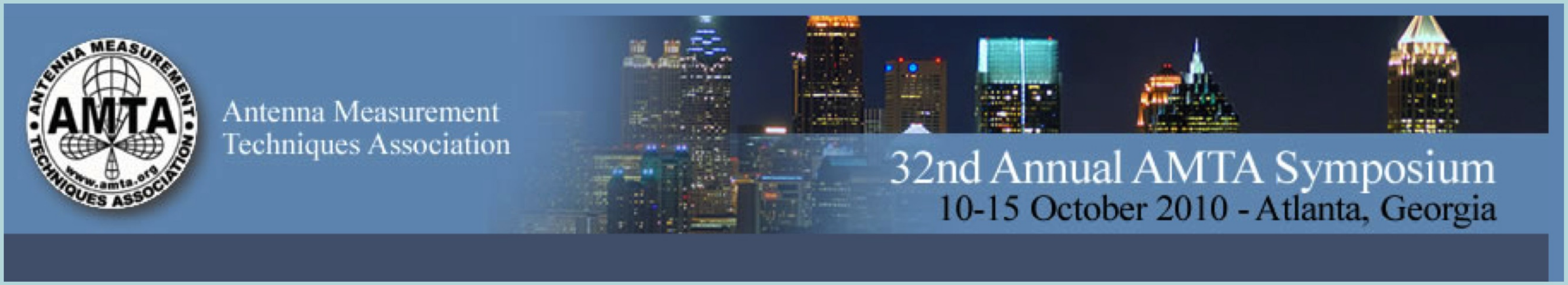
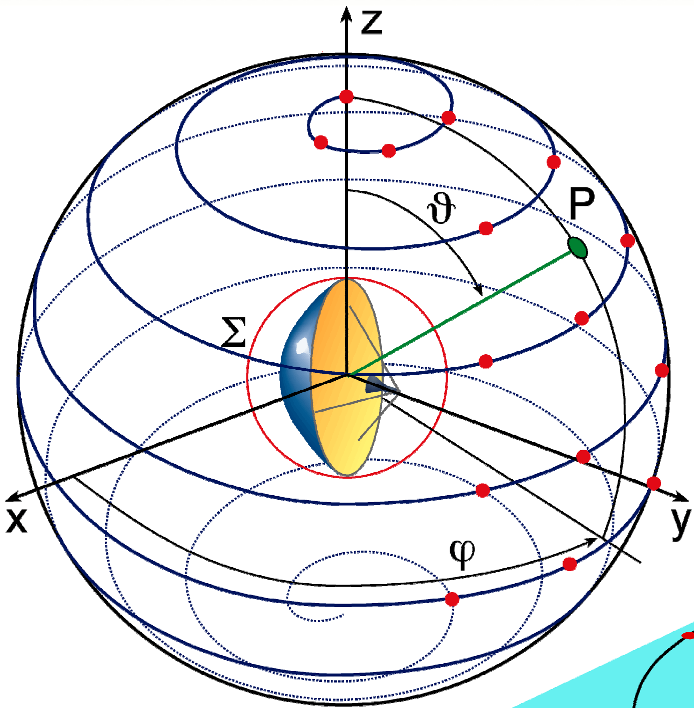


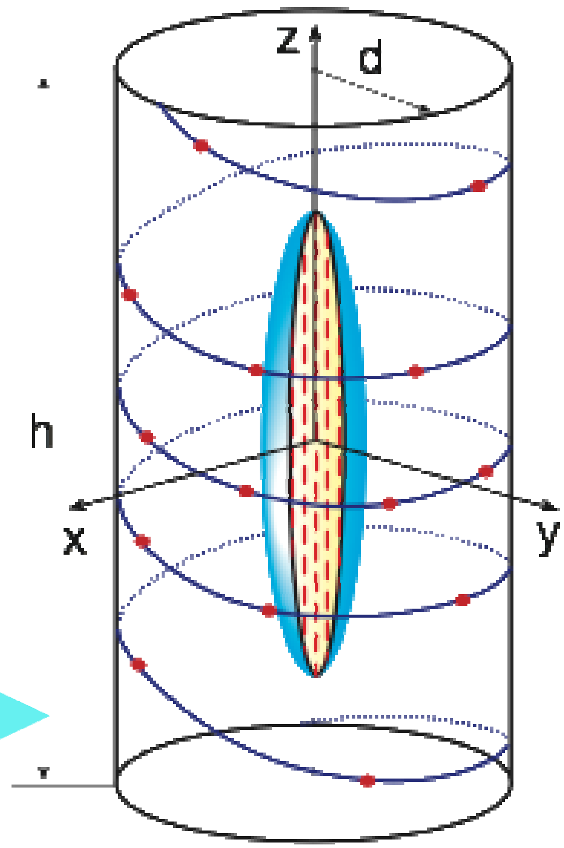
LABORATORY RESULTS ON THE COMPENSATION OF PROBE POSITIONING ERRORS IN THE NF – FF TRANSFORMATION WITH HELICOIDAL SCAN

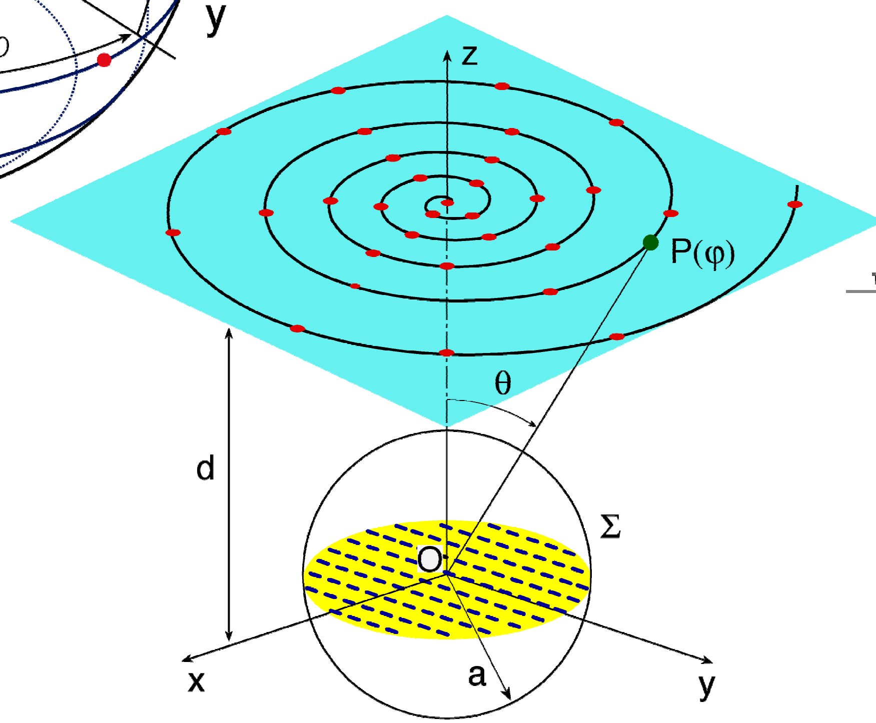
F. D'Agostino, F. Ferrara, C. Gennarelli, R. Guerriero, M. Migliozzi
University of Salerno, Italy

Jeff Fordham
MI Technologies

THE SPIRAL SCANS





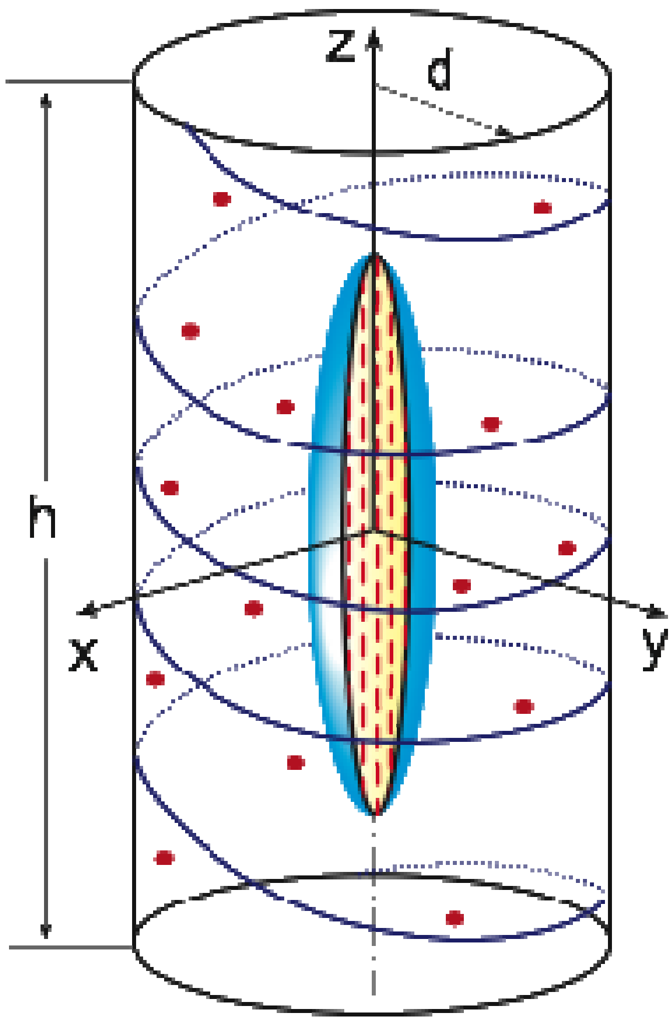


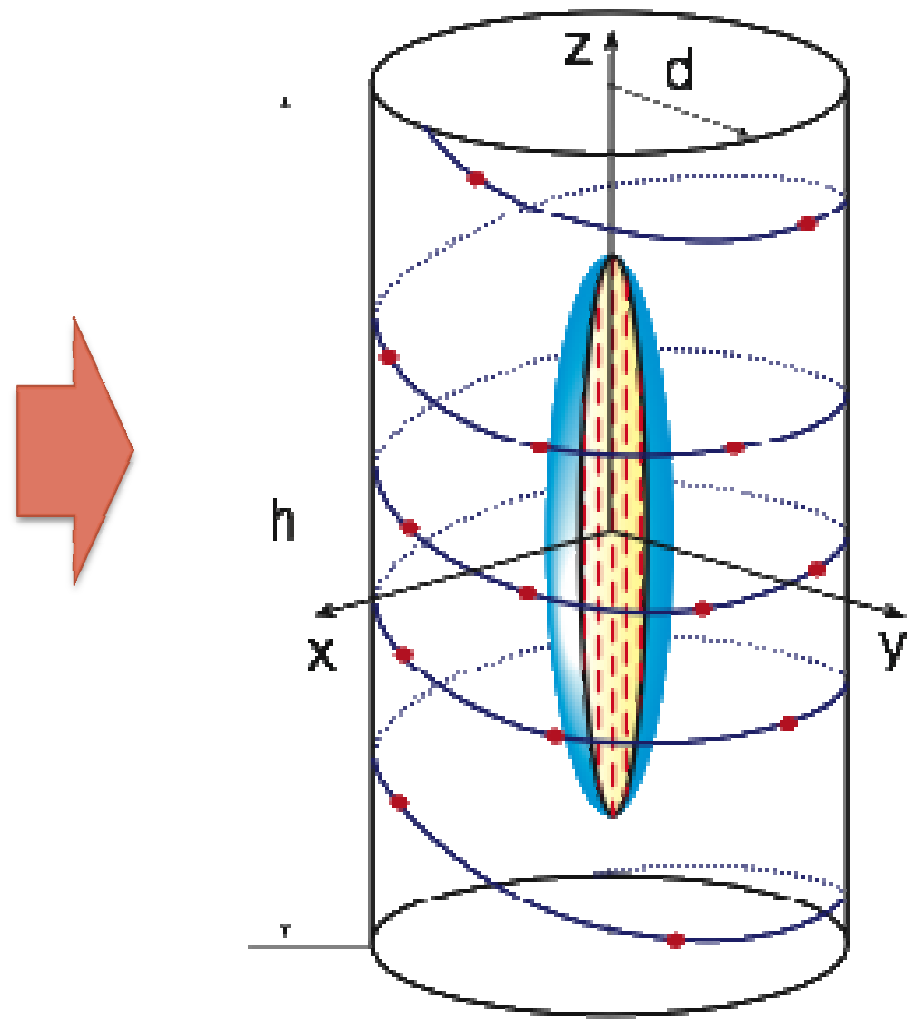
DATA NEEDED

Technique		NF data for the considered AUT
Classical approach [J.E. Hansen, 1998]		130562 (257)
Spiral scan	spherical AUT modelling	57550 (210)
	rounded cylinder AUT modelling	14162 (152)

"A NONREDUNDANT NF–FF TRANSFORMATION WITH SPHERICAL SPIRAL SCANNING USING A FLEXIBLE AUT MODEL ", F.D'Agostino, et. al., AMTA 2009

THE PROBLEM





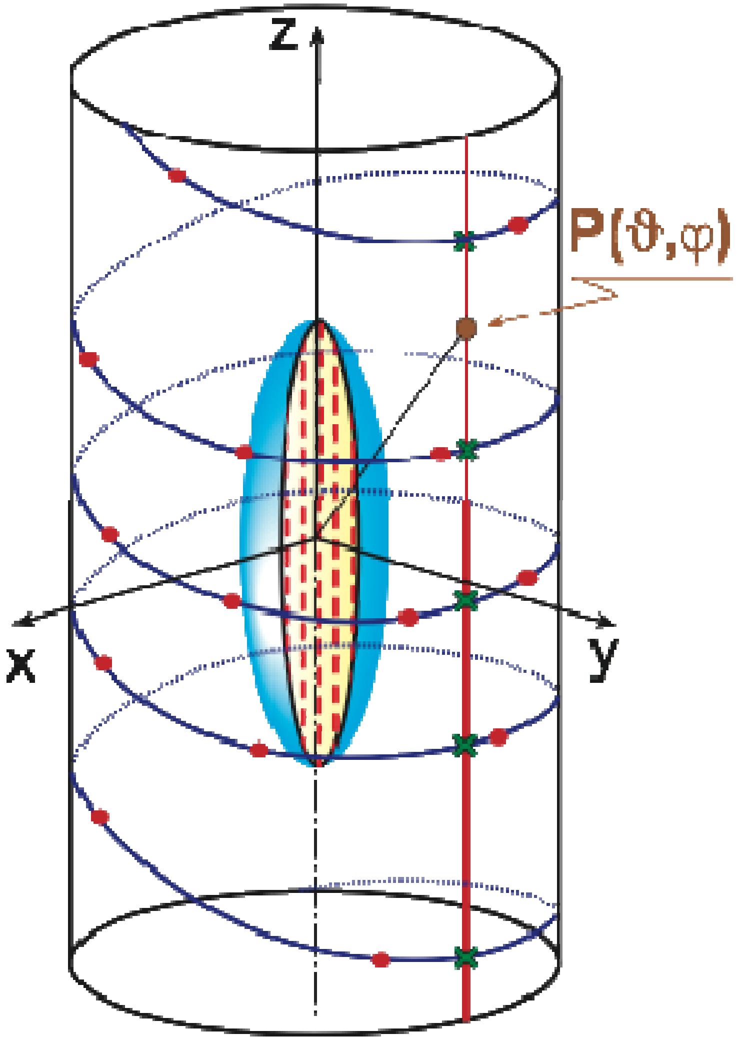
THE REDUCED FIELD

If the AUT is enclosed in a convex domain with rotational symmetry and any observation curve is described by a proper analytical parameterization $\underline{r} = \underline{r}(\eta)$, the **REDUCED VOLTAGE** can be approximated by a spatially bandlimited function, *[O.M.Bucci, G.Franceschetti, 1987]*

$$\tilde{V}(\eta) = V(\eta)\, e^{j\psi(\eta)}$$

$\psi(\eta)$ optimal phase function to be determined

$\underline{r} = \underline{r}(\eta)$ parameterization to be determined



To develop the voltage representation on the cylinder from a nonredundant number of its samples collected on a proper helix, it is necessary:

a) to determine the helix having the step such to intersect any generatrix at points having the sample spacing needed for the interpolation.

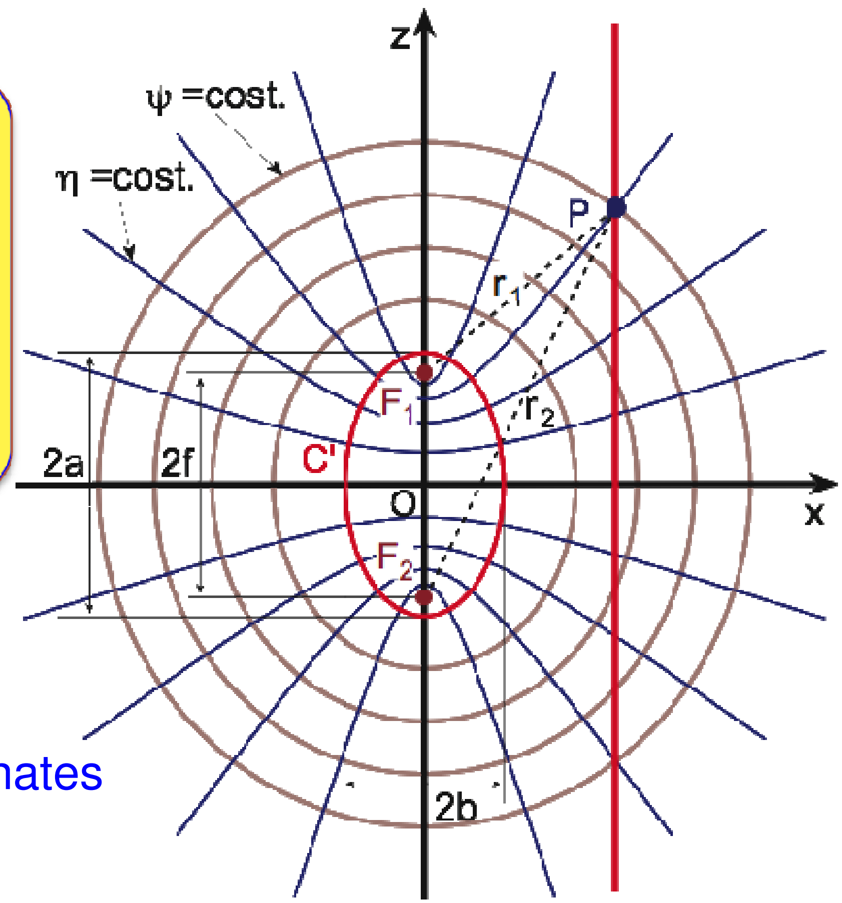
b) to determine a nonredundant representation along the helix;

THE HELICOIDAL SCAN (...)

The nonredundant representation along a generatrix when considering an ellipsoidal source modelling:

$$\eta = (\pi/2)\left[1 + E\left(\sin^{-1}u \,\middle|\, \varepsilon^2\right) / E\left(\pi/2 \,\middle|\, \varepsilon^2\right)\right]$$

$$\psi = \beta a\left[v\sqrt{\frac{v^2-1}{v^2-\varepsilon^2}} - E\left(\cos^{-1}\sqrt{\frac{1-\varepsilon^2}{v^2-\varepsilon^2}} \,\middle|\, \varepsilon^2\right)\right]$$



β = wavenumber

$\varepsilon = f/a$ = eccentricity of C’

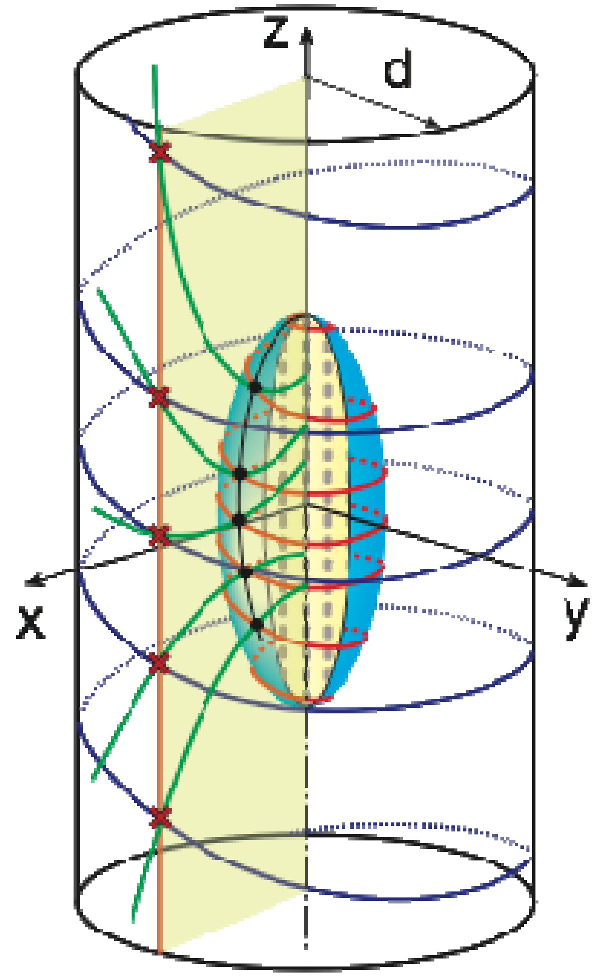
$u = (r_1 - r_2)/2f$ and $v = (r_1 + r_2)/2a$ = elliptic coordinates

$E(\bullet \mid \bullet)$ = elliptic integral of second kind

THE HELIX

The helix is obtained by projecting, along the curves at η = const., on the scanning cylinder a proper spiral wrapping the prolate ellipsoid and is defined by

$$\begin{cases} x = d\cos(\phi - \phi_s) \\ y = d\sin(\phi - \phi_s) \\ x = d\cot[\theta(\eta)] \end{cases}$$



ϕ = angular parameter describing the helix,

ϕ_s = ϕ value in a fixed point of the generatrix at φ=0

$\eta = k\phi$

where k determines the step.

THE HELIX (...)

The angular step of the helix is fixed equal to the sample spacing needed to interpolate the reduced voltage along a meridian:

$$\Delta\eta = \frac{2\pi}{2N''+1}$$

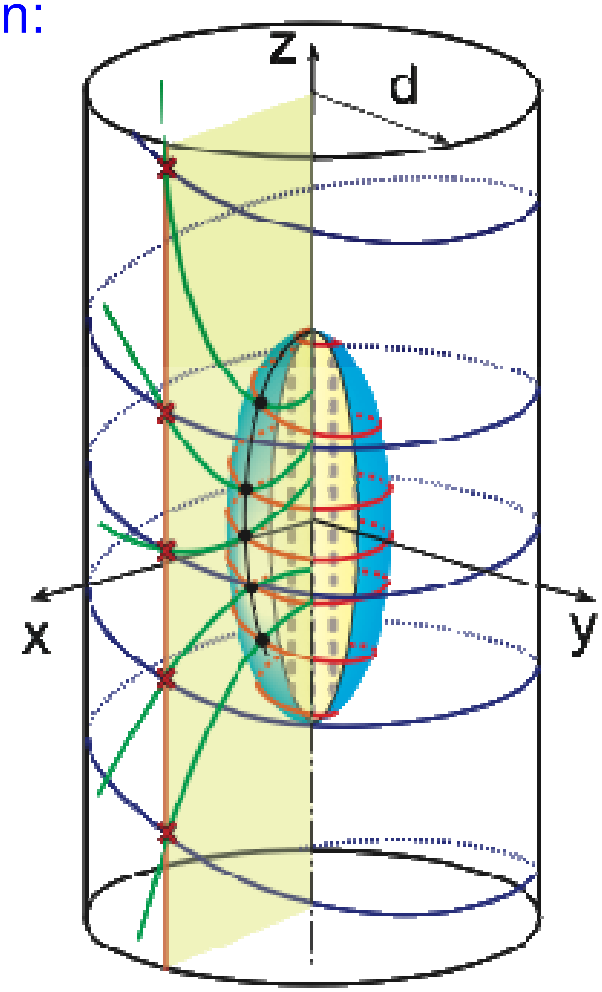
$$N'' = \mathrm{Int}(\chi N') + 1 \qquad N' = \mathrm{Int}(\chi' W_\eta) + 1$$

$\chi > 1$ is an oversampling factor;

$\chi' > 1$ is an enlargement bandwidth factor.

Being $\Delta\eta = 2\pi k$, it follows that:

$$k = \frac{1}{2N''+1}$$



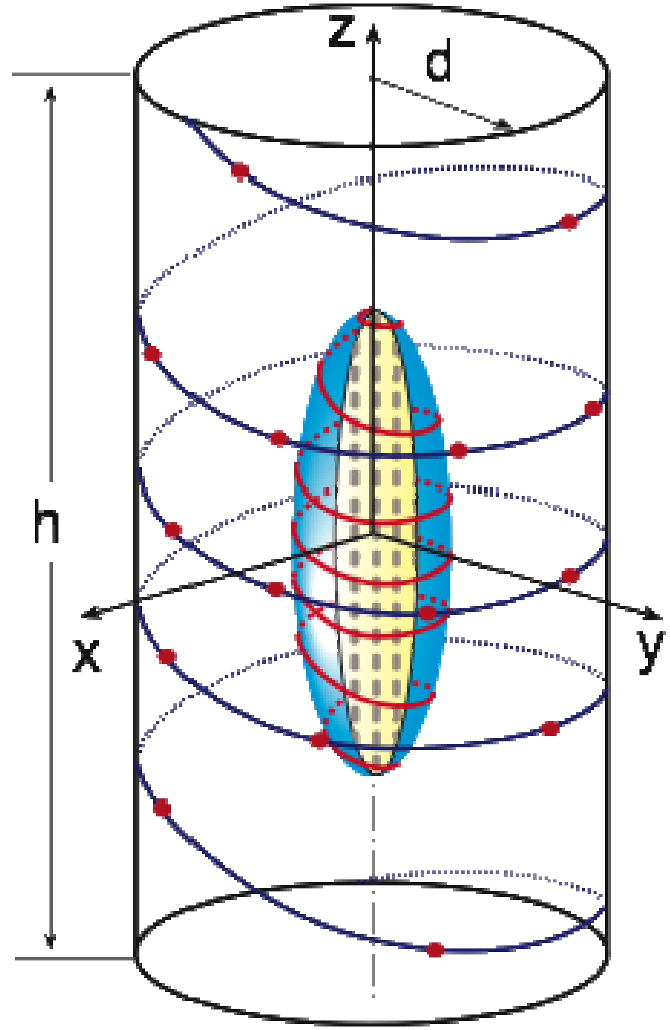
NONREDUNDANT SAMPLING REPRESENTATION

A nonredundant representation of the voltage along the helix can be obtained by choosing:

$$\gamma = \psi = \beta a\left[v\sqrt{\frac{v^2-1}{v^2-\varepsilon^2}} - E\left(\cos^{-1}\sqrt{\frac{1-\varepsilon^2}{v^2-\varepsilon^2}} \,\Big|\, \varepsilon^2 \right) \right]$$

$\xi = \beta / W_\xi$ times the arclength of the projecting point on the spiral wrapping the prolate ellipsoid.

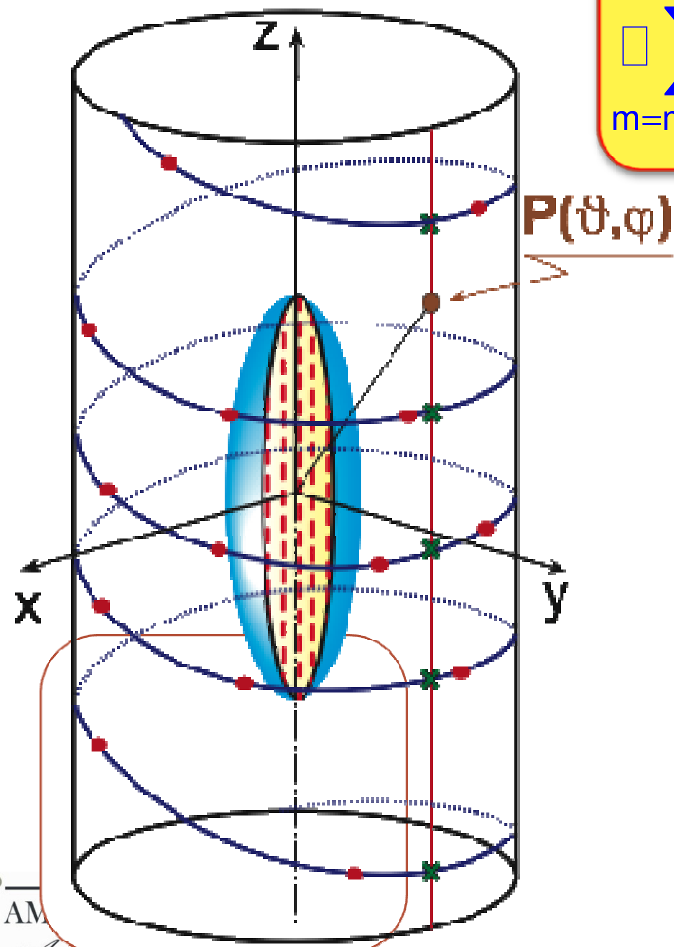
The bandwidth W_ξ is chosen equal to β/π times the length of the spiral wrapping the ellipsoid from pole to pole.



THE INTERPOLATION SCHEME

$$\tilde{V}(\vartheta,\varphi) = \sum_{n=n_0-q+1}^{n_0+q} \Big[\Omega_N\big(\eta(\vartheta)-\eta_n\big)\, D_{N''}\big(\eta(\vartheta)-\eta_n\big)\, \square$$

$$\square \sum_{m=m_0-p+1}^{m_0+p} \tilde{V}(\xi_m)\, \Omega_M\big(\xi(\eta_n)-\xi_m\big)\, D_{M''}\big(\xi(\eta_n)-\xi_m\big) \Big]$$



$$\xi_m = \xi(\phi_s) + m\,\Delta\xi = \xi_s + \frac{2\pi m}{2M''+1}$$

$$\eta_n = \eta(\phi_s) + k\varphi + n\,\Delta\eta = \eta_0 + \frac{2\pi n}{2N''+1}$$

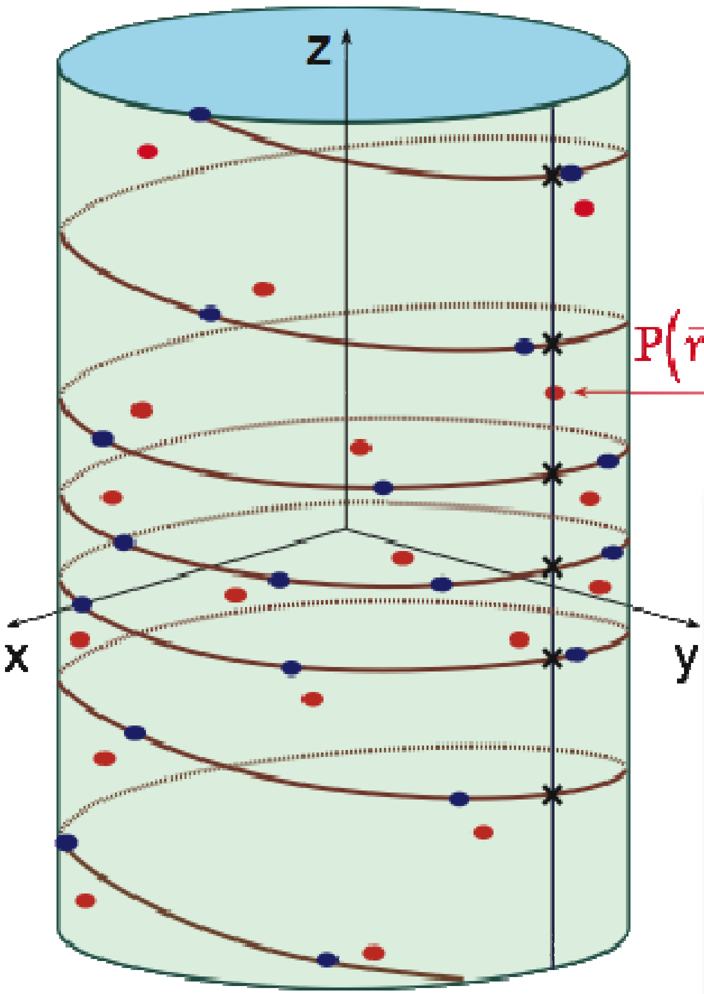
$$\begin{cases} M'' = \mathrm{Int}(\chi M') + 1 \\ M' = \mathrm{Int}(\chi' W_\xi) + 1 \end{cases}$$

$$M = M'' - M'$$

$$N = N'' - N'$$

$$n_0 = \mathrm{Int}\left[\frac{\eta-\eta_0}{\Delta\eta}\right] \qquad m_0 = \mathrm{Int}\left[\frac{\xi-\xi_s}{\Delta\xi}\right]$$

FROM NONUNIFORM TO UNIFORM SAMPLES



The reduced voltage at the nonuniformly distributed sampling points can be expressed as a function of the unknown values at the nearest uniform ones via the OSI expansion:

$$\tilde{V}(\bar{\eta}_i, \bar{\varphi}_i) = \sum_{n=n_0-q+1}^{n_0+q} \Big[D_{N''}(\bar{\eta}_i - \eta_n)\, \Omega_N(\bar{\eta}_i - \eta_n) \cdot$$

$$\cdot \sum_{m=m_0-p+1}^{m_0+p} \tilde{V}(\xi_m)\, D_{M''}(\xi(\eta_n) - \xi_m)\, \Omega_M(\xi(\eta_n) - \xi_m) \Big]$$

$$i = 1, 2, \ldots, Q$$

FROM NONUNIFORM TO UNIFORM SAMPLES

In the matrix form, it can be rewritten as $\underline{\underline{A}}\,\underline{x} = \underline{b}$

where $\underline{\underline{A}}$ is the Q × Q sparse banded matrix whose elements are given by:

$$A_{im} = D_{N''}(\bar{\eta}_i - \eta_n)\,\Omega_N(\bar{\eta}_i - \eta_n)\,D_{M''}(\xi(\eta_n) - \xi_m)\,\Omega_M(\xi(\eta_n) - \xi_m)$$

$\underline{x}$ is the vector of the unknown uniform samples and $\underline{b}$ is the vector of the acquired nonuniform ones.

Now, by splitting the matrix $\underline{\underline{A}}$ in its diagonal and nondiagonal parts, $\underline{\underline{A}}_D$ and $\underline{\underline{\Delta}}$ respectively, we get:

$$\underline{x} = \underline{\underline{A}}_D^{-1}\underline{b} - \underline{\underline{A}}_D^{-1}\underline{\underline{\Delta}}\,\underline{x}$$

THE ITERATIVE SCHEME

Thus, the following iterative scheme results:

$$\underline{x}^{(\nu)} = \underline{x}^{(0)} - \underline{\underline{A}}_D^{-1} \underline{\underline{\Delta}}\, \underline{x}^{(\nu-1)}$$

where $x^{(0)} = \underline{\underline{A}}^{-1}_D b$ and $x^{(\nu)}$ is the vector of the uniform samples estimated at νth step. Necessary conditions for the convergence of such an algorithm are:

$$A_{ii} \neq 0, \ \forall i \qquad\qquad |A_{ii}| \geq |A_{im}| \quad \forall m \neq i$$

These conditions are certainly satisfied in the here assumed hypothesis of biunique correspondence between each uniform sampling point and the "nearest" nonuniform one.

THE ITERATIVE SHEME (...)

By straightforward evaluations, it results

$$\tilde{V}^{(\nu)}(\xi_i) = \frac{1}{A_{ii}} \left\{ \tilde{V}(\overline{\eta}_i, \overline{\varphi}_i) - \sum_{n=n_0-q+1}^{n_0+q} \left[\Omega_N(\overline{\eta}_i - \eta_n) D_{N''}(\overline{\eta}_i - \eta_n) \cdot \right. \right.$$

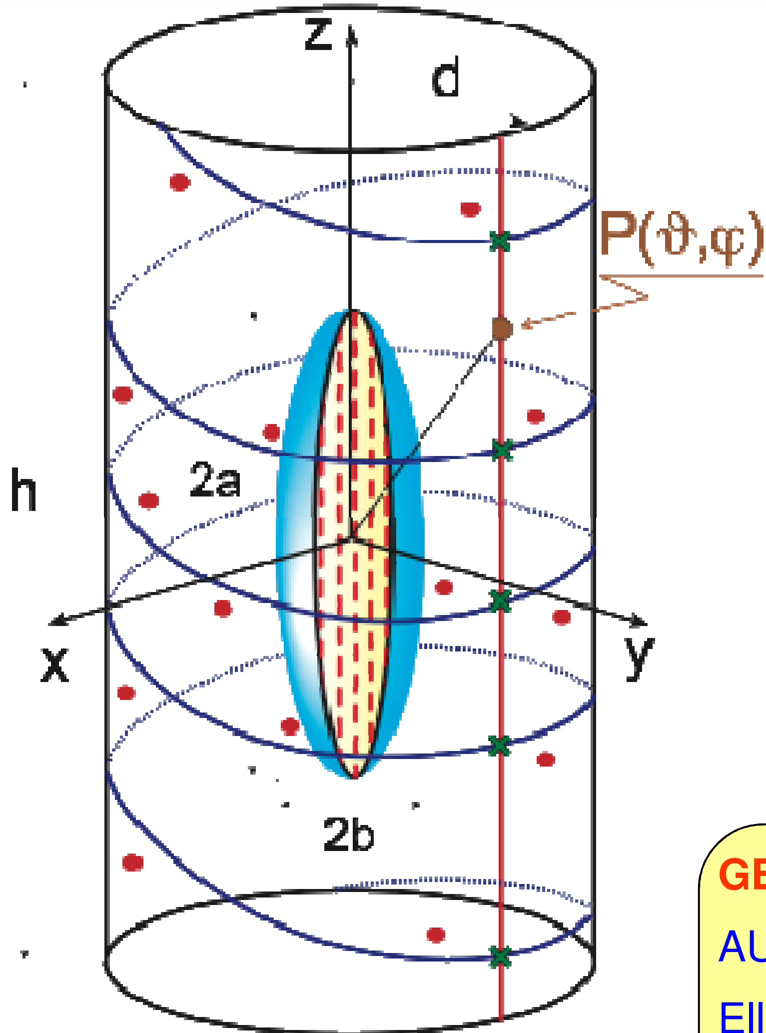
$$\left. \left. \cdot \sum_{\substack{m=m_0-p+1 \\ m \neq i}}^{m_0+p} \tilde{V}^{(\nu-1)}(\xi_m) \Omega_M(\xi(\eta_n) - \xi_m) D_{M''}(\xi(\eta_n) - \xi_m) \right] \right\}$$

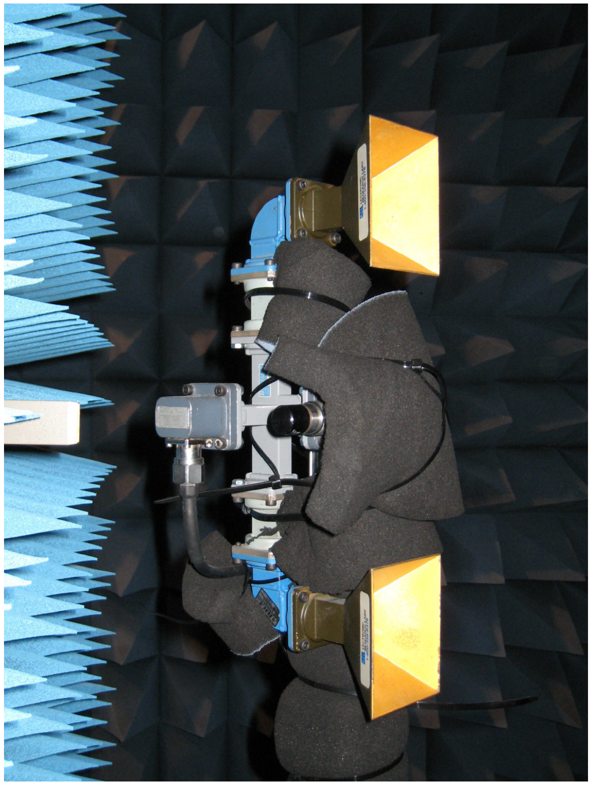
$$i = 1, 2, \ldots, Q$$

$$A_{ii} = D_{N''}(\overline{\eta}_i - \eta_{n_i}) \Omega_N(\overline{\eta}_i - \eta_{n_i}) D_{M''}(\xi(\eta_n) - \xi_m) \Omega_M(\xi(\eta_n) - \xi_m)$$

$$n_i = \mathrm{Nint}[(\overline{\eta}_i - \eta_0)/\Delta\eta]$$

LABORATORY TESTS





GEOMETRY OF THE PROBLEM

AUT: H-plane monopulse antenna @ 10 GHz

Ellipsoidal AUT modelling: (a = 5 cm, b= 27 cm)

Scanning cylinder: (h = 240 cm, d = 17.5 cm)

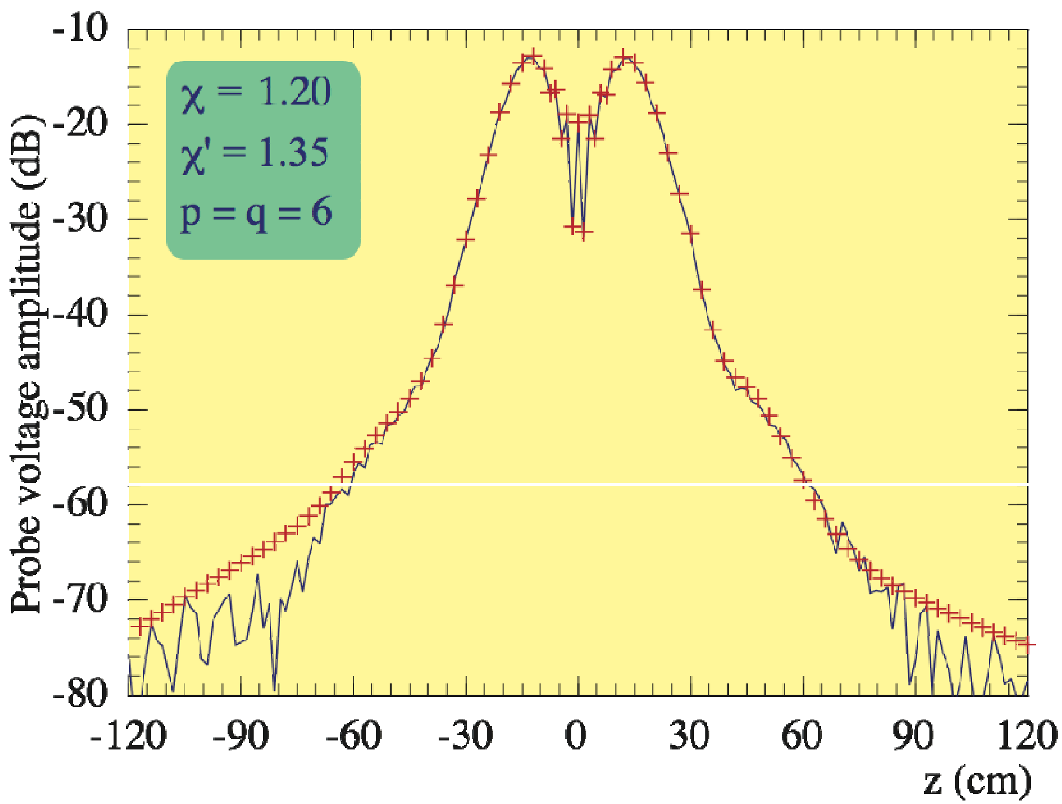
Maximum displacements: (± 0.3Δξ, ± 0.3Δη)

AMTA 2010 Atlanta

Antenna Characterization Lab

- Located at the University of Salerno in Italy
- 8 m x 5 m x 4 m
- 2.4 m Cylindrical Scanner

EXPERIMENTAL VALIDATION

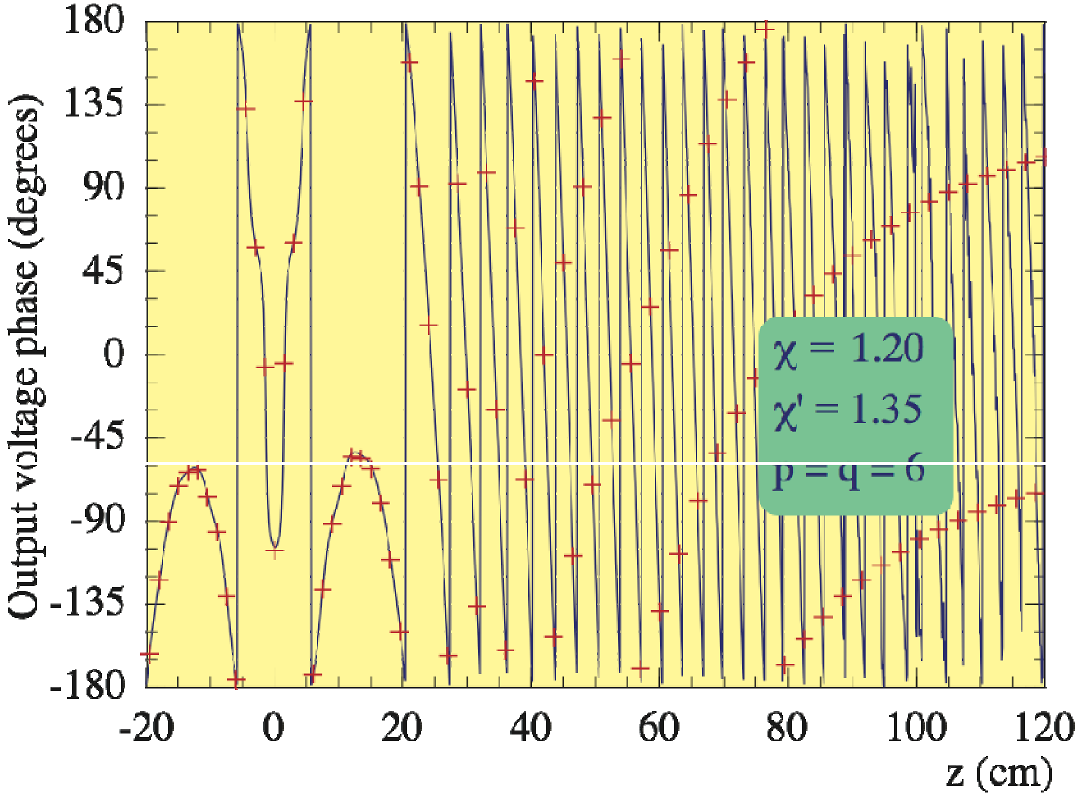


Amplitude of the probe voltage V on the generatrix at φ = 0°

Solid line: directly measured

Crosses: recovered from irregularly spaced NF data at the iteration 10

EXPERIMENTAL VALIDATION (...)

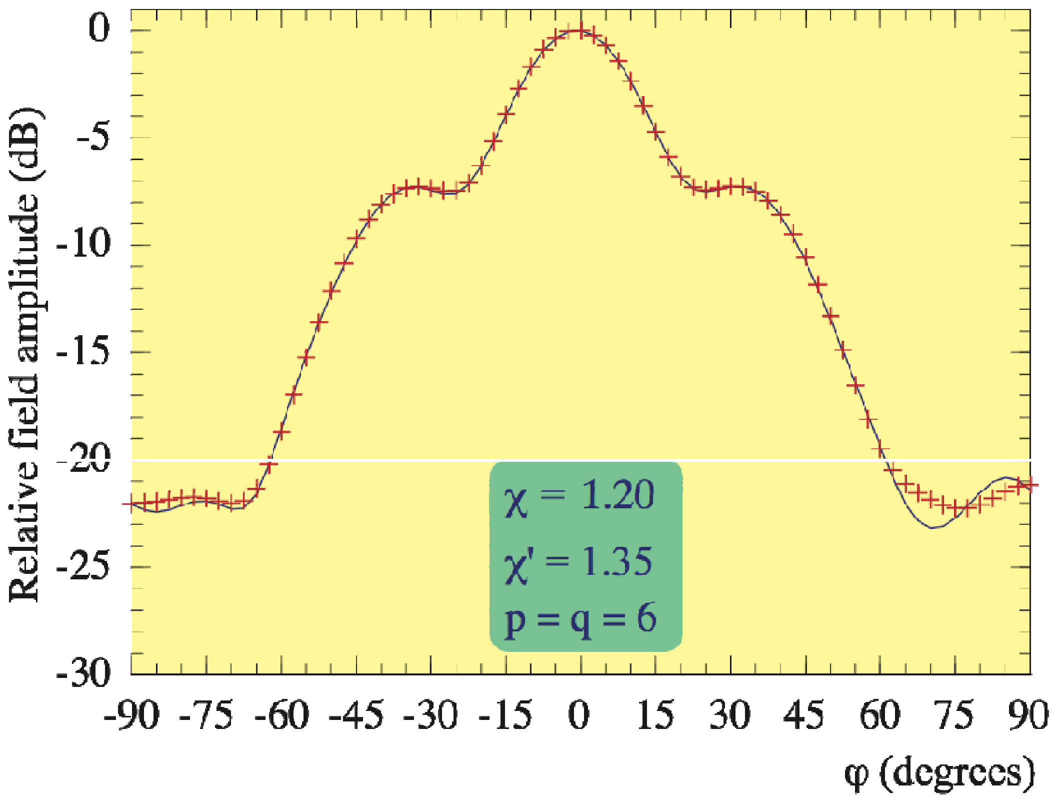


Phase of the probe voltage V on the generatrix at φ = 0°

Solid line: directly measured

Crosses: recovered from irregularly spaced NF data at the iteration 10

EXPERIMENTAL VALIDATION (...)

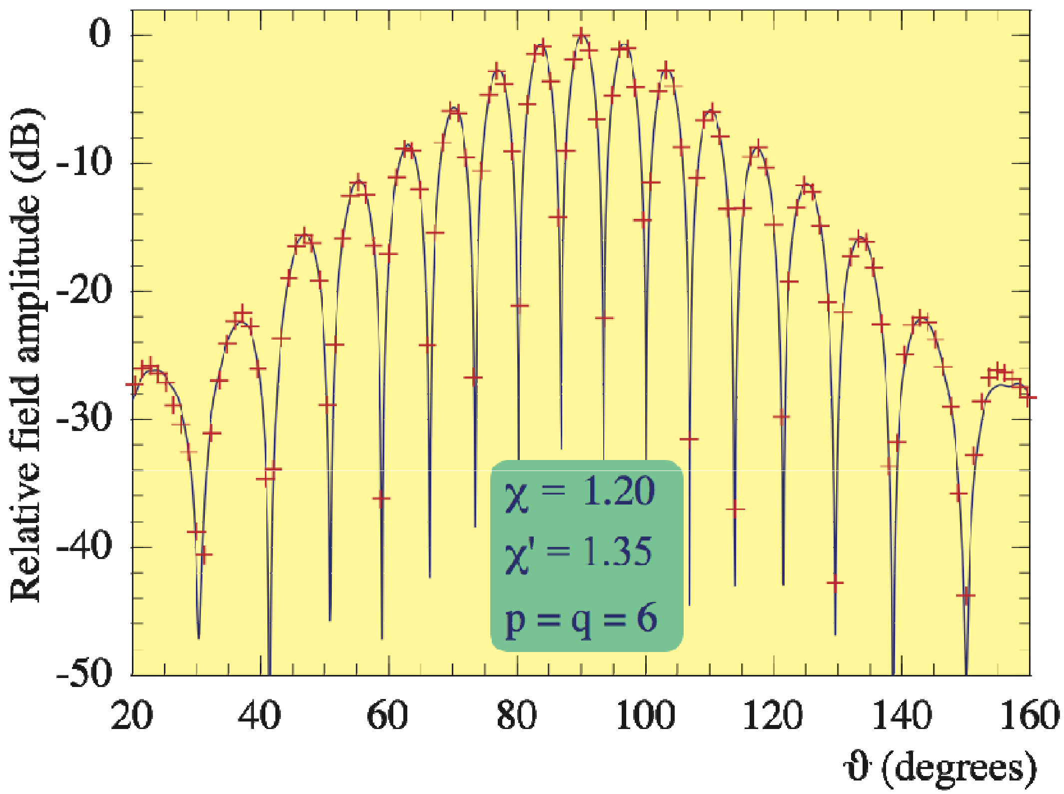


E-plane pattern

Solid line: reference (<u>5760 samples</u>)

Crosses: recovered from the irregularly spaced helicoidal samples (<u>1620 samples</u>)

EXPERIMENTAL VALIDATION (...)



H-plane pattern

Solid line: reference (<u>5760 samples</u>)

Crosses: recovered from the irregularly spaced helicoidal samples (<u>1620 samples</u>)

NEEDED DATA

Technique	NF data for the considered antenna
Classical approach [Paris et alii, 1973]	5760
The proposed helicoidal scanning	1620

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Conclusions

The science of spiral scanning techniques have been advancing for a number of years at the University of Salerno.

Experimental validation of the technique has been presented in the past couple of years validating the techniques.

This latest work shows the ability to compensate for known residual errors in the probe position.