting obstacles such as buildings, trees, brush, fence rows, power or telephone lines, vehicular traffic or parking, sizeable surface discontinuities, etc. If practicable, between the source and receive sites this latter region should extend from the corners of the secondary region at the source site to points defined by the width between first nulls of the horizontal-plane source-antenna directivity pattern at the receive site. The cleared region should then continue as a rectangular area beyond the receive site to a distance approximately equal to one-half of the range length.

Figure 14.15 summarizes the criteria developed above.

With reference to (14.74), note that for a fixed frequency and a fixed height of the center of the test aperture, and assuming operation in the first interference lobe of the illuminating field \( h_t = \lambda R / 4h_r \), we can write the derivative

\[
\frac{d}{dR} (\tan \psi) \approx \frac{d}{dR} (\psi) \approx \frac{d}{dR} \left( \frac{h_r}{R} + \frac{\lambda}{4h_r} \right) = \frac{-h_r}{R^2}
\]

(14.76)

for \( R >> (h_r + h_t) \). It is seen that the grazing angle, hence the grading tolerance, can be reduced only slightly by increasing the range length. It can also be shown that the area within the primary reflection region as defined above is approximately proportional to the three-halves power of the range length, so that any apparent savings in initial grading tolerance and subsequent surface maintenance due to increased range length can be more than offset by the increased range area and volume of earthwork required prior to surface finishing. Many range users have found it economical to install processed surfaces over the primary region, such as concrete, asphalt, compacted crushed aggregate, etc. In such cases the electrical properties of the selected material should be reasonably matched to the surrounding soil, or the boundaries of the primary region should be formed into serrations in order to suppress diffraction effects at the interface.
FIGURE 14.15 Possible ground-reflection range layout.
14.3 ANTENNA RANGE EVALUATIONS

An experimental evaluation of an antenna test facility is usually necessary to ensure that the facility is capable of performing the desired measurements. The relative merit of an evaluation of the electromagnetic environment of a test facility must be considered in view of (1) the consequences dependent upon the tests to be performed on the range, (2) the need for documentation of the measurement accuracies, and (3) the desired level of confidence in measurement results.

Sources of error on antenna test ranges include diffraction and reflection from mounting structures, extraneous reflections from the range surface and scattering from obstacles on or near the range surface, radio frequency interference, and many others. The greater the accuracy required, the more important an evaluation program becomes, for even small perturbations in the total field can seriously impede high accuracy measurements.

For the purposes of an evaluation it is convenient to consider the total electromagnetic field at an aperture under test to be created by the superposition of (1) the incident field, which includes the direct-path wave from the source antenna and all secondary waves produced by reflection or diffraction of the radiated energy in the region generally to the front of the test aperture, and (2) the wide-angle field, which includes all radiated, reflected, or diffracted energy which arrives at the test aperture from extremely wide angles or from the rear.

This section discusses several experimental techniques employed in antenna range evaluations, presents representative experimental data, and correlates the data with basic theoretical models of interference phenomena. The techniques are discussed in terms of outdoor antenna test facilities, but are equally valuable in evaluations of other configurations such as anechoic chambers, compact ranges and reflectivity ranges.

14.3.1 Incident Field Assessments - Assessment of the incident electromagnetic field as defined above may be accomplished with field probe measurements over a plane normal to the line of sight and adjacent to the expected location of the test antenna. We shall define this plane as the test aperture. These measurements yield data indicative of both the nominal level and the
general location of sources of extraneous energy in the region to the front of
the test aperture.

Typically, the measurements are made with a field probe mechanism similar
to the one shown schematically in Figure 14.16. The probe mechanism is
basically an I-beam support along which a carriage-mounted probe antenna
system is driven by remote control to measure the incident field strength as
a function of position. A synchro signal, indicating the position of the probe
antenna, allows continuous automatic plotting of the received field amplitude
versus aperture position. The data are usually presented as decibel plots,
using high resolution potentiometers in recorders or other sensitive detectors
to expand the ordinate (relative power) scale.

In order to discriminate against reflections arriving from the field probe
structure and its mounting fixture, moderately directive probe antennas are
employed in combination with an absorbing baffle mounted behind the antenna.
Tests with such a configuration are deliberately limited to assessments of
the field incident on the test aperture from the forward direction. Commonly
used types of probe antennas include periodic structures and pyramidal horns
with 3-decibel beamwidths in the E and H planes which satisfy the criterion

\[ \theta_\theta \approx 2 \tan^{-1} \left( \frac{4 h_{z}}{R} \right) \]

where \( h_z \) and \( R \) are test height and range length, respectively. This criterion
ensures that the major portion of the range surface is seen within the half-
power region of the probe pattern.

Few antenna test facilities are intended to be operated continually at a single
polarization of the transmit and receive antennas. In anticipation of this fact,
evaluation programs normally include aperture field probe measurements at
both horizontal and vertical polarization of the transmit and probe antennas.
This procedure reflects the fact that the level of extraneous energy reaching
the test aperture may be significantly different for these orthogonal polarizations.

Before considering specific applications of aperture field probe measurements,
FIGURE 14.16 Schematic diagram of field probe mechanism.
it is necessary to establish an approximate quantitative correlation between the field probe data and the theory of extraneous signal interference. The mechanism by which an extraneous signal interferes with the desired direct-path signal and distorts the incident field is demonstrated in the following idealized example. Consider the case of a direct-path plane wave of amplitude $E_0$ which is normally incident on a test aperture as shown in Figure 14.17(a). Let an extraneous plane wave of amplitude $E_r$ enter the aperture at an angle $\theta$ from the normal. The radial line defined by the unit vector $\vec{u}$ is the intersection of the plane containing the directions of propagation of the direct-path and extraneous signals and the plane containing the aperture. The line having unit vector $\vec{v}$ is at some general angle $\alpha$ from $\vec{u}$, in the plane of the test aperture. At any given time, $t$, the phase of the direct wave is constant across the aperture and the magnitude of the direct-path field may be expressed in phasor notation as

$$E_0 = E_0 e^{j(\phi + \omega t)}.$$  (14.78)

The phase of the postulated extraneous plane wave will vary across the aperture so that the magnitude of the extraneous field along the line $\vec{v}$ or any line parallel to $\vec{v}$ is given by

$$E_r = E_r e^{j(\phi' + \omega t + \frac{2\pi}{\lambda} v \sin \theta \cos \alpha)}.$$  (14.79)

In (14.78) and (14.79), $\phi$ and $\phi'$ are constants, $\lambda$ is the wavelength, and $v$ is a distance measured along the radial line $\vec{v}$ or along some other line parallel to $\vec{v}$. The magnitude of the total field measured as a function of distance along $\vec{v}$ will be proportional to

$$E(v) = E_0 + E_r \sin \left( \frac{2\pi}{\lambda} v \sin \theta \cos \alpha \right).$$  (14.80)

---

*The discussion here assumes a separation, $R$, between source and receiving antenna equal to or greater than $2D^2/\lambda$, where $D$ is the maximum dimension of the receiving aperture. It is further assumed that the ratio $D/R$ is small in comparison with the half-power beamwidth of the source antenna's pattern, so that plane wave approximations are meaningful.*
FIGURE 14.17 Relation of range coordinates and aperture field components for a single extraneous signal.
The distance between successive peaks of the resultant sinusoidal field is

\[ P = \frac{\lambda}{\sin \theta \cos \alpha} \]  \hspace{1cm} (14.81)

This spatial period or pitch has a minimum value of

\[ P_{\text{min}} = \frac{\lambda}{\sin \theta} \]  \hspace{1cm} (14.82)

along the direction \( \hat{u} \), as shown in Figure 14.17(b).

While the spatial periods of the measured field fluctuations are functions of the position of the radial line in the aperture for which a recording of the field is made, the peak-to-peak amplitude of these fluctuations is a constant proportional to

\[ (E_0 + E_R) - (E_0 - E_R) = 2E_R \]  \hspace{1cm} (14.83)

For typical logarithmic patterns, the relation of \( E_R \) and \( E_0 \) would be

\[ \frac{E_R}{E_0} (\text{dB}) = 20 \log \left[ \frac{-1 + \text{antilog} (\sigma/20)}{1 + \text{antilog} (\sigma/20)} \right] \]  \hspace{1cm} (14.84)

where \( \sigma \) is the difference in decibels between maxima and minima of the measured pattern. A plot of \( E_R/E_0 \) versus \( \sigma \) is given in Figure 14.18.

The preceding example demonstrates the manner in which extraneous signals would distort an otherwise planar wavefront. In a more realistic case, neither the direct wave nor the extraneous wave would be strictly planar, and there would be many sources of extraneous signal which could contribute to the total aperture field. However, the simple model presented above is quite useful in interpreting field probe data for the typical case where one extraneous signal is somewhat stronger than others which are present.

**Probe Measurements on Elevated Ranges**

The field probe technique allows rapid and systematic experimental evaluation of those elevated-range...
FIGURE 14.18 Magnitude of cyclic perturbations in the incident field versus the ratio of reflected signal to direct signal strengths for an aperture-probe cut in the plane of $E_a$ and $E_b$. 
parameters which affect the level of suppression of the extraneous energy coming from the range surface. These parameters typically include sources of extraneous energy on the range surface, the alignment and orientation of the source antenna, and the location and configuration of diffraction fences.

The coherent interference of an extraneous signal with the direct-path signal will produce a well defined interference pattern at the test aperture if the level of the extraneous signal relative to the direct-path signal is sufficiently large. Probing the aperture field along a few radial lines usually provides sufficient data to establish the relative levels of the major contributors of extraneous signals distorting the incident field and the angles of the sources from the line of sight.

If the level of extraneous signal is excessive, some remedial action such as the use of absorbing material or the complete screening or removal of some obstacle will be required. The field probe can be used during this procedure to determine how successful the various steps of remedial measures have been in suppressing the extraneous signal.

Diffraction fences are often placed on the surface of an elevated antenna range to screen certain areas from direct illumination by the transmitted wavefront. This in effect should decrease the level of reradiated energy reaching the test aperture. The fine tuning of the location and configuration of the diffraction fences in order to enhance the suppression of reflected energy can be accomplished through the use of the field probe mechanism. Having placed the fences on the range surface, field probe cuts are made along vertical and horizontal lines in the aperture to establish the initial interference pattern resulting from the summation of the direct path signal and residual extraneous signals from the range surface and/or the fence edges. Slight changes in the orientation or configuration of the diffraction fences may then be made, each time taking field probe cuts and comparing the result against the previous data for interpretation of the effect that the modification had on the resultant field received at the test aperture. This process is continued methodically until a satisfactory placement of the fences is accomplished. Typical modifications to basic

* See Section 14.2.4.
diffraction fence placements include movement along the range axis, skewing off the range axis, tilting the fences from the normal to the range surface, and installation of serrations along the top of the fences.

Figure 14.19 demonstrates the interference of direct-path and extraneous energy from the surface of a ground reflection range which was being tested for operation as an elevated range. These measurements were made on the NASA-MILA Boresight Test Range at Kennedy Spacecraft Center. The range is 1000 feet long and the height of the center of the test aperture above the range surface is 30 feet. In Figure 14.19a the range surface is unscreened while in Figure 14.19b it has been screened by the use of two diffraction fences. The measurements were made at a frequency of 10 GHz, and the diameter of the transmitting antenna was 4 feet. The fences were 12 feet and 8 feet in height and were located at the mid-point of the range and 250 feet from the source tower, respectively. In subsequent measurements made with only the center fence, very little difference could be seen in the extraneous energy level compared with that of Figure 14.19b. The interference pattern of Figure 14.19a represents a single source of reflection because the measurements were made over a precisely graded ground reflection antenna test range. When the measurements are made under conditions which are not as nearly ideal, the interference pattern will not usually have this classic character and both the pitch and amplitude of the ripple may vary with position in the test aperture.

Figure 14.20 shows the effect that diffraction fences can have on the received field at the test aperture over an ungraded range surface. The data in Figure 14.20 represent vertical field probe cuts made on an X-band range at horizontal polarization. The lower cut was made with no fences on the range surface. The upper cut was made with a single diffraction fence in place near the center of the range. The slow pitch ripple in the lower curve, corresponding to energy from the range surface, represents reflection levels as large as -25 decibels with respect to the direct path signal toward the bottom end of the cut, but more typically about -30 decibels. These reflections were suppressed to about -45 decibels by the diffraction fence as can be seen in the
FIGURE 14.19 Interference from the surface of a ground reflection range being tested for operation as an elevated range. Figure (a) is with no fences placed on the range surface. Figure (b) is with two diffraction fences screening the specular region of the surface.
FIGURE 14.20  Vertical field probe cuts made on an X-band range at horizontal polarization. The lower cut was made with no fences on the range surface. The upper cut was made with a single fence in place near the range center.
upper curve. The higher pitched interference at the bottom end of the cut resulted from diffraction from the edge of the receive tower. The diffracted energy was at a relative level of about -38 decibels with respect to the direct-path radiation.

Aperture field probe techniques are also useful in performing the adjustments necessary to correct any misalignment of the source antenna. If the azimuth or elevation pointing angle (squint) of the source antenna positioner is not properly aligned, the principal beam of the source antenna's radiation pattern will not be centered on the test aperture. Excessive asymmetrical amplitude taper across the test aperture may result. It is also desirable to align the beam axis of the source antenna with the roll axis of its positioner to allow for changes in polarization without further adjustment of the pointing angle. This is not only desirable, but necessary if the source antenna is to be continuously rotated to make polarization measurements. The procedure for alignment of the source antenna in both the azimuth and elevation planes is the same, and in the following paragraph only the adjustments in the azimuth plane will be discussed.

To establish the azimuth squint of the source antenna necessary for the peak of the radiation pattern in the azimuth plane to be centered on the test aperture, horizontal field probe cuts are taken across the test aperture. In utilizing the field probe data to properly squint the antenna, it is convenient to choose equal power points on the patterns as a reference and to adjust the antenna such that the geometric mean of the horizontal location of these two points occurs at the point on the pattern corresponding to the center of the aperture. Having adjusted the squint of the source antenna, the source antenna is then rotated through 180 degrees and another horizontal field probe cut is taken. If the resulting pattern is coincident with the preceding, the beam axis is coincident with the positioner roll axis. Should the patterns be displaced, consideration of the two field probe patterns will indicate the azimuth displacement of the beam axis with respect to the positioner roll axis. This may be corrected by an angular displacement of the source antenna with respect to the positioner mounting surface or, for optical antennas, by transverse alignment of the feed system. The procedure is continued until both the azimuth squint and alignment of the source antenna are adjusted such
that the main beam of the source antenna is symmetric about its roll axis and the roll axis is coincident with the line of sight to the center of the test aperture.

An evaluation program for an elevated antenna range should include investigation of the polarization characteristics of the incident field received at the test aperture. For many test problems, the polarization characteristics of the incident field can be adequately assessed simply by rotating a linearly polarized sampling antenna in the plane of the aperture while synchronously recording the received field. In most cases the same antenna used for the field probe measurements can be used as the sampling antenna. The field probe mechanism can be equipped with a probe rotator or polarization positioner to facilitate the measurements. It is important that any adjustments to the alignment of the source antenna be completed before making the polarization measurements.

The recorded pattern obtained from rotating a linearly polarized sampling antenna in the plane of the aperture is called the polarization pattern. From this polarization pattern, the axial ratio and tilt angle of the polarization ellipse associated with the incident field can be determined. The sense of rotation of the polarization is not indicated by this method, but the intended rotation is likely to be known from knowledge about the source antenna. The polarization of the incident field is normally assessed at various points within the test aperture in order to establish an indication of the off-axis polarization characteristics. If the source antenna is equipped with a capability for fine adjustments to its polarization, the data obtained from the polarization patterns allow adjustments of the source antenna to obtain a desired polarization.

Figure 14.21 is a typical polarization pattern (rectangular display) that was taken on an X-band range at assumed horizontal polarization of the transmitter. For truely linearly polarized transmit and sampling antennas, the null depths in the figure would be infinitely deep and the axial ratio would be infinite. However, practical antennas do not give true linear polarization; the approximate axial ratio in this case is 37.5 decibels (40-dB potentiometer in the recorder). The tilt angle is seen to be 0 degrees (horizontal) in Figure 14.21).

Figure 14.21 is a typical polarization pattern (rectangular display) that was taken on an X-band range at assumed horizontal polarization of the transmitter. For truely linearly polarized transmit and sampling antennas, the null depths in the figure would be infinitely deep and the axial ratio would be infinite. However, practical antennas do not give true linear polarization; the approximate axial ratio in this case is 37.5 decibels (40-dB potentiometer in the recorder). The tilt angle is seen to be 0 degrees (horizontal) in Figure 14.21.

Probe Measurements on Ground Reflection Ranges - - - The design considerations for a ground-reflection antenna range are directed toward the controlled

*See Chapter 10.
\[ \alpha = \text{POLARIZATION ROTATION ANGLE (360°)} \]

**FIGURE 14.21** Polarization pattern of the incident field at the center of the test aperture on an X-band elevated range. The source antenna was horizontally polarized and the sampling antenna was rotated about the line of sight.
use of energy reflected from the range surface, in contrast to the suppression of such energy for an elevated antenna range. The source antennas utilized on ground-reflection ranges are oriented such that an essentially smooth range surface is illuminated with energy at approximately the same level as the direct-path signal. In order to produce a broad interference pattern centered on the test aperture, the source antenna is operated at a particular height for which the direct-path and specularly reflected waves arrive at the test aperture in phase. From the previous discussion on design criteria for ground-reflection ranges, equation (14.31) gives the approximate source height setting necessary to align the peak of the first interference lobe of the illuminating field with the center of the test aperture for a given range length and receiver height at a given frequency: \( h_t = \frac{\lambda R}{4h_r} \) (14.31).

This expression assumes that the effective complex reflection coefficient of the range surface has an argument of exactly \(-\pi\), which for practical antenna ranges is not always a reasonable assumption. In practice, experimental optimization of the incident field usually requires a source height setting somewhat different from the theoretical value, due to effective phase shifts slightly greater or less than 180° at the range surface.

Positioning the source antenna to create an optimum incident field across the test aperture is of primary importance in an evaluation of a ground-reflection range. The actual source height necessary to produce the optimum interference pattern at the test aperture is found by varying the source antenna about the height given by (14.31) and monitoring the incident-field configuration with field-probe cuts over the test aperture. Symmetry about a maximum received signal at the center of the aperture indicates the proper height of the source antenna. The optimum source height setting will be different for different polarizations of the source antenna since the complex reflection coefficient of the range surface is dependent on polarization. In general, the optimum source height setting for horizontal polarization is lower than that for vertical polarization, since the corresponding phase retardations are greater than and less than 180°, respectively.

It has also been found that for most cases small changes in the elevation squint...
angle of the source antenna do not cause the interference pattern to vary appreciably. 8

Just as in the case of an elevated antenna range the symmetry of the incident field and the alignment, or focusing of the feed system, of the source antenna should be considered for a ground-reflection range. Since small changes in the source-antenna beam pointing direction in elevation have little effect on the interference pattern produced at the test aperture, accurate alignment of the source positioner and feed system can only be accomplished with horizontal field probe measurements. The same general procedure as was discussed for elevated ranges is applicable for making adjustments to the alignment of the source positioner and feed system on a ground-reflection range, except that now the adjustments have to be made using horizontal field probe data at vertical and horizontal polarization of the source antenna. If the source antenna is to be operated at only one polarization for all tests, then the only adjustment necessary to establish symmetry in the incident field at the test aperture is adjustment of the azimuth squint of the source antenna utilizing horizontal field probe data.

Extraneous signal interference can introduce errors into the measurements made on ground-reflection ranges just as were exemplified for elevated ranges. An assessment of extraneous signals reaching the test aperture can be accomplished as on elevated ranges with the field probe technique. Field probe cuts taken throughout the test aperture (normally horizontal, vertical, and ±45 degree cuts) yield data which are used to identify sources of extraneous energy and to establish the level of any impurities in the incident field. The periodicity and magnitude of any amplitude ripple noted on the field probe cuts, when substituted into the equations (14.82) and (14.84) yield the relative level and approximate source location (measured from the line-of-sight between the center of the test aperture and the apparent source location of the array formed by the transmit antenna and its image) for an extraneous signal distorting the incident field.

Polarization measurements made on a ground-reflection range must take into account the effect that the range surface has on the effective polarization characteristics of the incident field. In order to obtain accurate polarization patterns of the incident field, the linearly polarized sampling antenna must be
pointed at the center of phase of the array formed by the transmitting antenna and its image in the reflecting surface of the range. As discussed previously, it can be shown that the height of the center of phase is approximated by

\[ h_t' = \left( \frac{1-k}{1+k} \right) h_t \]  \hspace{1cm} (14.39)

where \( k \) is the amplitude ratio of the specularly reflected wave to the direct-path wave and \( h_t \) is the transmitter height. (In more general ground-reflection antenna range applications, the phase center may be assumed to be located at the intersection of the range surface with the vertical line joining the centers of the transmit antenna and its image.) Once the sampling antenna is pointed at the apparent source of radiation, rotation of the antenna and synchronous recording of the received field gives the polarization pattern of the incident field, from which axial ratio and tilt angle can be determined. The field probe antenna is moved to several different positions in the test aperture and polarization patterns are taken in order to establish off-axis polarization characteristics.

When it is desired to operate a ground-reflection range at circular polarization, polarization patterns are valuable aids to precision adjustment of the circularity of the incident field. Variations of the relative amplitude and phase of the vertical and horizontal field components of the source antenna, whose effects are monitored with the polarization patterns obtained with the sampling antenna, allows the circularity to be adjusted quite accurately. This capability permits compensation for the difference in effective reflection coefficient of the range surface for the horizontal and vertical field components. By transmitting in the proper elliptical polarization, the axial ratio can easily be adjusted to less than 0.1 decibel at a given position in the test aperture.

14.3.2 Wide Angle Field Assessments - - - The previous discussion of incident field assessment describes methods of evaluating the characteristics of the field at the test aperture produced by energy arriving from regions near the range axis. To completely describe the electromagnetic environment of the test antenna, however, the energy arriving from wide angles with respect to the range axis must also be determined. Evaluation of wide angle fields
is most easily accomplished by pattern comparison techniques and longitudinal field probe measurements, as discussed below.

**Pattern Comparison Measurements** - Pattern comparison measurements are based on the fact that any two patterns recorded with the same geometrical relationship between the incident field and the receiving antenna will be identical if no extraneous signals are present. When an extraneous signal is present, a pair of patterns which are measured with identical transmitter-receiver geometry but with differing scatter-receiver geometry, or vice versa, will differ due to the effects of the extraneous signal. Primary interest in this technique is directed toward the wide-angle structure of the pattern lobes, as opposed to the field-probe technique in which the main lobe of the receiving antenna is consistently directed toward the source of radiation. Changes in the minor lobe structure of pattern pairs are indicative of error caused by sources of extraneous signal both along the transmission path and at wide angles; thus, an evaluation program which includes the pattern comparison method serves as a more complete indication of the pattern measuring capability of an antenna measurement facility than does the field probe method alone.

The level of extraneous signals can best be determined if the test antenna can be positioned such that the direct-path and reflected signals at the test aperture are of the same phase for one pattern and of opposite phase for another. This can be accomplished if there is a capability for motion of the test antenna along the range axis and the longitudinal travel is sufficient to obtain inphase and out-of-phase conditions for the direct-path and reflected signals. By recording 360° azimuth patterns for incremental changes in the separation between source and receive antennas, the approximate level of extraneous energy may be determined. The change in pattern levels and the level at which they occur relative to the pattern peak must be considered to establish the level of extraneous signals.

Consider, for example, the azimuth patterns shown in Figure 14.22 which were made on a ground-reflection range for incremental changes in the separation between source and receive antennas. The difference of approximately 12 decibels in the patterns which occurred at an azimuth angle of
FIGURE 14.22 Azimuthal pattern comparisons (360° cuts) for incremental longitudinal displacements of the center of rotation.
120° represents an extraneous signal 4.5 dB below the direct-path signal as sensed at the receiving-antenna terminals. However, this excursion occurs at a point approximately 30 dB below the pattern peak, so that the extraneous energy could have been received with 30-decibel gain relative to the direct-path signal. The excursion thus represents an extraneous signal, whose level is not less than -34.5 dB relative to the direct-path signal. The pattern comparison also indicates that the source of extraneous energy is in a region located at an angular separation of approximately 120° relative to the range axis, since, for this comparison, 0 degrees represented the direction to the source antenna. In general, the maximum pattern differences will occur at an azimuth angle where the main lobe of the test antenna is directed toward the primary source of extraneous energy.

Pattern comparison data may also be collected by recording several conical-cut patterns about a fixed center of rotation for the receiving antenna. Suppose, for example, a 360° azimuth pattern of the probe antenna is made; the probe antenna is then rolled 180° in polarization, and the previously made 360° pattern is repeated, this time in the opposite direction so that the same pattern is made as before. The direction of travel of the abscissa on the chart recorder is reversed between cuts and the synchros readjusted so that the patterns will correspond. If a source of extraneous energy exists it should evidence itself as a difference in signal level between the two cuts, unless it is symmetric about the range axis. In that case another measurement, such as longitudinal-displacement comparisons, would be necessary to detect its presence.

Figure 14.23 is an illustration of an azimuth pattern comparison measurement about a fixed center of rotation. The change in phasing of the extraneous signal evidences itself as a change in pattern level of the minor lobe structure of the radiation pattern. At the peak of the first sidelobe the levels differ by 1.3 dB which represents an extraneous signal 22 dB below the direct-path signal as sensed at the receiving-antenna terminals. However, since this excursion occurs at a pattern level 26 dB below the peak of the main lobe, the extraneous signal level could be as low as -48 dB relative to the direct path signal.

Another possibility for a pattern comparison measurement is to take identical conical cuts at symmetric azimuth pointing directions with respect to the range axis. For example, one could turn the azimuth axis to a given position,
\[ \phi = \text{AZIMUTHAL PATTERN ANGLE (360°)} \]

FIGURE 14.23 Azimuthal pattern comparisons (360° cuts) for a fixed center of rotation. The test antenna was rotated 180° in polarization between cuts.
which we will call θ, and make a 360° pattern with the polarization axis of the test positioner. He could then turn the azimuth axis to a setting of -θ, roll the polarization axis 180° to achieve the same starting position, reset the synchros so the pattern will correspond, and repeat the pattern.

An illustration of a conical comparison of this type is shown in Figure 14.24. The comparison was accomplished by rotating a linear antenna at azimuth settings of ±5° relative to the range axis. The indicated difference of about 2 dB at the peak of the first secondary lobe corresponds to an extraneous signal approximately 30 dB below the direct-path signal.

**Longitudinal Field Probe Measurements** - Extraneous energy arriving at the test location from wide angles may also be detected and analyzed by moving a probe antenna longitudinally along the range axis and recording the received signal as a function of probe position. This technique is most effectively accomplished by use of a longitudinal counterpart of the previously described field probe mechanism for this purpose.

Provided the length of probe travel is sufficient to detect variable phasing of the direct path and extraneous signals, the existence of extraneous energy will be evidenced by periodic amplitude variations of the received signal.

Figure 14.25 illustrates the geometrical relationships of the direct path and extraneous signals, and the line of probe travel. The approximate longitudinal probe distance, \( p_l \), between lines of constructive interference of the direct path and extraneous signals is given by

\[
p_l = \frac{\lambda}{2 \sin^2(\theta/2)}
\]  

(14.85)

where \( \theta \) is the angle between the direction to the source of extraneous energy and the direction to the source antenna. Thus by noting the period of the amplitude variation of the received signal, the approximate direction to the source of extraneous energy may be determined.

The level of the interfering signal relative to the desired direct-path signal may be approximately determined by considering the peak-to-peak amplitude variation of the received signal in conjunction with the radiation pattern.
$\phi = \text{AZIMUTHAL PATTERN ANGLE (360')}$

Figure 14.24 Conical-cut pattern comparisons (360° cuts) for symmetrical angular displacements of the $\phi$-axis from the line of sight.
characteristics of the probe antenna. In order to obtain the best definition of extraneous energy, several longitudinal cuts may be desired, with the probe antenna positioned at various azimuth angles relative to the line of sight. Thus, the probe antenna gain may be used to accentuate the extraneous signal level. However, for each longitudinal pattern, the relative gain of the probe antenna in the directions of the desired and interfering signals must be considered as well as the amplitude variation of the received signal.

FIGURE 14.26 Geometrical relations for longitudinal probe technique.

Figure 14.26 shows a longitudinal field probe pattern which was made at a frequency of 2.0 GHz. The period of the amplitude variations of approximately one wavelength indicated that the source of the extraneous energy was at an angle of approximately 90 degrees as measured from the line of sight to the
test antenna. The peak-to-peak amplitude variation of approximately 0.1 decibel would correspond to an extraneous signal level of -45 decibels relative to the direct-path signal if both were received with equal gain. However, for this particular measurement, the gain of the receive antenna in the direction of the source of extraneous energy was approximately -17 decibels relative to the gain in the direction of the source antenna; thus, the level of extraneous energy was approximately -28 decibels relative to the direct-path signal.

FIGURE 14.26 Longitudinal field probe pattern.
### 14.4 SUMMARY OF BASIC CRITERIA

The major criteria discussed in this chapter are summarized below in tabular form. The symbols, some of which are introduced here for conciseness, are defined in paragraph 14.4.3.

#### 14.4.1 Elevated Ranges

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Parameter</th>
<th>Chapter Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R &gt; 10\lambda$</td>
<td>Inductive Coupling</td>
<td>Equation (14.1)</td>
</tr>
<tr>
<td>$R &gt; KD^2/\lambda$</td>
<td>Phase Curvature</td>
<td>Figure 14.2</td>
</tr>
<tr>
<td>$R &gt; 10L$</td>
<td>Axial Amplitude</td>
<td>Equation (14.19)</td>
</tr>
<tr>
<td>$\Delta E &lt; 0.25dB$</td>
<td>Mutual Coupling</td>
<td>Paragraph 14.2.1</td>
</tr>
<tr>
<td>$d &lt; 0.37KD$</td>
<td>Amplitude Taper</td>
<td>Equation (14.24)</td>
</tr>
<tr>
<td>$\theta_h &lt; 2h_r/R$</td>
<td>Sidelobe Illumination</td>
<td>Equation (14.58)</td>
</tr>
<tr>
<td>$h_r d &gt; 1.5KD^2$</td>
<td>Sidelobe Illumination</td>
<td>Equation (14.61)</td>
</tr>
<tr>
<td>$h_r &gt; 4D$</td>
<td>Composite Criterion</td>
<td>Equation (14.63)</td>
</tr>
<tr>
<td>$A_e &gt; w_{30}l_{30}$</td>
<td>Surface Screening</td>
<td>Paragraph 14.2.4</td>
</tr>
<tr>
<td>$\theta_3 &gt; 2 \tan^{-1}(4h_r/R)$</td>
<td>Probe Beamwidth</td>
<td>Equation (14.77)</td>
</tr>
</tbody>
</table>

#### 14.4.2 Ground-Reflection Ranges

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Parameter</th>
<th>Chapter Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R &gt; 10\lambda$</td>
<td>Inductive Coupling</td>
<td>Equation (14.1)</td>
</tr>
<tr>
<td>$R &gt; KD^2/\lambda$</td>
<td>Phase Curvature</td>
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</tr>
<tr>
<td>$R &gt; 10L$</td>
<td>Axial Amplitude</td>
<td>Equation (14.19)</td>
</tr>
<tr>
<td>$d_n &lt; 0.37KD$</td>
<td>Horizontal Taper</td>
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</tr>
<tr>
<td>$h_t \doteq \lambda R/4h_r$</td>
<td>Interference Lobe</td>
<td>Equation (14.31)</td>
</tr>
<tr>
<td>$h_r &gt; 4D$</td>
<td>Vertical Taper</td>
<td>Page 14-20</td>
</tr>
<tr>
<td>$\Delta h &lt; \lambda/M \sin \psi$</td>
<td>Surface Smoothness</td>
<td>Equation (14.73)</td>
</tr>
<tr>
<td>$A_1 \doteq Rw_{30}$</td>
<td>Primary Surface</td>
<td>Page 14-39</td>
</tr>
<tr>
<td>$A_2 \doteq 2.5A_1$</td>
<td>Secondary Surface</td>
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</tr>
<tr>
<td>$A_3 &gt; A_2$</td>
<td>Cleared Area</td>
<td>Figure 14.15</td>
</tr>
<tr>
<td>$\theta_3 &gt; 2 \tan^{-1}(4h_r/R)$</td>
<td>Probe Beamwidth</td>
<td>Equation (14.77)</td>
</tr>
</tbody>
</table>
14. 4. 3 Symbols

R = range length
λ = wavelength
K = a constant
D = test diameter
L = active region
ΔE = amplitude variation
d = source diameter
θ_m = mainlobe width
h_t = test height
A_s = screened area

w_{20} = 20th-zone width
l_{20} = 20th-zone length
θ_3 = 3-dB beamwidth
h_t = source height
Δh = surface perturbation
M = smoothness factor
ψ = grazing angle
A_1 = primary surface
A_2 = secondary surface
A_3 = cleared area
REFERENCES
CHAPTER 14


CHAPTER 15
ANTENNA TEST EQUIPMENT
Charles H. Currie, William L. Tuttle, and Joseph H. Pape

15.1 INSTRUMENTATION SUBSYSTEMS FOR ANTENNA TESTING

Instrumentation Systems designed specifically for antenna testing can be divided into five distinct subsystems. These subsystems are described in the subsections which follow.

15.1.1 Transmitting Subsystem -- A functional block diagram of a typical antenna measurement system is shown in Figure 15.1. The transmitting subsystem includes the source antenna, signal source, and the signal source control.

The characteristics of the source antenna and its location must be compatible with the tests being performed and the range design criteria as presented in Chapter 14.

15.1.2 Receiving Subsystem -- The receiver is usually located in the system control console with the local oscillator unit (LO) and mixer remotely located near the test antenna (see Figure 15.1). In cases where the physical configuration of the system does not require the LO Unit to be remotely located, it is installed in the receiver.

An amplitude only receiving system consists of one RF channel and mixer (solid lines in Figure 15.1). A signal from the test antenna to the signal-channel mixer is converted to an intermediate frequency suitable for processing.
A dual-channel phase/amplitude receiver consists of a signal-channel mixer, an APC channel mixer, an LO unit, and a receiver mainframe. This type of receiver is most often used for antenna measurements when either (1) phase and amplitude data are required, (2) multiple-channel amplitude data are required, or (3) when the higher sensitivity afforded by a narrow-bandwidth, phase-locked receiver is desirable. In order to phase-lock the receiver, a sample of the transmitted signal must be supplied to the APC-channel mixer. When phase-locked, the receiver converts signals supplied to the signal-channel and APC-channel mixers to coherent intermediate frequencies for data processing in the receiver.
15.1.3 **Positioning Subsystem** -- The antenna under test (AUT) which is usually tested in the receive mode (Figure 15.1) is mounted on a multi-axis positioner. The positioner is controlled by the positioner control unit located in the system control console and is equipped with synchros or encoders to provide position information to the control console. The position information is used to drive position displays and drive the position coordinate of an antenna pattern recorder.

The range source antenna is mounted on a positioning mechanism which provides source antenna polarization adjustment, vertical and horizontal alignment, and height adjustment. These functions are usually remotely controlled at the system control console with position displayed by angle indicators.

15.1.4 **Recording Subsystem** -- The pattern recorder is mounted in or near the system control console (see Figure 15.1). The pattern recorder accepts digital or analog amplitude data directly from the receiver or from a digital amplitude display unit that interfaces the receiver to the pattern recorder. The coordinates of the current pattern recorders may be programmed to perform either a rectangular or polar plot. These pattern recorders can accept either analog or digital amplitude inputs. Although other functions are available to permit recording of linear signal amplitude or power, signal strength is usually recorded as a logarithmic function. The position axis of the pattern recorder is slaved to the test positioner by either analog signals from synchros, the digital output of a synchro-to-digital display unit, or by digital inputs derived from encoders installed in the test positioner.

15.1.5 **Control Subsystem** -- The antenna measurement control system may be either manual or automatic. The control center for a manual system consists of a system control console containing the control subunits of the system. Typical subunits are the receiver, pattern recorder, signal-source control unit, test positioner control unit, test positioner angle displays, source tower controls and source tower angle and position indicators.
A typical automatic control console (Figure 15.2) includes subunits of the manual system that are capable of operating in an automatic or manual mode. A central processor with disk storage is accessed by a keyboard display and provides the automatic control capability. The system includes a printer-plotter to enable hard copies of program listings, tabulated data and plots of radiation-pattern contours.

FIGURE 15.2 Typical automatic antenna range control console
15.2 ANTENNAS

15.2.1 Source Antennas -- The antenna which transmits to or receives RF energy from the test antenna is referred to as the source antenna. The characteristics of the source antenna such as gain, beamwidth, and polarization depend on the specific measurement requirement and the antenna range design (See Chapter 14).

Types of antennas used as range source antennas include:

1. Log-Periodic Arrays--This type antenna, usually employed at frequencies of 1.0 GHz and lower, has moderate gain, relatively wide beamwidth and an extremely wide bandwidth (typically a decade). The gain of decade bandwidth units is typically 8.0 dB, while narrower bandwidth units have gains of 10 dB or more. The log-periodic array is fabricated either as a single linearly polarized antenna (see Figure 15.3) or as a dual-polarized antenna pair (see Figure 15.4).

2. Paraboloidal Reflectors with Log-Periodic Feeds--Antennas of this type are used as source antennas at frequencies from 400 MHz to above X-band. The low frequency limit is determined by a practical reflector diameter (a 10-foot reflector is typical at 400 MHz). Log-periodic feeds may be either linearly polarized, dual polarized, or circularly polarized. The gain of a complete feed and reflector assembly is largely dependent on the reflector diameter. The gain of a typical log-periodic feed and 6-foot reflector assembly operating over the 1.0 GHz to 6.0 GHz frequency range is from 22 dB at 1.0 GHz to 36 dB at 6.0 GHz while the comparable gains of a similar assembly with an eight-foot reflector are 25 dB at 1.0 GHz and 40 dB at 6.0 GHz. A dual-polarized log-periodic feed is shown in Figure 15.5.

3. Paraboloidal Reflectors with Waveguide Horn Feeds--These antennas are used as source antennas over the frequency range from approximately 3.0 GHz to above 100 GHz. The lower frequency limit is established by the largest practical physical size of the waveguide feed and reflector. The waveguide feed horn may be linearly polarized, orthogonal dual-linear-polarized, or circularly polarized. The frequency
FIGURE 15.3 A linearly polarized log-periodic array
for operation over the 100 MHz to 1.0 GHz frequency range

FIGURE 15.4 A dual polarized log-periodic array
for operation over the 100 MHz to 1.0 GHz frequency range
FIGURE 15.5 A dual-polarized log-periodic feed

FIGURE 15.6 An X-band source antenna assembly with waveguide horn feed
range of a paraboloidal reflector-waveguide horn feed assembly is established by the frequency range of the waveguide feed. Very high gains are possible through the use of large paraboloidal reflectors with waveguide feeds.

The nominal gain and beamwidth of a typical antenna assembly consisting of a four-foot paraboloidal reflector and an X-band waveguide feed is 42 dB and 1.5 degrees respectively. An X-band source antenna assembly complete with 4-foot reflector, waveguide feed, feed support assembly and a support bracket (spider), is shown in Figure 15.6.

Waveguide Horns--The source antenna employed may be a waveguide horn when the required directivity of the source antenna is relatively low. Waveguide horns with half-power beamwidths of slightly less than 10 degrees are practical, although smaller beamwidths can be achieved at the expense of horn length or by use of a collimating lens in the aperture. A typical X-band waveguide horn is shown in Figure 15.7.

![FIGURE 15.7 An X-band standard gain waveguide horn](image)

15.2.2 Reference Antennas -- Reference antennas serve as gain and polarization standards and may be either purchased pre-calibrated or constructed and calibrated. Standard gain antennas are employed as source antennas but are intended primarily as calibration standards to enable antenna gain to be measured using the method of substitution. Reference antennas as polarization standards should be provided with a reference axis index identified on the antenna and with appropriate test data to describe the polarization characteristics with respect to the operating frequency range. Several types of reference antennas are discussed in the following paragraphs.
1. **Standard Gain Dipole**—The standard-gain dipole is employed as a gain reference antenna at frequencies from a few megacycles to approximately 600 MHz. The typical standard-gain dipole has adjustable elements that allow adjustment of the dipoles to a half wavelength at the operating frequency. Calibration of the standard gain dipole (gain versus frequency) is generally performed over a frequency band which is 60 percent of an octave. A standard gain dipole for use over the 350 MHz to 600 MHz frequency range is shown by Figure 15.8.

![Figure 15.8 A standard gain dipole for use over the 350 MHz to 600 MHz frequency range](image)

2. **Standard Gain Horns**—The gain standard most often used when operating in the 350 MHz to above 100 GHz frequency range is the standard gain horn. An X-band standard gain horn is shown in Figure 15.7. Standard gain horns operate over standard waveguide bands, and calibration data (gain versus frequency) are required with each horn. Most standard gain horns are fabricated to mechanical dimensions specified in NRL Report No. 4433, and the calibration data are taken from data furnished in this report.

3. **Polarization Standards**—Antennas which have well-defined, stable polarization characteristics are required for reliable antenna polarization measurements. They are often employed either as accurately positioned source antennas or as polarization standards in comparative or substitution type measurements. Linearly polarized standard gain dipoles and standard gain horns are employed as linear polarization standards. These antennas are relatively easy to fabricate; the tilt
angle is precisely defined by the sharp null of the polarization pattern and the polarization is likely to be independent of frequency. Linear polarization standards cannot be used to indicate sense of elliptical or circular polarization. An independent method must be used for this purpose; often two orthogonal circularly polarized antennas of nearly equal gain are used.

Calibration of polarization standards must be performed using a rigorous three antenna method. See chapters 3 and 10 for a discussion of polarization and methods of calibration of polarization. Calibration is often ignored in practice and the final "standard" used is a waveguide horn or linear dipole which is assumed to have precise linear polarization due to its construction.

15.3 SIGNAL SOURCES

15.3.1 Performance Requirements --

1. **Frequency Control**—Most of the characteristics of antennas are sensitive to frequency and must be tested at several frequencies. This requires that the signal source oscillator be tunable over the range of test frequencies. Tuning may be either mechanical or electrical depending on the type of oscillator used. Signal sources used with phase-amplitude microwave receivers must have a maximum tuning rate compatible with the frequency tracking rate of the phase-locked receiver. If the signal source slew-rate exceeds the tracking capability of the receiver the receiver will not remain phase-locked.

2. **Frequency Stability**—Antenna testing may require amplitude-only measurements or both phase and amplitude measurements. When performing measurements requiring only amplitude data or phase-amplitude data taken during a short interval, frequency variations on the order of 0.01% to 0.1% may be acceptable. For more accurate phase and amplitude measurements, a frequency stability of one part in 10^6 or better is required.

3. **Spectral Purity**—Spectral impurities commonly generated in signal sources may be subdivided into harmonics of the fundamental frequency and spurious or non-harmonically related outputs. Many signal sources tune over a frequency range of an octave or more making it difficult to eliminate all harmonics using fixed-tuned filters.
Broadband crystal detectors used in antenna measurement systems operate over several frequency octaves with little change in sensitivity. When used to determine the characteristics of a broadband antenna, the actual antenna gain may be higher at the second harmonic than at the fundamental test frequency. The result is considerable distortion of the antenna test pattern. The harmonic problem can be eliminated by employing filtering between the antenna and detector or by using a selective receiver to discriminate against spurious frequencies.

When harmonics are present in the sample of the signal source output supplied to the automatic phase lock input of a harmonic mixing receiver and are below the fundamental by 20 dB or less, false locking of the receiver to a harmonic may occur. False locking can be eliminated if the harmonics are suppressed by suitable filtering in the path of the signal source sample that is coupled to the phase-lock (APC) channel of the receiver or by special harmonic sensing circuits in the receiver.

4. **Power Level**—The power level required from the signal source is dependent on specific test conditions. Significant test conditions include range length, source and test antenna gains, receiver sensitivity, and transmission line losses. Mounting the signal source directly behind the source antenna will minimize the transmission line loss. Output power levels available from common antenna range signal sources typically fall in the 0 dBm to +30 dBm range and may be increased by the use of solid-state or TWT power amplifiers.

5. **Modulation**—When test requirements warrant modulation, modern signal sources accomplish this by high speed pin diode switches. The most common switch is the reflective type which attenuates by reflecting energy back toward the load. When the reflective pin diode switch is used, an isolator must be placed between the signal source oscillator and switch to absorb the reflected power and minimize frequency pushing of the oscillator.
15.3.2 Signal Source Types --

1. Solid State Oscillators--Mechanically-tuned and/or voltage-tuned solid-state oscillators are currently being employed in the majority of antenna range signal sources. These devices are widely used because of their small size, low power drain and high reliability. Available power levels range from 1.0 watt in the low microwave frequency range (1 to 4.0 GHz) to 100 milliwatts at 94 GHz.

There are several common types of solid-state devices for generating microwave power. These may be classified as follows:

- microwave transistor
- transistor driven harmonic multiplier chains
- transferred electron (Gunn effect) devices
- Impatt (avalanche) diodes
- LSA (limited space-charge accumulation) diodes
- Impatt (anomalous mode avalanche) diodes

Transistor Oscillators--The power output and frequency range of operation of transistor oscillators have been continually increased. For narrow-band operation, greater than 10 watts can be obtained from a single Si bipolar transistor. The power output of the Si bipolar transistor falls off to a few hundred milliwatts in the 5-6 GHz range. The typical power output of octave-tuned Si bipolar transistor oscillators is 100 milliwatts at L band and from 5 to 20 milliwatts at S band.

The GaAs FET is now being employed in oscillators and amplifiers up to frequencies of 40.0 GHz and higher. Narrow-band GaAs Power FET oscillators are capable of power levels as high as 1.0 watt at 18.0 GHz. GaAs FETs are now being employed in oscillators which are continually tunable over an octave or more and are capable of power outputs of ½ 80 milliwatts up to 18.0 GHz.

Transistor oscillators (Si bipolar and GaAs FET) can be tuned over frequency ranges of an octave or more by either varactor tuning or YIG (yittrium iron garnet) tuning. The power output from octave or multi-octave tuned Si bipolar or GaAs FET oscillators is substantially less than can be obtained at a fixed frequency. For general antenna range requirements, broad-band, voltage-tunable operation is desirable. Varactor-tuned oscillators can deliver more than 250 milliwatts over L-
band and approximately 100 milliwatts over S-band. YIG-tuned oscillators are available which deliver 80 milliwatts or more within the 1 to 18 frequency range. Higher power levels are available from these broadband tunable oscillators when the YIG-tuned oscillator is followed by a solid-state amplifier which serves to isolate the oscillator output from the load.

The frequency of the varactor-tuned oscillator (VTO) is dependent on the change in varactor capacitance with voltage, which varies from type-to-type and includes stray capacitance. This results in a different tuning curve for every band.

The resonant frequency of the YIG-tuned oscillator (YTO) obeys the basic law of ferromagnetic resonance and the tuning curves for all bands are linear. This characteristic makes the YIG-tuned oscillator the popular choice for use in voltage-tuned sweepers and programmable signal sources.

**Harmonic Generators**—Transistors can be used as multiplier-amplifiers and oscillator-multipliers to produce useful power levels at frequencies well above their maximum fundamental frequency of operation. The non-linear element primarily responsible for harmonic generation is the capacitance of the base-collector junction. The junction performs similarly to the junction of a varactor diode. High output power levels can be achieved using transistor driven harmonic-generator chains that employ varactors as the multiplying elements. Although octave-tuned multipliers are available with lower output power levels, these type sources are generally fixed-tuned or tunable over narrow bandwidths.

**Characteristics of Bulk-Effect Oscillators**—Transferred electron (Gunn-effect) devices produce CW power levels on the order of 100 to 200 milliwatts at frequencies in the 4.0 to 40 GHz range and typical power levels of 25 milliwatts at 100 GHz. For pulsed operation, the peak power varies from 100 watts at L-band to 5 watts at X-band for a 0.1 percent duty cycle. When these devices are YIG-tuned over an octave or greater frequency ranges, the output power is generally less than available from fixed frequency or narrow-band Gunn-effect oscillators. The Gunn-effect devices operate at low voltages (typically 5 to 10 volts). The performance of the Gunn diode at millimeter-wave frequencies is continually being improved, and power outputs approaching the 50 to 100 milliwatt levels are anticipated in the near future.
Impatt (avalanche) diodes are generally used for fixed-frequency or mechanically-tuned CW operation at or above C-band. The impatt diode can deliver considerably more power than the Gunn-effect oscillator but with a considerably higher FM noise. For this reason, impatt oscillator sources which are not phase-locked to a stable source are not recommended for use with phase-amplitude receivers. Narrow-band impatt sources are available which supply CW output power levels from 100 to 250 milliwatts in the 26.5 to 100 GHz frequency range. The reliability of the solid-state sources when operated in the proper environment has proven to be excellent. These devices have been operated for many thousands of hours with little or no change of power output or frequency. Since power supply voltage requirements are low, difficulties encountered with high voltage circuits are not a factor.

2. **Triode Cavity**—Triode cavity oscillators usually utilize planar triodes with dual (plate-grid and grid-plate) cavities. Frequency tuning resolution on the order of one part in $10^5$ or better is typical, while power outputs greater than 1 watt are common. These oscillators are capable of octave band operation at frequencies in the 1.0 GHz to 8.0 GHz range. Frequency stability is typically 25 PPM per degree centigrade.

3. **Klystron**—Dual-cavity klystrons are mechanically tunable over approximately 20% of their frequency range. The extended-interaction klystron oscillator (EIO) is a multi-cavity klystron oscillator having a structure which includes coupling between multiple cavities. The EIO was developed to improve the power handling capability of the klystron. EIO's are available for operation from the microwave frequency range to the millimeter-wave frequency range (100 GHz and above). These tubes are capable of supplying CW power outputs from hundreds of milliwatts to watts and peak pulse powers in the kilowatt range. The tubes exhibit excellent frequency stability and low AM and FM noise when the power supply ripple is low. Tuning is generally limited to 1% or less of the carrier frequency. Tuning can be accomplished in two ways; mechanically, by changing the cavity dimensions, and electronically by controlling the beam voltage.

The reflex klystron is a single-cavity device with frequency determined by the cavity dimensions and the voltage applied to a repeller element. The reflex
klystron can be square-wave modulated by the application of a square-wave voltage to the repeller. The square-wave drives the klystron in and out of oscillation. If the modulation waveform has zero slope when the klystron is oscillating, frequency modulation is minimized.

Noise components are produced in reflex klystrons due to shot, partition and microphonic effects. Typical power to noise ratios for a band 1.0 MHz wide removed from the fundamental frequency by 30 MHz are on the order of 94 to 104 dB. Microphonic noise can present a problem unless special mounting precautions are taken. It is common to shock mount klystrons to eliminate fan vibrations.

4. **Backward Wave Oscillators**--Backward wave oscillators (BWOs) of the "O-type" design (having collinear electric-magnetic fields) are voltage tunable, vacuum tube signal sources capable of octave bandwidths and low to moderate power in the microwave and millimeter wave regions. Although special and experimental designs have been built which operate at frequencies as high as 700 GHz, most commercially available O-type BWOs fall within the 0.5 to 90 GHz frequency range.

Minimum output power always occurs at the low end of the frequency band. The total power variation from the low to the high end of the band depends on the tuning bandwidth and can be from 1 dB to 10 dB.

The frequency stability of backward wave oscillators is largely determined by the stability of the power supply since any variation of voltage applied to the helix (and to a lesser degree, to the grid and anode) affects the output signal. The requirements for voltage regulation and ripple depend on the amount of FM the system can tolerate and can be computed readily from the known FM sensitivities of the BWO elements. BWOs are relatively insensitive to frequency shifting due to load VSWR and phase. Values of ±1 MHz are typical for a load VSWR of 1.5:1 (any phase).

15.3.3 **Frequency Control and Stabilization** -- Frequency control of a signal source may be either by manual local controls or by analog or digital remote control. Signal source tuning is accomplished by mechanical adjustment of the electrical dimensions of the oscillator tuned circuit or by adjustment of the oscillator tuning voltage. Mechanically-
tuned sources may be controlled remotely by a tuning motor capable of being speed and direction controlled.

Digital control data is widely used for the control of signal source frequency in modern antenna ranges. Precise frequency programming is possible when digital frequency control is employed with a synthesized signal source. Digital control data is commonly transmitted serially from the controller to the signal source and back.

The frequency stability of an oscillator may be increased by any of several means: (1) by control of the operating environment, such as immersion of the oscillators in a temperature controlled oil bath; (2) by coupling the oscillator to a stable, high Q cavity; (3) by electronically referencing the oscillator to a microwave cavity (the Pound System)\(^6\); (4) by electronically referencing the frequency to a stable source (automatic frequency control or AFC); or (5) by phase-locking the frequency to a stable source (automatic phase control or APC).

The degree of frequency stabilization by an AFC system and APC system differ considerably. In an AFC system (see Figure 15.9), the error signal which operates the frequency control loop is derived from a small frequency difference between the stable frequency reference and the controlled frequency. When the gain of the AFC control loop is large, this difference can be very small. However, a frequency difference must exist to generate an error signal. The APC system error is derived from a small phase

\[ \text{FIGURE 15.9 Improvement of oscillator stability by means of an AFC loop} \]
error that exists between the reference oscillator and the controlled oscillator (see Figure 15.10). Since the error voltage is derived from a phase difference between the stable reference frequency and the controlled frequency, no frequency difference exists between these signals.

FIGURE 15.10 Improvement of oscillator stability of an APC loop

The inherent stability of the phase-locked oscillator is not compatible with continuous tuning. Frequency stabilities on the order of one part in \(10^6\) are typical of non-temperature stabilized quartz crystal oscillators and one part in \(10^8\) for temperature stabilized quartz crystal oscillators. Tuning a phase-locked source over the microwave frequency range requires locking to successive harmonics of a crystal reference oscillator. A synthesized signal source permits incremental frequency stepping of the output frequency while retaining the stability characteristics of a phase locked source.

A photograph of a synthesized signal source is shown in Figure 15.11. A block diagram of a synthesized signal source is shown by Figure 15.12. This synthesized system consists of two phase-locked loops with a common reference crystal oscillator. The first phase-locked loop establishes the signal frequency supplied to the system mixer. A sample of the voltage tuned oscillator is coupled through a programmable frequency divider to a phase detector. Part of the crystal reference is also supplied to this phase detector. The programmable divider establishes the ratio of the oscillator frequency to the reference frequency when the system is phase-locked.
FIGURE 15.11 A wide frequency range, 40 GHz, programmable, synthesized signal source designed for antenna measurements

FIGURE 15.12 A synthesized signal source
The voltage tuned oscillator frequency is multiplied \((x N)\) to the microwave range and supplied as the input signal to the mixer. A sample of the signal source voltage-tuned oscillator output is supplied as the local oscillator input to the mixer. The programmable frequency step increments of this first phase-lock loop are generally broad \((50 \text{ MHz to } 400 \text{ MHz})\).

The second phase-lock loop establishes the operating intermediate frequency of the system by programming the programmable divider following the IF amplifier. The frequency resolution of the synthesizer is determined by the reference frequency supplied to the IF phase detector and by programming the intermediate frequency in increments corresponding to the reference frequency. The intermediate frequency is programmed over a frequency range which corresponds to the frequency interval between successive lock frequencies of the first phase lock loop. This process provides continuous, wide-frequency range programming of the signal source. Typical synthesizer frequency step-intervals (resolution) applicable to antenna testing are 0.1 MHz and 1.0 MHz.

15.3.4 Amplitude Control and Stabilization -- The control of the RF output level of a signal source may be controlled by any of several level control devices located in the RF output path; (1) A mechanically variable attenuator; (2) a digitally programmable step attenuator; (3) a current controlled PIN diode attenuator.

The use of a mechanically adjustable attenuator is common in non-programmable signal sources. Modern programmable signal sources employ either the digital programmable attenuator, or if leveled, employ the PIN diode attenuator of the amplitude leveling loop to adjust the signal source output. This is accomplished by adjustment of the leveling loop reference voltage. See Figure 15.13. The output level of most signal sources is adjustable only over a limited dynamic range of 15 dB to 30 dB.
In many antenna measurement situations, small signal source power level variations on the order of ±0.05 dB are not significant. For measurements, such as very precise insertion-loss measurements or absolute gain comparison measurements, power level stabilities on the order of ±0.01 dB or ±0.02 dB are required for short periods of time. This order of power level stability can be achieved by the use of an automatic power-leveling feedback loop (see Figure 15.13). The signal source output is coupled through an isolator, PIN attenuator, and directional coupler to the RF output port. The PIN attenuator is a current-controlled diode network which attenuates by reflecting power toward the source. The isolator located between the signal source and the PIN attenuator absorbs the power reflected by the PIN attenuator eliminating frequency pulling of the signal source oscillator. A sample of the signal source output is coupled to the diode detector from the directional coupler. The detected signal is filtered and applied as a dc input to the comparator. Peak detection is employed for pulsed signals. The comparator compares the detected dc level with a stable dc reference level and supplies either a zero, positive, or negative output signal dependent on the amplitude of the detected voltage relative to the reference voltage. The error signal is amplified by the error amplifier and supplied as a current control to the PIN attenuator to greatly reduce amplitude variations present at the signal source output.
It is common practice to locate the directional coupler and detector portion of the power leveling loop near the source antenna to eliminate changes of transmission line attenuation due to frequency and temperature sensitivity. This exposes the temperature sensitive detector to environmental effects. These effects can be minimized by the use of stable temperature-compensated detectors.

15.3.5 **Remote Control** -- Remote control of signal source functions is convenient for the operator and necessary when automatic antenna measurements are being made. Signal source functions which are generally controlled in non-automatic systems are CW frequency, swept-frequency, modulation ON-OFF, and power ON-OFF. An automatic antenna test system typically includes digitally-programmed frequency. Remote control of signal source output power level is omitted in many antenna measurement systems since the maximum signal source power output level is usually required for low noise measurements using the lowest practical source antenna gain.

Remote control up to distances of 4,000 feet or more is accomplished by means of either analog or digital control signals transported to and from the signal source and the signal source control unit by means of any of the following: (1) multi-conductor shielded control cable; (2) radio link with a modem pair; (3) telephone line or coaxial line link with a modem pair; (4) or fiber optics link with a modem pair. A complete automatic-frequency control system is shown in Figure 15.14.
15.4 RECEIVING SYSTEMS

15.4.1 Basic Types -- Most antenna-development facilities are required to perform measurements over an extremely-wide RF spectrum. The spectrum ranges from very low frequencies to the microwave and millimeter-wave regions. For economy and efficiency, it is desirable that receiving systems operate over all or most of the spectrum. Typical operating frequency ranges of antenna microwave measurement test receivers are from 30 MHz to above 110 GHz. Three general types of receiving systems are employed for antenna range measurements. These are the diode detector, bolometer detector, and superheterodyne receiver.

The diode detector and bolometer detector are broadband, low-sensitivity detectors which deliver outputs having a square-law characteristic. These detectors have the advantage of simplicity; however, their application is limited to amplitude-only antenna measurements requiring limited sensitivity, dynamic range, and filtering. Sensitivity and dynamic range can be maximized at the expense of measurement speed using post-detection bandwidths of 1 Hz or less. For measurements over a narrow range of input frequencies, a preselector can be placed before the diode or bolometer detector to exclude broadband RF noise and spurious signals.
Although the superheterodyne receiving system is a more complicated receiving system than the direct detection systems, it is widely used because of its many advantages. The superheterodyne receiving system is capable of a wide operating frequency range, improved sensitivity (60 dB or more over direct detection systems), narrow-band filtering, and a wide continuous dynamic-range of operation.

15.4.2 Direct Detection Systems --

Diode Detectors--Point-contact crystal rectifiers and hot-carrier (Schottky-barrier) diodes are metal-semiconductor devices which provide rectification due to the nature of the semiconductor contact. Point-contact diodes are fabricated by pressing a fine metal point (whisker) into the surface of a semiconductor. Hot-carrier diodes, fabricated by depositing a metal film on the surface of a semiconductor, are called planar diodes, hot-carrier diodes, or Schottky diodes (see Section 4.3).

For antenna measurements, microwave diode detectors are operated at low input signal levels which result in square-law output characteristics. This results in a 2:1 decibel ratio of the signal level change measured at a diode detector output compared to a given signal level change measured in decibels at the diode detector input. This output-to-input ratio of dynamic range results in an 80 dB dynamic range of output signal level for a 40 dB dynamic range of input signal level. This large output dynamic range must be measured accurately in a typical antenna test system. The practical input dynamic range of an antenna test system employing a square-law detector is limited to slightly more than 40 dB.

Figure 15.15 shows a typical diode detection circuit. The received signal (RF input) is amplitude modulated (typically square-wave at 1000 Hz). $C_2$ located in the detector output circuit is a filter capacitor chosen to bypass RF frequencies but allows the detected square-wave to appear at the detector output. The narrow-band amplifier (bandwidth 1.0 Hz to 30 Hz) amplifies and filters the detected signals and supplies a high level signal output for recording. Typical narrow-band amplifier bandwidths fall in the 1.0 Hz to 30 Hz range.
Amplified - Filtered

Amplitude Modulation

Modulated Demodulated

Carrier Carrier

Detector

RF Narrow-Band Output to Input Amplifier Recorder

Figure 15.15 A typical diode detection system

Bolometer Detectors—A bolometer is a device which detects an RF signal by changing resistance due to the heating effect of the applied RF power (see Section 4.3). Bolometers are usually biased at a resistance of either 100 ohms or 200 ohms. The resistance of commercial bolometers, constructed using a fine platinum wire mounted in a scaled container, is directly proportional to the temperature of the element and the power dissipated in the element. The output voltage is directly proportional to the input power (square-law characteristic) providing the input power is small compared to the dc power.

Although the sensitivity of the diode detector may typically be 20 dB better than that of the bolometer detector, bolometer detectors are capable of square-law operation at higher power levels (0 to +6 dBm) and over a greater dynamic range than diode detectors. Bolometer detectors are generally preferred over diode detectors for detecting 1.0 kHz modulated test signals because of their accuracy, stability, and dynamic range capability.

A simplified schematic of a bolometer detection system is shown by Figure 15.16. The system shown demodulates and amplifies a square-wave modulated RF signal. A constant, direct-current bias supplied to the bolometer establishes the operating point
(resistance) of the bolometer. The ON/OFF application of the RF signal to the bolometer element causes the bolometer resistance to change during the signal ON period and return to the biased resistance during the signal OFF period (see Figure 15.16). The thermal lag of the bolometer element limits the usable modulation frequency. A 1000 Hz modulation rate is common.

15.4.3 Heterodyne Receiving Systems --

Heterodyne Principle--Heterodyne reception is a process in which a received signal is combined in a non-linear device with a local-oscillator signal resulting in output signals having frequencies which are the sum and the difference of the input frequency and the local oscillator frequency. Filtering at the output terminal of the non-linear device enables selection of the desired output signal. The output can be at a specific frequency either higher (up-conversion) or lower (down-conversion) than the signal frequency by the proper selection of the local-oscillator frequency and the output filter. A heterodyne receiving system which results in a converted output frequency that is superaudible is referred to as a superheterodyne receiver.
Although more complex than a direct detection receiving system, the superheterodyne receiver is widely used for antenna measurements because of its several advantages: (1) tunable selectivity, over a wide frequency range with narrow-band filtering for any selected input frequency, is provided by the bandpass characteristics of the IF system; (2) simplified amplification with high system gain can be provided at the desired intermediate frequency at reduced cost when compared to the cost of wide range RF amplifiers; (3) simplification of gain or signal level control of wide range receivers by providing level control at the intermediate frequency; and (4) versatile translation of frequency bands (see Figure 15.17). The system shown uses a tunable L-band first local oscillator with a dual conversion system to enable conversion of the 0.1 to 1.0 GHz frequency range to a 45.0 MHz intermediate frequency. Systems such as illustrated by Figure 15.17 are employed to extend the frequency range of microwave receivers to cover frequencies below 1.0 GHz using the same receiver local oscillator employed for conversion of microwave frequencies of 1.0 GHz and higher.

**FIGURE 15.17** A dual conversion superheterodyne receiving system which converts the 0.1-1.0 GHz band to 45.0 MHz using a 1.28 to 2.18 GHz first local oscillator

**Harmonic Mixers**—Harmonic mixing and sampling type mixers are commonly used to enable microwave receiver spectrum analyzers and network analyzers to operate over extremely wide frequency bands using a single local oscillator that is tunable over approximately an octave. The expression for frequency translation for harmonic mixing
\[ f_{RF} = n f_{LO} \pm f_{IF} \]  

where

- \( f_{RF} \) is the RF signal,
- \( f_{LO} \) is the local oscillator signal,
- \( n \) is the harmonic mixing number, and
- \( f_{IF} \) is the intermediate frequency.

Wide frequency coverage is achieved by mixing using values of \( n \) from 1 to 50.

The mixing efficiency is the conversion loss which is defined as the ratio of the IF output power to the RF input power. The actual loss encountered will depend on the match of the source to the mixer and the mixer to the IF amplifier.

The sensitivity of a superheterodyne receiver is determined by its noise figure and noise bandwidth. For a receiver without amplification preceding the converter, the noise figure is given by

\[ F = L_c \left( t + F_{IF} - 1 \right) \]

where \( t \) is the noise temperature ratio of the mixer, \( F_{IF} \) is the IF amplifier noise figure, and \( L_c \) is the mixer conversion loss. The typical conversion loss of a broadband harmonic mixer employing the fundamental local oscillator signal is approximately 10 dB. As the harmonic number employed increases the mixer conversion loss increases (approximately 6 dB each time the harmonic mixing number doubles). When operating at microwave and millimeter-wave frequencies using the local oscillator harmonics, other factors such as mixer mismatch and increased noise, due to conversion of broadband RF input and local oscillator noise to IF by lower order harmonics of the local oscillator signal, can reduce the sensitivity of a receiving system using a harmonic mixer. The conversion loss of a 1.0 GHz to 18.0 GHz harmonic mixer using a 1.0 GHz to 2.1 GHz local oscillator input is shown by Table 1.
<table>
<thead>
<tr>
<th>Frequency Range GHz</th>
<th>Harmonic Mixing Number</th>
<th>Typical Conversion Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 to 2.0</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>2.0 to 4.0</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>4.0 to 8.0</td>
<td>4</td>
<td>23</td>
</tr>
<tr>
<td>8.0 to 12.4</td>
<td>6</td>
<td>26</td>
</tr>
<tr>
<td>12.4 to 18.0</td>
<td>9</td>
<td>32</td>
</tr>
<tr>
<td>18.0 to 26.5</td>
<td>13</td>
<td>37</td>
</tr>
<tr>
<td>26.5 to 40.0</td>
<td>20</td>
<td>42</td>
</tr>
<tr>
<td>40.0 to 60.0</td>
<td>29</td>
<td>52</td>
</tr>
<tr>
<td>60.0 to 90.0</td>
<td>43</td>
<td>57</td>
</tr>
</tbody>
</table>

Mixers designed specifically for antenna test systems are designed as two-port mixers. A two-port mixer is configured to have a separate RF input port and a common local oscillator, IF and mixer current port. This arrangement is advantageous since a single coaxial cable can connect a remote mixer to the receiver. A typical 1.0 GHz to 18.0 GHz two-port mixer is shown by Figure 15.18.
**Sampling-Type Mixers**—Sampling-type mixes are commonly employed in complex impedance measuring instruments (network analyzers). The sampling technique is a special case of heterodyning related to harmonic mixing but employing different techniques and circuitry. If the conduction angle in a harmonic mixer is made small compared to the period of the highest signal frequency, the mixer may be called a **sampler**. The sampling mixer samples the signal at the local oscillator rate, and the local oscillator frequency is adjusted so the signal frequency and a harmonic of the sampling frequency differ by the intermediate frequency. In practice the local oscillator signal is a narrow-pulse applied to the mixer in order to gate the mixer diode on at the local oscillator pulse rate. Successive sampling pulses sample the signal envelope at different points resulting in a reproduction of the signal envelope at a lower frequency (see Figure 15.19).

**Figure 15.19** An idealized sampler and sampling waveforms

The basic difference between sampling and harmonic mixing is the sampling pulse associated with the sampling process is short compared to the period of the RF signal. Both the mixer and the sampler produce a linear frequency translation of the signal spectrum to the IF spectrum with amplitude linearity and phase coherence maintained.
Most antenna range measurement systems employ harmonic mixers rather than samplers. This is because of the greater achievable sensitivity with harmonic mixers, the higher operating frequency capability of harmonic mixers, and the ability to conveniently remote a single mixer from a second mixer or from the receiver.

**Receiver Local Oscillators**—The local oscillator for a typical antenna measurement receiver is generally housed in a remotable module which may be located either in the receiver or in a weatherproof enclosure remotely located from the receiver. The most common local oscillator frequency ranges are 1.0 GHz to 2.18 GHz and 2.0 GHz to 4.1 GHz. The 1.0 GHz to 2.18 GHz range has two advantages. First, the fundamental receiver frequency range can begin at 1.0 GHz. Second, the attenuation of long coaxial cables is lower for the 1.0 GHz to 2.18 GHz band than for the 2.0 GHz to 4.1 GHz band. The chief advantage of the 2.0 GHz to 4.1 GHz local oscillator frequency range is that the order of harmonic mixing is reduced by two, thus resulting in higher receiver sensitivity.

The YIG-tuned local oscillator has rapid tuning-rate capability and excellent tuning linearity. These characteristics are highly desirable in programmable antenna measurement systems.

**Intermediate Frequency Circuits**—Antenna measurement receivers usually employ a double conversion system which involves two intermediate frequencies. A common first intermediate frequency is 45.0 MHz. This frequency which is in the vicinity of the television receiver intermediate frequency band was selected because of the freedom from television transmission interference. IF bandwidths of single channel CW receivers vary from 100 kHz to several MHz. Phase/amplitude CW receiver IF bandwidths can vary from a few Hz to several kHz.

The gain of antenna measurement receivers is nearly always controlled by varying the gain of the IF amplifiers. A total IF gain control range from 40 dB to 50 dB is provided in most receivers.
15.4.4 Single-Channel Microwave Receivers -- The single-channel microwave receiver is employed in antenna measurement systems instrumented for measurements requiring amplitude data only. A block diagram of a typical single-channel antenna measurement receiver is shown by Figure 15.20. Figure 15.21 shows a typical single-channel microwave receiver. The microwave receiver shown includes the following:

1. A broadband harmonic mixer
2. A voltage-tunable (2.0 to 4.1 GHz, YIG-tuned) local oscillator
3. An automatic frequency control loop
4. A mixer current leveling loop
5. A dual conversion IF system
6. A swept, second-local oscillator and narrow-band IF amplifier which provides modulation of a CW received signal
7. A square-law (bolometer) output detector
8. A panoramic oscilloscope display

IF System, AFC System--CW signals received at the harmonic mixer (Figure 15.20) are downconverted to 45.0 MHz. These signals are amplified and coupled to the second mixer. The 3-dB bandwidth of the main IF amplifier which follows the second mixer is 0.5 MHz centered at 10.0 MHz. The second local oscillator is swept at a 1,000 Hz linear rate (from 54 MHz to 56 MHz) to provide an IF difference frequency swept from 9 MHz to 11 MHz. The results of the swept IF signal passing through the narrow-band main IF amplifier is an IF output signal modulated at a 1,000 Hz rate. This signal is detected by a bolometer detector and supplied as an output for pattern recording. The signal is also detected by a video detector with the detector output supplied as the signal input to the AFC amplifier and the vertical input of the panoramic display. The phase reference for the APC amplifier and the horizontal sweep for the panoramic display are coupled from the 1,000 Hz sweep generator. The AFC circuit consists of a 1,000 Hz phase detector and amplifier which provides correction for drift of the received signal frequency.
FIGURE 15.20  A typical single-channel antenna measurement receiver

FIGURE 15.21  A typical single-channel microwave receiver
Panoramic Display—The panoramic oscilloscope display provides visual indication of the detected second IF signal with respect to the 1,000 Hz sawtooth sweep supplied to the oscilloscope. The position of the peak of the displayed signal along the horizontal axis of the display represents the signal frequency. The total sweep width represents a 2.0 MHz band (9.0 MHz to 11 MHz).

60-dB Systems—The dynamic range of the receiver employing a bolometer detector for the data output is limited to a little more than 40 decibels. The dynamic range of the receiver-recorder combination is often extended to 60 dB by introducing a voltage to the receiver IF amplifier to vary its gain as a function of either the recorder pen displacement or the detected signal level. The action of the feedback system is to increase the receiver gain to effect a total receiver gain increase of 20 dB for a 40 dB signal level decrease to the recorder. This feature provides an accurate 60 dB dynamic range of recording when the feedback is calibrated at 15 dB intervals.

Low Frequency Reception—Coverage of frequencies below the lowest local oscillator frequency (2.0 GHz) is provided by a frequency up/down converter. The up/down converter employs the 2.0 to 4.1 GHz receiver local oscillator as the local oscillator for the up-converter, then down converts the signal to supply a 45.0 MHz. The up/down converter accepts an input signal ranging from 0.03 to 2.0 GHz. Figure 15.17 shows a similar up-converter using a 1.28 to 2.18 GHz local oscillator.

15.4.5 Receivers for Phase and Amplitude Measurements — When the evaluation of antennas or antenna systems requires both phase and amplitude data to be collected and evaluated, it is necessary that a phase and amplitude measurement receiver be used. Phase-amplitude receivers using heterodyne techniques for phase measurements are possible because of the one-to-one correspondence between the change in phase of the intermediate frequency signal to the change in phase of the radio frequency input signal.

The heterodyne process results in linear conversion of the RF signal to the intermediate frequency. Thus, the amplitude of the IF signal is proportional to the amplitude of the microwave signal.

Figure 15.22 illustrates a single channel, double conversion heterodyne circuit as used in typical microwave measurement systems. This circuit in the form shown makes use of
FIGURE 15.22 A single-channel, phase-locked, double-conversion receiving system

harmonic mixing (see section 15.4.3) to convert the input signal to an intermediate frequency of 45 MHz. The input signal is coupled directly to a harmonic mixer, combined with the first local oscillator output, and converted to the first intermediate frequency.

The second intermediate frequency is set at 1.0 KHz because this frequency is compatible with most recording equipment. Heterodyning to such a low second intermediate frequency is made possible by the use of phase-lock techniques.

The phase-lock circuit causes the intermediate frequency to be synchronous with (locked-to) the frequency of a 45.000 MHz reference oscillator. Since the reference oscillator is a crystal oscillator with a virtually pure spectrum, the intermediate frequency signal contains almost no frequency modulation components within the passband of the phase-lock loop. This permits the IF signal to be converted to 1 KHz by means of a (45.001 MHz) crystal oscillator, which is phase-locked, so that its frequency differs from the 45 MHz reference oscillator by precisely 1 KHz. The bandwidth of the first IF amplifier is of the order of 10 MHz while the bandwidth of the (1 KHz) second IF amplifier is of the order of 100 Hz.
This narrow IF bandwidth results in a sensitivity which is in the order of 25 decibels better than that of a conventional receiver employing square-law detection and predetection and post-detection bandwidths equal to those of the phase-locked receiver (10 MHz and 100 Hz). A typical phase-locked receiver is shown in Figure 15.23.

**FIGURE 15.23** A phase-locked, phase-amplitude receiver

**Sensitivity**—A simplified block diagram of a phase-lock, dual conversion system is shown by Figure 15.24. The 1,000 Hz SIG CH and APC CH outputs shown are not detected but are intermediate frequencies resulting from the dual conversion process. The center frequency of the 1,000 Hz low intermediate frequency is maintained by the frequency tracking capability of the receiver phase-lock loops. The IF bandwidths of modern phase-lock receivers may be selected to be in the range from 7 Hz to 1,600 Hz. An often used bandwidth is 100 Hz. The effective noise bandwidth in this case will be greater (approximately 400 Hz) since image noise is summed with signal channel noise in both the first and the second mixers.

The noise bandwidth of a receiver employing square-law detection is approximately \( \sqrt{2B_1B_2} \), where \( B_1 \) is the predetection system noise bandwidth and \( B_2 \) is the post-detection system noise bandwidth. Comparing the sensitivities of the
two receiver types (with identical noise figure, the phase-lock receiver with a 100 Hz effective noise bandwidth, and the square-law detection receiver with a 0.25 MHz predetection noise bandwidth and a 50 Hz post detection noise bandwidth), results in the phase-locked receiver having a sensitivity 17 dB better than the square-law detection receiver. If the information bandwidth of both receivers were reduced to 1 Hz (phase-lock receiver noise bandwidth = 2.0 Hz and post-detection bandwidth of square-law receiver = 1.0 Hz), the sensitivity of the phase-lock receiver is 25.5 dB better. The sensitivity of the square-law detection receiver can be increased by reducing the predetection bandwidth, but the sensitivity is only proportional to the square-root of the predetection bandwidth.

Data Measurement Accuracy -- The first intermediate frequency of the phase-locked receiver is identical to that of the crystal-stabilized oscillator used as the phase-lock reference. This results in virtually zero error due to intermediate frequency drift. The amplitude stability of IF amplifiers can be held to a minimum by use of IC amplifiers employing feedback or discrete amplifiers employing feedback together with the use of temperature compensated components. Other sources of drift such as changes of first mixer crystal current can be eliminated by leveling the crystal current. Frequency instability of the signal source can be eliminated by the use of a synthesized source.
Modern phase lock receivers employ programmable IF calibration which automatically corrects the system for small gain and phase changes due to temperature variations.

A technique commonly referred to as the shared-mixer mode of operation (see Figure 15.25) employs a synchronous RF/IF switch arrangement to direct two RF signal inputs to two separate IF channels with both signals time-sharing a common signal channel mixer, mixer-to-receiver coaxial cable, and IF preamplifier. This system enables phase to be measured between the two signal-channel inputs with no phase error contribution due to the signal channel mixer, mixer to receiver cable, or IF preamplifier. Since phase changes in these system subunits and cables are common to both channels, they cancel.

Linear Output Function—The linear output of the phase-locked, double-conversion receiver delivers a 100:1 output voltage ratio for a 100:1 input voltage ratio. For this reason, typical phase-locked receivers have a continuous operating dynamic range of at least 60 dB and up to 80 dB in some models.

Phase Lock Reference—One factor that limits the usefulness of a phase-locked receiver compared to the single-channel, non-phase-locked receiver is the requirement for an input reference signal for operation of the phase-lock-loop.

FIGURE 15.25 Shared mixer mode RF/IF system configuration
The typical sensitivity or phase-lock threshold of the reference (APC) channel is 40 dB to 60 dB lower than the sensitivity (signal power = noise power) of the signal channel. The sensitivity difference is due to the wide bandwidth of the automatic phase-lock loop required to permit the receiver to track signal frequency deviation rates of 10 GHz/second or greater. In practice, the reference is obtained from a reference antenna or a cable connected to sample the signal source output.

**Signal Processing**—The final output intermediate frequency of phase-locked receivers designed specifically for antenna measurements is either 1,000 Hz, 1,025 Hz or 5,020 Hz. The 1,000 Hz output is used when the power line frequency is 60 Hz and the 1025 Hz output is used when the power line frequency is 50 Hz. These frequencies are selected to maximize the separation between harmonics of the power line frequency and the final intermediate frequency. The 5,020 Hz output is used in some phase lock receivers to enable higher data-acquisition-rates to be employed. The 5,020 Hz frequency is selected to be approximately equally spaced between harmonics of both 50 Hz and 60 Hz power line frequencies.

The direct IF output of the phase-lock receiver can be applied to the input of an antenna pattern recorder configured to accept an analog input. Antenna range installations which use automatic or semi-automatic techniques require the receiver output to be supplied in digital form. The processing of the signal for some phase-locked receivers is done externally in digital amplitude processors and in phase processors. Other phase-locked receivers include this signal processing internally.

The processing of the two or more IF signal outputs of a typical phase-locked, phase-amplitude receiver includes the following: (1) conversion of the data of each output channel to a digital form; (2) conversion of amplitude data of each channel to either of several selectable functions such as linear, logarithmic or ratio; (3) processing of channels to measure phase; (4) providing selectable phase and amplitude offsets; (5) providing digital phase and amplitude outputs (these are generally supplied as byte-serial outputs); and (6) provisions for averaging phase data and amplitude data.

**15.4.6 Programmable Receivers**

Many antenna range installations incorporate either automatic or semi-automatic techniques. These installations are capable of performing automatic antenna range tests
simply by providing the system controller with the proper instructions. Antenna range receivers which operate in automated range systems must be fully programmable.

A block diagram of the programmable phase-locked, phase-amplitude receiver depicted in Figure 15.23, is shown by Figure 15.26. The receiver shown is capable of shared-mixer operation and has three IF channels with a final output frequency of 5,020 Hz. The receiver is capable of providing output data for three-amplitude channels and two-phase channels at a rate of 100 data points-per-second. The receiver interfaces with the system controller by means of the IEEE-488 Standard Bus. The receiver is composed of five basic subunits as follows: (1) the RF switch unit which is employed for shared-mixer operation; (2) the remotable LO unit which contains the RF system and IF preamplifiers; (3) the IF unit which contains three IF amplifier channels and all phase-lock circuitry; (4) the phase/amplitude processor which processes the three 5,020 Hz IF channels; and (5) the control unit which provides receiver programming control and the IEEE-488 Standard Bus interface.

All programmable functions of the receiver are controllable through the IEEE-488 Standard Bus (see Figure 15.27). During operation, the receiver is able to "talk" and "listen" and can be operated with other bus compatible instruments.

A front panel alpha-numeric display indicates operating status (frequency, mixer currents, modes, etc.) and output data (three amplitude channels and two phase channels). Servicing of the digitally controlled receiver is facilitated by built-in service modes.
FIGURE 15.26 Block diagram of a programmable phase-amplitude receiver

FIGURE 15.27 A programmable microwave receiver shown connected to the IEEE-488-1978 standard bus
Antenna Test Positioners -- Most antenna test ranges utilize a single or multi-axis positioner to orient an antenna relative to a stationary signal-source antenna. The direction of the signal source relative to the antenna under test is described in terms of angles $\alpha$ and $\theta$ of the antenna's spherical coordinate system as described in Chapter 5.

Commercially available antenna positioners are offered in a variety of sizes and configurations. The small portable azimuth positioner in Figure 15.28 weighs only 55 pounds and may be easily transported from site to site. The heavy-duty elevation positioner shown in Figure 15.29, although less than four feet high, is capable of supporting and moving an antenna weighing several thousand pounds. Figure 15.30 is an example of the very versatile and popular azimuth-over-elevation type of positioner. The single axis azimuth positioner (Figure 15.28) and the positioner of Figure 15.30 usually have the antenna under test mounted on a bracket or pylon that is attached to the azimuth or $\theta$ axis.

FIGURE 15.28 Small azimuth positioner designed for making measurements of lightweight antennas.
FIGURE 15.29 Heavy duty elevation positioner.
The massive antenna positioner illustrated in Figure 15.31 has azimuth elevation and lower azimuth axes. This positioner has the capability of supporting and rotating test antennas up to fifty feet in diameter and weighing many thousand pounds. A positioner of the type shown in Figure 15.32 is frequently used to support and rotate the source antenna. In addition to rotation about the polarization axis, axes are provided for adjustment in azimuth and tilt to more easily position the source antenna beam relative to the antenna test site. Source positioners are also available with motorized linear actuators to facilitate control of the source transmitter, which may be several hundred feet away from the control room.

The model tower of Figure 15.33 has as its lower axis an azimuth rotator of conventional design. An offset arm supports a fiberglass-reinforced plastic tube on which a head or roll axis is mounted. The head axis is provided with a collet to which a model may be mounted. A fiberglass-reinforced torque tube inside the main support tower provides the drive force for the head rotation. Motors and the necessary reduction gears are in the offset arm at the base of the model tower mast.
FIGURE 15.31 Large azimuth-over-elevation positioner.

FIGURE 15.32 Positioner designed for supporting and rotating source antennas. The positioner is mounted on a mounting fixture which provides for adjustment in azimuth and tilt.
The relatively flat azimuth section of the positioners in Figure 15.30 and Figure 15.31 is possible because of the utilization of large diameter flat bearings. A typical partial section of a four point contact bearing used in azimuth positioner applications is shown in Figure 15.34. This bearing, although only one and one-half inches high overall, is slightly over 29 inches in diameter. The outer race of the bearing is provided with counter sunk holes to facilitate mounting the bearing race into a housing. The inner race is provided with tapped holes to permit a turntable to be attached directly to the inner race of the bearing. The inner race also has an integrally machined internal gear, which is the main drive gear of the azimuth axis. A unique feature of this ball bearing is the deep groove provided for the ball and the shape of the two raceways to provide for four points of contact on the balls, two points on the inner race, and two points on the outer race. The bearing provides large thrust and radial load capabilities. Furthermore, the large diameter gives the bearing a large bending moment capability.

Another bearing type which can support radial loads, thrust loads, and bending moment loads at the same time is the crossed roller bearing. The crossed roller bearing utilizes a square raceway with rollers alternately crossed so that they resist up and down thrust loads and axial loads.

FIGURE 15.33 Model tower mounted on azimuth-over-elevation positioner. Crank on front of base operates linear actuator for positioning model over $\theta$ axis. See Figure 5.23.
Figures 15.35 through 15.38 show an azimuth-over-elevation positioner in a partial state of assembly to illustrate specific mechanical features which are typical of heavy duty antenna positioners. Figure 15.35 pictures the massive sector gears of the elevation axis that are driven by twin pinions. The twin pinions are driven from both sides by secondary spur gears that are nested inside the support housing. These gears are normally protected by covers, which have been removed on the positioner shown.

The dc motor which drives the elevation axis is partially visible. DC motors are almost universally utilized to provide the main driving power to the axes of antenna positioners because of the wide controllable speed range provided by the dc motor using a variable voltage motor controller. The photograph of Figure 15.36, which was taken from the opposite side of the positioner, more clearly shows the secondary spur gears and the driving pinions, which are rotated by the totally enclosed, oil bath type worm gear reducer that is normally a double reduction reducer for a positioner of this type.
The underside view of the azimuth housing is shown in Figure 15.37 and illustrates the compact arrangement of the azimuth reducer and motor. The azimuth reducer unit shown is a two-stage reducer which has a worm gear type output reduction stage and a helical spur input reduction stage. The small housing between the motor and reducer is the synchro transmitter assembly which provides the azimuth position information. A top view of the azimuth housing is shown in Figure 15.38. This view shows the flat, large diameter bearing with its integral, internal gear. The drive pinion is in the upper left corner and the gear which drives the synchro transmitter unit is in the center. The synchro drive gear is a spring loaded, antibacklash gear having a six to one ratio with respect to the azimuth turntable. Additional gearing, consisting of a 6:1 step up and a 6:1 step down that is encased in the synchro assembly housing (Figure 15.37), provides the 36:1 and the 1:1 ratio synchros for position information.

Nearly all of the section which is visible in Figure 15.38 is enclosed by the turntable, which is mounted to the holes that are visible in the inner race of the bearing. The white ring just outside the bearing OD is a flexible gasket which seals the bearing chamber from outside weather and contamination. When the turntable and the side covers are in place on this partially disassembled positioner, its external appearance is somewhat similar to that shown in Figure 15.30.
Source Positioners -- Directional source antennas are commonly used at the transmitter end of an antenna measurement range in order to obtain increased power at the test antenna station and to discriminate against reflections. Because of the directivity of the source antenna, it is desirable to provide a means for angular adjustment of the antenna beam maximum in order to center the illumination upon the test antenna. This is accomplished by squint adjustments in elevation and azimuth planes. Squint angle variation will normally be of the order of ±5 degrees for each axis, and the adjustment mechanism is commonly provided in a mounting fixture used to interface the antenna or polarization positioner to the source tower.

Squint adjustment may be manual or motor actuated. If motors are used, the control system will normally be an open loop dc type. Synchros are geared to the motor shaft for remote position indication. The squint mounting fixture may be installed directly to the source tower or to a carriage which provides vertical height adjustment capability. When motorized hoist devices are installed on source towers, they are generally actuated with an open loop control system and use synchro feedback for remote position indication.
The source tower is sometimes installed on a source cart to provide range length adjustment. The source cart may be equipped with either rubber wheels for movement along properly prepared surfaces, steel wheels for movement along properly prepared surfaces, or steel wheels for movement along tracks. The former provides a larger range of movement without producing the possibility of errors due to reflection from the tracks but at the cost of reduced rigidity. This is typically overcome by using jacks to partially remove the load from the rubber tires. Steel rails are often used for either relatively small changes in range length, or, as in an anechoic chamber, where absorptive material can readily be placed over the steel rails. The source tower may be guyed to prepared guy points along the length of the range or to the source cart if it is large enough in comparison with the size of the tower. Motorized source carts usually use open-loop control systems operated in a local mode only. This provides maximum personnel safety and eliminates the requirement for remote position indication.

Polarization Positioners -- Many antenna measurement requirements include testing in two or more polarization modes. Polarization rotation is obtained by mounting the source antenna on a polarization positioner. This unit usually has a reversible motor with an open loop control system. Limit switches are often used to limit antenna rotation to 90 degrees. When remote control is employed, indicators are usually employed on the remote unit panel, which illuminate when the limit switch is activated. Feedback devices such as synchros or encoders are used to provide continuous position indication.

Field Probes -- Field probes are special positioners designed primarily for antenna range evaluation. They are usually installed on the test antenna positioner and provide the capability of moving a small probe device across the expected aperture of antennas to be tested on the positioner. The field illumination provided by the source antenna is probed across the aperture usually in a horizontal and vertical plane. The use of the field probe in antenna range evaluations is described in Chapter 14. Care must be taken in the design of field probes to suppress residual variations of the aperture field due to reflection and diffraction of the illuminating wavefront from the probe structure.

Radome Boresight Positioners -- A radome measurement system detects and records certain electrical effects caused by placing a dielectric radome over the associated radar antenna to provide data for prediction of the operational performance of the antenna-radome configuration. Adequate simulation of the operational environment and compliance with typical specifications require the design and fabrication of high-
performance positioning and signal-processing equipment and a careful design of the test facility into which such equipment is to be integrated. Radome measurements are described in Chapter 12, and radome positioners are described in Chapter 5.

15.5.2 Positioner Control Units

Open Loop Controls -- Open loop control systems commonly consist of an on-off driver, a motor with a suitable gear reduction ratio, and limit switches at each end of the desired range of travel. The motor and driver may operate on alternating or direct current. Typically, the control is arranged so that the motor can run in either direction until a limit switch is encountered. After a limit is reached, the direction of drive may be reversed, and the motor will drive back away from the limit. Limit switches can usually be preset to accuracies of the order of one degree. Multiple point operation may be obtained by adding additional limit switches with overrides for each switch. Open loop control systems may also include some type of remote position indication from either synchros geared to the positioner shaft or encoders assembled as an integral part of the positioner axes, while continuous position control is provided manually by the operator. Accuracy is limited by readout accuracy, rate of movement, inertia and operator reaction time.

Open loop controls are commonly used for making continuous antenna patterns, for transmitter site polarization positioners, elevator controls for hoist equipped source towers, cart position controls for motorized carts, and similar applications where precise positioner controller is not required.

Alternating current systems may be as simple as a forward-off-reverse switch controlling the application of 115V primary power to a motor and phase shift network. These have no speed control and are usually geared to run very slowly in order to achieve reasonable position accuracy through the use of remote indicators. Direct current control units may use a variable auto-transformer (Figure 15.39) or silicon controlled rectifier (SCR) (Figure 15.40) to produce a smoothly varying rate of positioner movement. Speed ranges of 10 to 1 are common. If a wider range of speed control is desired, tachometer rate feedback may be utilized. Speed ranges of 50 to 1 are common and 500 to 1 can be achieved.
FIGURE 15.39 Open loop controlled system using variable auto-transformer control.

FIGURE 15.40 Rate loop with SCR power control.
Servo Control Systems — The rate loop is the simplest form of positioner servo control system. This type of system may use the minimum amount of loop compensation by using relays energized by a forward-off-reverse switch to reverse the direction of rotation of the positioner.

Position control may be added to a rate loop by using feedback from a tachometer geared to the positioner shaft (Figure 15.41). This rate signal is applied to a differential error amplifier where it is compared with the commanded speed voltage. The resulting error signal is amplified by power amplifiers and applied to the servo motor. The effect is to maintain a constant speed of the positioner throughout varying load conditions.

This type of position loop may be used with a bi-directional or a uni-directional system (in the latter forward and reverse relays are used to reverse the direction of drive). The latter requires that the servo loop be over damped in order to prevent forward and reverse relay chatter with hunting between the respective relay thresholds. This type of system is relatively slow in response time, but will achieve position accuracies of the order of ±0.01 degrees. The bi-directional system does not use reverse relays. It has a positive drive at all times with a much tighter control than the uni-directional system.

FIGURE 15.41 Rate or position loop with tachometer and synchro feedback. The position accuracy is about the same, but the bi-directional system has a much faster response.
Incremental Control Loops -- With the advent of digital control techniques the desirability of a digital motor became obvious. Stepper motors are constructed such that sequential variation of the polarity of voltage across segments of the control winding produces a step of shaft rotation. The amount of rotation is dependent on the number of poles and the coil arrangement. Any error produced by angular misalignment of coils is nonaccumulative. The power capability of stepper motors is low, restricting their use as direct drives to very small positioners or very slow rotation for larger positioners. These motors lend themselves extremely well to digital programming techniques. The use of set-reset pulse generators and digital counters allows the application of a known number of pulses to the motor driver circuit to give a known number of degrees of rotation. Reversal of direction of rotation is accomplished by applying drive pulses to either a forward or reverse line to the motor driver. The motor's holding torque between steps is equal to the drive torque so that no braking action external to the motor is required to maintain a particular position. The major limitations of this control technique are lack of power and the possibility of resonant vibration caused by starting and stopping the system with each pulse. Accuracies are limited by the gear train compliance and backlash. Accuracies of the order of ±0.05 degrees have been obtained.

Digital control techniques may be obtained for higher power requirements by gearing the stepper motor shaft to the control transformer shaft in the synchro feedback servo system discussed above. With this combination of techniques, very large positioners can be controlled to accuracies of the order of ±0.05 degrees or better. Both rate and position modes can be obtained by supplying an analog signal and switching between the demodulator and analog signals.

15.5.3 Position Indicators -- Digital readout of synchro position transducers can be obtained by instruments such as the one shown in Figure 15.42. This multi-channel instrument provides angular display for synchro transducers as well as providing range, display absolute, and position offset functions. The displayed angular information is also used for input to the chart portion of a pattern recorder.
Digital encoders have been used for many years to provide very accurate readout of antenna position. These encoders are generally binary encoders in which a circle is divided into $2^n$ parts, with $n$ referred to as the number of bits. Encoders with resolution of 20 bits or greater and accuracies of a few arc seconds or better are in use. The resolution of a binary encoder in terms of the number of bits is given in Table II.

### Table II
Resolution of Binary Encoders

<table>
<thead>
<tr>
<th>No. of Bits (&lt;n&gt;)</th>
<th>Divisions (&lt;2^n&gt;)</th>
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<th>Minutes</th>
<th>Seconds</th>
<th>Seconds</th>
<th>Radians</th>
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<td>1,296,000</td>
<td>6.28318</td>
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<td>19.78</td>
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<td>0.0824</td>
<td>4.94</td>
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<td></td>
</tr>
<tr>
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<td>0.000687</td>
<td>0.0412</td>
<td>2.47</td>
<td>0.000012</td>
<td></td>
</tr>
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<td>0.0206</td>
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<td>0.000006</td>
<td></td>
</tr>
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<td>0.0052</td>
<td>0.31</td>
<td>0.0000015</td>
<td></td>
</tr>
</tbody>
</table>
Decimal encoders which measure directly in degrees are also in use.

Precision digital encoders are coupled directly to the rotating member of the antenna positioner. This method of coupling avoids gear errors. Where less accuracy suffices, a relatively inexpensive encoder can be geared to the positioner. The resolution can be increased by gearing the encoder so that it makes several revolutions per revolution of the positioner.

Synchros are the most commonly used devices in the measurement of position on antenna test ranges. Appropriately geared synchro packages, in combination with synchro indicator units, provide readout of linear travel or positioner angle. Dual-speed synchro systems are generally used to improve readout accuracy.

Advancements in digital techniques have enhanced the advantages of synchro-to-digital converters \(^{10}\) over digital encoders. Synchro converters provide the easiest retrofit for existing systems since suitable synchro sources are usually already available and because of the simplicity of the interface. Synchro converters require fewer lines connecting to the positioner. The cable lengths can be longer than for digital encoders unless amplifiers and drivers are included in or near the encoder.

Multi-speed resolvers can also be used for indicating positioner rotation. Multi-speed resolvers are multi-pole devices that effectively provide a geared-up resolver output, at a ratio of 16:1 or 36:1 for example, but do not contain gearing. Thus, these resolvers can be mounted directly to the rotating member of the antenna positioner in the same manner that digital encoders are mounted. Accuracy of a few arc seconds can be achieved with multi-pole resolvers. A single-speed resolver can be included to resolve the ambiguity of the multi-speed unit.
The digital position indicator shown in Figure 15.43 was developed specifically for the antenna range application. This multi-channel unit is programmable via the IEEE-488 interface and is compatible with 1:1, 36:1, and encoder transducers commonly employed in antenna systems. The unit includes visual position indication as well as controls and digital outputs for recording and positioner control purposes. The display range can be either 0 to 360 degrees or ±180 degrees (angular), or 0 to 36 or ±18 (linear). A programmable offset is included so that the display angle may be easily referenced to the coordinate system of the antenna under test.

15.6 ANTENNA PATTERN RECORDERS

15.6.1 General Types -- Antenna pattern recorders are employed to plot signal strength (amplitude) and relative phase angle as a function of angular position of the antenna under test. The resulting plots graphically represent the beam pattern, gain, and/or polarization characteristics of the antenna. The signal plotted may be either an analog signal coupled from the output of a diode or detector, a superheterodyne receiver, or it may be a digital signal (amplitude or phase) coupled from the output of a signal processor.
which is part of a receiving system. Chart position information fed to the recorder may be either analog coupled from a synchro transmitter attached to the antenna positioner, coupled digitally from an encoder attached to the antenna positioner, or a digital signal from a system controller.

Typical antenna pattern recorders are electro-mechanical devices employing servo systems to drive both the pen and the chart axes. There are three basic pattern recorder configurations: (1) the rectangular coordinate antenna pattern recorder; (2) the polar coordinate antenna pattern recorder; and (3) the dual-coordinate system (polar or rectangular) antenna pattern recorder.

The rectangular coordinate system consists of a Y-axis (ordinate) driven by the signal input and an X-axis (abscissa) driven by the chart system angular input. This type of system is commonly referred to as a strip chart recorder. The polar coordinate chart system consists of a circular chart with the signal amplitude represented by the distance of the plot from the chart center and positioner angle represented by angular chart position. Dual-coordinate system recorders are recorders which can plot either in the rectangular or polar coordinate systems. These recorders typically use a rectangular type strip-chart system when making rectangular plots and employs an X-Y positioning system to provide the polar plot. When the X-Y positioning system is used, the polar coordinates of the inputs (signal data and chart position data) are electronically or digitally converted to rectangular coordinates and a polar plot is generated by the combined action of the pen system and chart system.

The rectangular coordinate system is generally preferred when plotting the characteristics of narrow-beamwidth antennas. Polar coordinates are often preferred for plotting patterns of antennas that are not highly directional. The polar coordinate format is useful for visualizing the power distribution in space since position angles are measured directly as angles on the polar plot. A typical rectangular plot is shown by Figure 15.44A and a typical polar plot is shown by Figure 15.44B. Figure 15.45 shows a modern dual-coordinate system antenna pattern recorder capable of plotting in the rectangular or polar coordinates, analog or digital input data.
FIGURE 15.44 Typical antenna patterns

FIGURE 15.45 Antenna pattern recorder capable of plotting in polar and rectangular coordinate systems
15.6.2 Chart Systems -- The rectangular coordinate system employs the chart position of a strip-chart recorder as the X-axis (abscissa). This coordinate system permits narrow-beam antenna patterns to be plotted in finer detail than possible with a polar plot because the pattern does not become crowded in regions of low signal levels. To provide adequate resolution in a rectangular display of patterns of different beamwidths, selectable chart scales are generally provided. In a servo-driven chart system, the chart scale may be expanded by deriving the angular information from a synchro transmitter geared to the positioner so that the synchro shaft rotates a number of times for one revolution of the positioner. A common ratio employed is 36 to 1 resulting in one synchro revolution for each 10 degree of positioner rotation. This results in a 10 degree chart scale. Digital chart systems driven from an encoder attached to the positioner or from a synchro-to-digital converter provide chart scale expansion by digital programming. Common rectangular chart scales employed on modern digital antenna pattern recorders are 9, 18, 90 and 360 degrees per chart cycle. Many older rectangular antenna pattern recorders use chart scales of 10, 60, and 360 degrees.

A block diagram of the chart system of a modern dual-coordinate system antenna pattern recorder is shown by Figure 15.46. The chart drive motor drives both the chart and the chart position encoder. Position and rate information of the chart system that is derived from the encoder in digital form are processed by the microprocessor (CPU). The information is then converted to analog form and supplied to the servo amplifier to complete the servo loop. The system has the capability of interfacing with either an analog synchro input or a digital encoder input. In practice, only the desired input interface is supplied in a pattern recorder. The common system bus interfaces both the chart system digital circuitry and the pen system digital circuitry. A chart system such as the one shown by Figure 15.44 is capable of chart speeds up to 30 inches-per-second.

15.6.3 Pen Systems -- Most antenna pattern recorders provide a selection of amplitude functions. This is because the recorder pen system input can be derived from various signal detection systems; some which supply square-law outputs and some which supply linear outputs. Most recorders provide a variety of pen function characteristics such as linear, logarithmic, square and square-root. These choices make it possible to plot field
FIGURE 15.46 Block diagram of a microprocessor-controlled antenna pattern recorder chart system

strength (linear plot) from a square-law detector output utilizing the "square-root" pen function. Also, power may be plotted from a linear input function by using the "square" pen function, and various logarithmic responses can be plotted from either a square-law or linear signal input. Typical logarithmic ranges provided are: 10 dB, 20 dB or 40 dB when the pen system input signal has a square-law characteristic and 20, 40 or 80 dB when the input signal has a linear characteristic.

Recorder chart scales are available for linear, square-law, logarithmic, and related polar plots. Chart scales linearly calibrated in decibels are particularly useful since antenna gains and pattern levels are generally specified in decibels. The decibel scale provides constant resolution over the entire display range and allows a wider dynamic range to be displayed than with other formats. Comparison of patterns is easier with a decibel scale since a difference in system gain is equivalent to a constant offset of the recorded data. Recording dynamic ranges vary from 40 dB for a square-law detector to 80 dB for a linear detection system. In both cases, the antenna pattern recorder pen system must respond to an 80 dB range of input voltage.
A narrow-band crystal bolometer amplifier accessory available for most pattern
recorders permits operation from a detector receiving a square-wave modulated
carrier. The crystal/bolometer amplifier is a low-noise narrow-band amplifier capable of
operation from the output of a diode detector or from a bolometer detector. When
operating with a bolometer detector, the bolometer bias current is supplied by the
crystal/bolometer amplifier. Most recorders can also be configured to operate from a dc
(non-modulated) input. This is accomplished with an accessory dc amplifier which
modulates the dc input at the desired rate (typically 1,000 Hz) and passes the modulated
signal through a 1,000 Hz narrow-band amplifier.

A linear pen response may be obtained with a conventional linear servo system. The
logarithmic response may be obtained by electronically performing the logarithmic
operation on the output of the crystal/bolometer amplifier or dc amplifier to obtain an
output signal directly proportional to the input signal in decibels. This output is fed to a
conventional linear position servo system. The logarithmic servo had been the principal
means of providing logarithmic conversion in pattern recorders employing analog (non-
digital) design techniques. The ac signal (typically 1,000 Hz) is coupled from the output
of the crystal/bolometer amplifier or dc amplifier to a precision potentiometer whose
attenuation in decibels is proportional to the potentiometer shaft rotation, and the shaft
is driven by the servo system as shown by Figure 15.47. The output of the potentiometer
is given by:

\[ e_o = e_i 10^{-kr} \]  (10)

where \( r \) is the angle of shaft rotation. Taking the logarithm of both sides gives:

\[ kr = \log \frac{e_i}{e_o} \]  (11)
The constant $k$ is determined by the maximum angle of rotation of the potentiometer shaft and by the dynamic range to be recorded. It seems from Figure 15.47 that the servo system will operate to maintain a constant dc level at the rectifier output. This requires $e_o$ to be held constant and the pen displacement becomes a function of the signal input ($e_i$) expressed in decibels. This method of conversion has the advantage that the recorder accuracy is primarily dependent on the accuracy of the potentiometer employed in the servo loop. This technique is generally limited to uses with input signal frequencies of 2,500 Hz or less because, at higher frequencies cross-coupling within the potentiometer will degrade the system accuracy.

A block diagram of the pen system of a digital dual-coordinate antenna pattern recorder is shown by Figure 15.48. The pen drive motor drives the recording pen and the pen position encoder. Position and rate information are derived from the encoder in digital form, processed by the microprocessor (CPU), converted to analog form, and supplied to the servo amplifier to complete the servo loop. The pen system has the capability of interfacing with either an analog or digital signal input. The analog input typically is either dc or 1,000 Hz. The typical digital input is compatible with byte-serial or parallel BCD signals. The common system bus interfaces with all pen system digital circuits as
well as chart system digital circuits. Recording pen speeds as high as 60 inches-per-second are practical with a properly designed pen drive system. The recording pen is generally a lightweight pen featuring a felt tip.

The microprocessor-controlled pen drive system is usually programmed to provide necessary pen functions such as linear, square, square-root, and the desired logarithmic scales. The non-linear pen functions are commonly generated by relating discrete signal levels to addresses in a look-up table stored in permanent memory. The desired function is generated by substituting data stored in the look-up table for the input data. This procedure can be employed to generate any desired pen function. In practice, a look-up table includes only a limited number of data points. When the input data is at a level which requires an output between points available in the look-up table the microprocessor interpolates the output data.

A part of the pen system shown by Figure 15.48 is the 60 dB option. The 60-dB option supplies an output voltage that varies linearly with recorder pen position. When the recorder is operated with a single-channel microwave receiver having a square-law output characteristic, the operating dynamic range of the receiver/recorder system can be increased from 40 dB to 60 dB by applying the voltage available at the 60 dB recorder

![Block diagram of a microprocessor-controlled antenna pattern recorder pen system](image)

**FIGURE 15.48** Block diagram of a microprocessor-controlled antenna pattern recorder pen system
The IEEE-488 Standard Bus interface is generally designed to be a recorder option. When IEEE-488 is included, complete remote programming of all recorder front panel control functions as well as measurement and position data can be supplied through the standard bus interface.

15.6.4 Antenna Pattern Recorder Systems Interface -- The recorder-to-data acquisition interface and resultant pattern characteristics which may be plotted are directly related to the type of data acquisition system involved. A block diagram of an antenna pattern recorder operating on-line with a direct detection (diode or bolometer detector) receiving system is shown by Figure 15.49. The recorder chart system is driven directly from the positioner synchro transmitter. An accessory crystal-bolometer amplifier is included as part of the recorder to interface with the diode or bolometer detector. Since a direct detector delivers a square-law output, a "linear" recorder pen function will plot received signal power. This plot is usable over a dynamic range of approximately 20 dB. A "square-root" pen function provides a plot of received signal strength usable over a dynamic range of approximately 40 dB. A "logarithmic" pen function provides a logarithmic plot with a maximum dynamic range of 40 dB.

A block diagram of an antenna pattern recorder operating on-line with a single-channel microwave receiver with a square-law 1,000 Hz output is shown by Figure 15.50. The recorder chart system input is coupled directly from the synchro transmitter located in the test positioner. The system includes a crystal-bolometer amplifier to interface with the bolometer detector in the receiver. The square-law receiver output results in a recorder plot of received power usable over an approximate 20 dB dynamic range if a "linear" recorder pen function is selected. A "square-root" pen function provides a plot of signal strength usable over a 40 dB dynamic range. A "logarithmic" recorder pen function will provide a maximum logarithmic dynamic range of 40 dB. A 60 dB logarithmic recording dynamic range is available on many recorder-receiver combinations.
FIGURE 15.49 Block diagram of an antenna pattern recorder operating with a direct detection system.

FIGURE 15.50 Block diagram of an antenna pattern recorder operating with a single-channel microwave receiver.
A block diagram of an antenna pattern recorder operating on-line with a phase-lock microwave receiver is shown by Figure 15.51. The recorder chart system input is coupled directly from synchro transmitters located in the test positioner. The microwave receiver provides a byte-serial output signal which represents either amplitude or phase data. Phase data is plotted on the recorder using a linear pen function. The receiver amplitude output data supplied to the recorder is linear. This results in a recorder plot of received field strength usable over approximately 40 dB of dynamic range if the "linear" pen function is used. A plot of received power usable over approximately 20 dB of dynamic range results if the "square" pen function is used. A total usable dynamic range of 80 dB is available when the maximum dynamic range "logarithmic" pen function is selected.

A block diagram of an antenna pattern recorder operating off-line with a programmable antenna test system interfaced through the IEEE-488 Standard Bus is shown by Figure 15.52. The controller directs the positioner, signal source, receiver, and the antenna pattern recorder through the IEEE-488 Standard Bus. Such a system can acquire multi-channels of phase and amplitude data at data rates far exceeding the recording rate of the antenna pattern recorder. To permit recording of selected data, the data is retained in a mass storage device and upon command is supplied to the off-line antenna pattern recorder at an acceptable data rate. The digital output of the programmable receiver is byte-serial and has a linear characteristic. This results in a plot of received field strength usable over approximately a 40 dB dynamic range if a "linear" recorder pen function is selected. A "square" pen function will provide a plot of received power usable over an approximate 20 dB dynamic range. The maximum logarithmic dynamic range available using the maximum range "logarithmic" pen function is 80 dB.
FIGURE 15.51 Block diagram of an antenna pattern recorder operating with a phase-locked microwave receiver

FIGURE 15.52 Block diagram of an antenna pattern recorder operating off line from stored data provided by a programmable antenna test system
15.7 AUTOMATIC INSTRUMENTATION SYSTEMS FOR ANTENNA MEASUREMENTS

15.7.1 General -- Automated instrumentation systems for antenna measurements provide distinct advantages over conventional manually operated measurement systems. With the large number of instruments available with compatible digital interfaces, an automated system may be easily configured from basic system components. Other complete test systems are available that satisfy specific antenna measurement requirements. Automated instrumentation configured with a system controller performs antenna measurements automatically with minimum operator interaction and with a high degree of repeatability, speed, accuracy, and efficiency. Typical automated systems include the capability for storing data and therefore, provide a convenient means for documenting test procedures and results. Standard bus interfaces such as the IEEE-488 and RS232C/449 provide adequate system interfaces for performing automated tests and for expanding system capabilities with a minimum amount of modification to the existing system software. A typical automated system configured for antenna measurements provides control for the transmitting, receiving, and recording subsystems.

A system processor with its associated software and peripherals serves as the controller in an automatic antenna measurement system. As a minimum, the system should be capable of acquiring and analyzing data as well as formatting the data for output. Figure 15.53 is a functional block diagram of a typical automatic antenna measurement system.

Software for automatic antenna measurements is designed to reduce operator interaction with the controller and allows the subsystems to perform their respective functions. Figure 15.54 shows the software architecture of a typical automated antenna measurement system.
FIGURE 15.53 Functional block diagram of a typical automated antenna measurement system

FIGURE 15.54 Automatic antenna measurement system software architecture
15.7.2 *Data Acquisition* -- Data acquisition in an automatic antenna measurement system typically utilizes operator selectable test routines. The acquired data are stored in uniquely identifiable files where they can be processed for analysis and display. Typical data storage devices include magnetic tape, hard disc, and floppy disc units. The use of removable storage devices provides a convenient means of storing data for comparison and archival purposes.

Preprogrammed data acquisition routines for operator selection may include raster scan, beam maximum search, boresight shift, principal plane cuts, polarization pattern, near-field scanning, and gain measurements. The system should provide for the generation and storage of other acquisition routines for specific applications using the system controller and available test file storage devices. An automatic antenna measurement system should provide an efficient means of performing antenna testing with the use of preprogrammed test routines while maintaining flexibility in testing with operator generated routines.

Automatic antenna measurement systems usually require test parameters to be input to the system controller by the system operator. These parameters are used to generate test file data which control the subsystems for execution of the data acquisition routines.

Parameters such as frequency and output level are programmed into the signal source subsystem. The receiving subsystem requires inputs that may include, IF bandwidth, tuning frequency, and mode of operation. Positioner subsystem parameters such as positioner coordinates, axes of motion, step/scan angles, record increments, and positioner speed are programmed into the system positioner so that the antenna under test (AUT) is positioned properly throughout the test. When the data acquisition routine is initiated, the system controller will establish communications with the subsystems, initiate default routines upon receipt of any errors or discrepancies based on the subsystem input parameters, execute acquisition routines, create test data files, and, after completion of the test, return control of the system to the operator.

15.7.3 *Data Output* -- The acquisition routines are used to produce data acquisition files that can be readily formatted for output. After the system controller has acquired and stored the test data, programmed data analysis and plotting routines can be run. As an example, planar pattern analysis can be executed to yield amplitude and phase data for both co-polarized and cross-polarized antenna patterns. Gain and directivity analysis
routines can produce partial gain, total gain, and peak directivity data. Boresight
analysis routines can produce boresight shift angle data. In addition, analysis routines for
monopulse antennas can be used to produce sum and difference data. Polarization
analysis routines produce data from which axial ratio and tilt angle can be evaluated.
Other routines may transform near-field data to far-field or far-field data to near-field.

Depending upon the peripherals interfaced to the system controller, various types of data
display routines can be executed to present the stored measurement data. Rectangular
pattern plots can be used to display amplitude, polarization, and phase data while polar
plots can be used to display amplitude data. Other output format routines such as
contour patterns, three-dimensional patterns, radiation distribution plots, gain analysis
plots, or raw listings of any data file can be chosen.

There are a multitude of features that a computer in an automatic antenna measurement
system can offer. The computer provides an efficient, versatile, and creative approach
to antenna measurement techniques. The addition of a computer in the antenna
measurement system provides the system with the necessary capabilities to
automatically acquire, analyze, and output the measurement data. The computer
processes the data for analysis and output in an efficient manner with a high degree of
speed, precision, and repeatability. The computer also features software for the system
operator that will allow him to tailor his testing requirements. Such features include
pattern overlays for comparison, 3-D and contour plots, and other complex antenna
patterns. The operator selected plots can be viewed on a monitor and processed or
modified until the desired presentation is achieved, then the presentation can be copied
to a plotter for permanent documentation.

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CONVERSION OF VOLTAGE AND POWER RATIOS TO DECIBELS

The basic chart below indicates the number of decibels (dB) ratio of 2,000:1 to dB, express 2,000 as 2 x 10^3; the number of those included in the chart, the ratio can be broken down into a ratio of 2,000:1 is approximately 30 dB + 3 dB = 33 dB. In the lower righthand corner of the chart dB values for voltage and power ratios of integral powers of 10 are given.

### Table: Conversion of Voltage and Power Ratios to Decibels

<table>
<thead>
<tr>
<th>Voltage Ratio</th>
<th>Power Ratio</th>
<th>(-\text{dB})</th>
<th>Voltage Ratio</th>
<th>Power Ratio</th>
<th>(-\text{dB})</th>
<th>Voltage Ratio</th>
<th>Power Ratio</th>
<th>(-\text{dB})</th>
<th>Voltage Ratio</th>
<th>Power Ratio</th>
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<td>1,000</td>
<td>1,000</td>
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<td>5,012</td>
<td>-100</td>
<td>4,467</td>
<td>19,550</td>
<td>4,475</td>
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<td>0.007</td>
<td>0.499</td>
<td>0.507</td>
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<td>4,467</td>
<td>19,550</td>
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<tr>
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<td>0.020</td>
<td>0.264</td>
<td>0.285</td>
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<td>4,467</td>
<td>19,550</td>
<td>4,475</td>
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<td>4,475</td>
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<td>4,475</td>
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<td>5,012</td>
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<td>4,467</td>
<td>19,550</td>
<td>4,475</td>
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<tr>
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<td>4,467</td>
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<td>4,467</td>
<td>19,550</td>
<td>4,475</td>
</tr>
</tbody>
</table>

Note: The table continues with similar entries for different voltage and power ratios.
NOMOGRAPH OF SPACE ATTENUATION AS A FUNCTION OF RANGE

If the nomograph does not cover the desired range of \( \lambda \) and \( R \), multiply both scales by \( n \), and multiply \( N \) scale by unity.

SOLUTION OF: \[ N = 20 \log \left( \frac{4\pi R}{\lambda} \right) \]