Application Note

Conceptual Analysis of Measurement on Compact Ranges

Scientific Atlanta
Instrumentation

CONCEPTUAL ANALYSIS OF MEASUREMENT ON COMPACT RANGES

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I. INTRODUCTION

The testing of microwave antennas or the measurement of radar backscatter usually requires that the antenna or target under test be illuminated by a uniform plane electromagnetic wave; however, the creation of such a wave is difficult. In practice, a uniform plane wave is approximated.

The conventional procedure for approximating a uniform plane wave is to locate a transmitting source antenna at such a distance that the incident wave can be considered to be planar. When the source antenna is located $2D^2/\lambda$ away from the test antenna (where D is the largest dimension of the test antenna aperture and λ is the wavelength), the spherical wavefront emitted from the source will produce a maximum phase taper of $\pi/8$ at the edge of the test antenna aperture. For most applications, such a phase taper is acceptable.

In a compact range [1-5], on the other hand, the incident plane wave is created by a range reflector and feed in the immediate vicinity of the test antenna. The basic principle of operation is illustrated in Figure 1. The diverging rays from the point-source feed are collimated by the range reflector, and a plane wave is incident on the test antenna or target. The incident wave has a very flat phase front but the feed-reflector combination introduces a small (but acceptable) amplitude taper across the test zone.

The principal advantage of a compact range is its small size; this allows it to be indoors and free from adverse weather effects. In research and development laboratories, a compact range can be located convenient to the design engineers. In manufacturing or rework facilities, a compact range can be located near an assembly line for use in final testing and adjustment. By placing the range in a shielded room, one can eliminate interference from external sources and provide a test site that is secured against monitoring by outside parties.

II. THE COUPLING BETWEEN THE COMPACT RANGE AND AN ANTENNA UNDER TEST

In this section a simple mathematical description of the coupling between a compact range and an antenna under test will be give from two separate points of view. The case in which the test antenna is receiving electomagnetic energy will be considered first, following the conventional picture usually referred to in explaining the operation of a compact range. Second, the case in which the test antenna is transmitting and the compact range is receiving electromagnetic energy will be considered. It will be seen that the same transmission equation results, from both derivations, as one expects for two reciprocal antennas. The importance of the idea of the operation of the compact range as an angle filter for the wave spectrum results from these considerations.

Compact Range Transmitting and the Test Antenna Receiving

The operational picture of the compact range given in introductory descriptions shows the compact range transmitting an electromagnetic field whose local form in the test zone region is a uniform plane wave field. From this picture one may easily develop a coupling or transmission equation that describes the passage of electromagnetic energy from the port of the compact range feed horn to the port of the antenna under test.

The development of the transmission equation is done in two steps. First, an equation for the power density in the planar wave illuminating field in terms of the power accepted by the feed horn is derived. Then the well known relationship between the effective area and gain of a reciprocal antenna is used to get an expression of the power received by the antenna under test. The details of this approach are given in the following paragraphs.

Consider the feed horn illuminating the compact range reflector; let the feed horn be characterized by its gain G_f and assume that G_f is the peak gain of the horn along the principal ray that passes through the center of the test zone. The compact range reflector is in the far field of the feed horn so that the power density in the wave that illuminates the reflector can be readily calculated from the geometry of the configuration. Let the point Q_o be that point on the reflector where the principal ray intersects the surface; and, let the point Q be the point in the center of the test zone illuminated by the principal ray in the absence of the test antenna. Let F denote the focal point of the paraboloidal surface and V denote the vertex of the paraboloid, as in the Figure of Appendix II. Then, the distance between the focal point and the principal ray intersection point can be denoted R_o which is the length of the line FQ_o.

The radiation power density S_f in the wave from the feed horn as it strikes the surface of the reflector at Q_o is given simply by the equation

$$S_f(Q_o) = G_f \cdot \frac{P_o}{4\pi R_o^2} \qquad (1)$$

where P_o is the power accepted by the feed horn and G_f is the gain of the feed. The effect of the reflector upon the wave from the feed horn is to change the curvature of the phase front from spherical to planar and to reverse its direction of propagation. The amplitude of the field along any particular ray is unaltered by the reflector, assuming a perfectly conducting surface. The propagation of the reflected field leaving the reflector is such that the power density at point Q in the test zone is the same as the power density at point Q_o on the reflector. This is due to the fact that the reflected wave is collimated; hence, there is no curvature to the reflected wave front and no dispersal of rays to give a loss of power density. One may write, therefore, that the power density in the field of the compact range at Q equals that at Q_o:

$$S_{cr}(Q) = S_f(Q_o) \qquad (2)$$

When an antenna to be tested is placed in the test zone of the compact range it is illuminated by a planar wave front having power density S_{cr}. One can readily say what the power will be at the port of the test antenna by utilizing the definition of the effective area of the test antenna:

$$P_{rvc}^{aut} = S_{cr} \cdot A_{eff}^{aut} \qquad (3)$$

Here, P_{rcv} is the power received and A_{eff}^{aut} is the effective area of the test antenna. This may be expressed in time of the gain G_{aut} of the test antenna by making use of the well known relationship between the gain and effective area of a reciprocal antenna:

$$G_{aut} = \left(\frac{4\pi}{\lambda^2}\right) A_{eff}^{aut} \tag{4}$$

Combining the first two equations gives an expression for the power density of the field in the test zone of a compact range:

$$S_{cr} = G_f \frac{P_o}{4\pi R_o^2} \tag{5}$$

Combining the second two equations gives an expression for the power received by an antenna in terms of its gain and the power density in the field illuminating it:

$$P_{rcv}^{aut} = S_{cr} \frac{\lambda^2}{4\pi} G_{aut} \tag{6}$$

When these are combined, one gets the simple result that:

$$\frac{P_{rcv}^{aut}}{P_o} = \left(\frac{\lambda}{4\pi R_o}\right)^2 G_{aut} G_f \tag{7}$$

This result is strongly reminescent of the Friis' transmission equation which applies to the coupling of two antennas in each other's far field. It is as if one is coupling the test antenna to the feed horn at a distance of R_o. This equation is often used to estimate the port-to-port loss of a compact range-test antenna comfiguration.

Compact Range Receiving with the Test Antenna Transmitting

In order to understand the operation of the compact range when the test antenna is transmitting and the compact range is receiving energy, one can make use of the concept of a plane wave spectrum, taking the point of view that the compact range acts as an "angle filter" for the plane waves emitted by the test antenna.

It is known from the theory of the plane wave analysis that the far field pattern of an antenna is identical to the plane wave spectrum of the field emitted by that antenna. This can be treated as an assumption for the heuristic development given here. One thinks of the field of an antenna as composed of plane waves travelling outward in all directions. In the far field of the antenna one observes the various plane waves one at a time and he may therefore directly measure the plane wave spectrum. The near field, on the other hand, is made up of the entire set of plane waves. In order to get the plane wave spectrum from the near field, one must utilize one of two methods. Either he must perform a Fourier analysis, as is done with data acquired from planar scanning, or he must employ an angle filter such as a compact range to pick out a specific component of the spectrum. One can think of the compact range as an analog device for performing a two-dimensional Fourier transform.

The plane wave spectrum differs in one essential aspect from the conventional idea of a frequency spectrum. The plane wave spectrum is two-dimensional whereas the conventional frequency spectrum is one dimensional.

Another difference is in the matter of units; conventionally one thinks of the various components of a frequency spectrum as being labelled by their respective frequencies. In dealing with the plane wave spectrum, one thinks of the various components as being labelled by their directions of propagation in space, as specified by angles. Because it takes two angles to specify a direction, one readily sees that the plane wave spectrum must be two-dimensional. Just as the spectral density function of a frequency filter might carry the units of $(\mathrm{Hz})^{-1}$, the spectral density function of an angle filter for plane waves would carry the units of inverse solid angle or $(\mathrm{steradian})^{-1}$.

One way of estimating the "angular width" of the plane wave spectrum that is accepted in the pass band of the compact range when it is receiving, is to examine the family of rays associated with plane waves that impinge onto a compact range under axial illumination. The plane wave associated with the principal ray will be focussed at the focal point of the paraboloid where the center of the feed horn is located. The plane waves impinging from directions that are slightly off axis will be focussed just off the center of the feed horn but will also cause a response provided they are focussed within a radius of the focal point equal to the radius of the feed horn's aperture.

From the geometry of the configuration one can see that these plane waves whose angle off axis is less than $\frac{1}{2}(\Delta\Theta)$ will be accepted by the feed providing

$$\tan \tfrac{1}{2}\Delta\Theta \lesssim \frac{a/2}{R_o} \tag{8}$$

Therefore, the angular extent of the pass band of the compact range is

$$\Delta\Omega \simeq (\Delta\Theta)^2 = (a/R_o)^2 \tag{9}$$

where a is the physical radius of the feed horn aperture and R_o is the distance from the focus to the reflector along the principal ray.

There is another way of estimating the spectral width of the compact range pass band which gives the same result and serves as a verification of this first method. Consider the feed plus reflector as an antenna whose pattern one wishes to consider. The aperture size of the antenna is D; its beam width is, therefore, roughly λ/D radian in each plane. Since the far field of this antenna is just its plane wave spectrum, one then knows that the spectrum is appreciable over an extent of solid angle equal to

$$\Delta\Omega \approx (\lambda/D)^2 \tag{10}$$

Now utilize the requirement imposed upon the feed by asking that it give nearly uniform illumination of the test zone; to accomplish this the angle subtended by the aperture of the reflector at the focus must equal the beamwidth of the feed horn; thus, the beamwidth $\Delta\Theta$ of the feed

$$\Delta\Theta \simeq \lambda/a \tag{11}$$

must be such that

$$\Delta\Theta \simeq D/R_o \tag{12}$$

where D/R_o is the angle subtended by the reflector. Combining these gives $\Delta\Theta$ expressed in terms of a and R_o as was done before:

$$\Delta\Omega = \left(\frac{\lambda}{R_o \lambda/a}\right)^2 = (a/R_o)^2 \tag{13}$$

Therefore, by two separate lines of though it has been shown that the passband of the angle filter represented by a compact range is

$$\Delta\Omega = (a/R_o)^2 \tag{14}$$

This expression for the width of the passband of the compact range angle filter can be utilized to estimate the power level of the signal received by the range, provided one knows the power spectral density in the wave impinging onto the compact range. One gets this as follows:

When the antenna under test is transmitting, denote the radiation intensity in the field by $dP^{aut}/d\Omega$. This quantity is the power per unit solid angle emitted in the radiated field. Considering that the radiated field is made up of a spectrum of plane waves, it is instinctively clear that the power spectral density of the plane wave spectrum must be given by the same quantity. The radiation intensity is related to the gain of the antenna under test by the well known relation

$$\Phi^{aut} \equiv \frac{dP^{aut}}{d\Omega} = G_{aut} \frac{P_o}{4\pi} \tag{15}$$

The compact range will accept only that portion of the plane wave spectrum that is within its passband. The passband of the compact range will be much narrower than the plane wave spectrum of the test antenna, and therefore, one may write that the power received by the compact range will be given by

$$P^{cr}_{rcv} = \left(\frac{dP^{aut}}{d\Omega}\right) \cdot \left(\Delta\Omega\right) \tag{16}$$

Therefore, in view of these last two equation one has

$$P^{cr}_{rcv} = G_{aut} \frac{\Delta\Omega}{4\pi} P_o \tag{17}$$

The previous expression for $\Delta\Omega$ contains a dependence on the dimension of the feed; this can be converted to a dependence on the gain of the feed by utilizing the relation derived in Appendix I;

$$G_f = \frac{4\pi}{\lambda^2} \; a^2 \tag{18}$$

Thus,

$$\Delta\Omega = (a/R_o)^2 = \frac{1}{4\pi} (\lambda/R_o)^2 \; G_f \tag{19}$$

is an alternative expression for the passband of the compact range angle filter. Using this equation and the equation above for the power received by the compact range one gets the desired result that

$$\frac{P^{cr}_{rcv}}{P_o} = \left(\frac{\lambda}{4\pi R_o}\right)^2 \; G_{aut} \; G_f \tag{20}$$

which has been derived by assuming that the test antenna is transmitting and the compact range is receiving. Compare this to the previous equation (7).

The fact that this result is exactly the same as one got in the case when the test antenna receives gives one confidence in the correctness of the basic concepts employed.

III. THE COUPLING BETWEEN THE COMPACT RANGE AND A RADAR TARGET

One of the most important applications of the compact range is its use as a tool in measuring radar cross sections. The concepts developed in the previous section can be applied to the case of cross section measurements to get a coupling equation in terms of the parameter of the range and the radar cross section of the target.

Power Loss from a Radar Cross Section Target

For the purpose of this discussion, we take as the definition of mono static radar cross section the following quantity

$$\sigma = 4\pi \frac{dP_{sc}/d\Omega}{dP_{in}/dA} \tag{21}$$

where dP_{in}/dA is the radiation density, or power per unit area, of the plane wave energy incident on the target and $dP_{sc}/d\Omega$ is the radiation intensity, or power per unit solid angle, of the electromagnetic energy scattered from the target, in the direction of the incoming wave. The quantity σ is dependent upon the direction of the monostatic angle relative to the target.

Imagine that a radar target is set in the test zone of the compact range and that it is illuminated by a planar wavefront transmitted by the range. Then the range can also be used to receive the scattered energy and hence to give a measure of the cross section of the target. To do this, a circulator can be used at the port of the feed horn to route the out-going and in-coming signals. The compact range is used as an angle filter to pick out only that small piece of the plane wave spectrum scattered by the target which corresponds to the wave coming directly back along the line of incidence. Let us now examine the process by writing equations that correspond to the successive steps.

When the target is illuminated with energy from the compact range, the power density of the incident plane wave is given by the expression previously derived:

$$dP_{in}/dA = S_{cr}(Q) = G_f \frac{P_o}{4\pi R_o^{\,2}} \tag{22}$$

The scattering process is represented by the definition of the radar cross section; the radiation intensity in the scattered wave is given by

$$dP_{sc}/d\Omega = \left(\frac{\sigma}{4\pi}\right)\left(dP_{in}/dA\right) \tag{23}$$

From the spectrum of plane waves scattered by the target the compact range accepts only those members of the spectrum that are within its passband; the power received by the range that is scattered from the target is

$$P_{rcv}^{sc} = (dP_{sc}/d\Omega)\ \Delta\Omega \tag{24}$$

where $\Delta\Omega$ is the width of the passband and is given by

$$\Delta\Omega = \frac{1}{4\pi}\left(\frac{\lambda}{R_o}\right)^2 G_f \tag{25}$$

Combining these last four equations gives

$$\frac{P_{rcv}^{sc}}{P_o} = \frac{\sigma}{\lambda^2}\ \frac{1}{(4\pi)^3}\ \left(\frac{\lambda}{R_o}\right)^4 G_f^{\ 2} \tag{26}$$

This equation is the same as the usual radar equation for an antenna whose gain is G_f and for a target placed a distance R_o away. This equation gives a means of estimating the signal level of a target return and is the basis fo designing a measurement system for radar cross sections based on a compact range.

In order to illustrate the use of this equation, an example follows in which the case of a target with σ = 0dBsm is worked out. The compact range radar equation alone is first put into logarithmic form

$$10\log\frac{P_{rcv}^{sc}}{P_o} = 10\log\left(\frac{4\pi\sigma}{\lambda^2}\right) + 10\log\left(\frac{\lambda}{4\pi R_o}\right)^4 + 10\log G_f^{\ 2} \tag{27}$$

Taking the following values, which are typical in cases of practical interest

$$\lambda = 3\text{ cm} = 1.18\text{ in}\ ,\ G_f = 8\text{ dB}\ ,\ R_o = 152\text{ in} \tag{28}$$

permits one to get a definite appreciation of the practical aspect of the problem.

The first term can be broken up as follows

$$10 \log \left(\frac{4\pi\sigma}{\lambda^2}\right) - 10 \log \left(\frac{\sigma}{1m^2}\right) + 10 \log \left(\frac{4\pi \ m^2}{\lambda^2}\right) \tag{29}$$

to permit an estimate to be adjusted for targets having cross sections of other than $1m^2$ or OdBsm. In this case, then the loss equation for a OdBsm target has terms with the following values

$$\begin{aligned} 10 \log \frac{4\pi \ m^2}{\lambda^2} &= +41.5 \text{ dB} \\ 10 \log \frac{\lambda}{4\pi R_o}^{4} &= -128 \text{ dB} \\ 10 \log (G_f)^2 &= +16 \text{ dB} \end{aligned} \tag{30}$$

and the loss between the transmitted signal at the port of the compact range and the received signal for the scattered energy is

$$10 \log P^{sc}_{rcv}/P_o = -71 \text{ dB}. \tag{31}$$

The peak radar cross section for a square plate is given by the expression

$$\sigma_{sq\ plt} = 4\pi \frac{A^2}{\lambda^2} \tag{32}$$

where A is the physical area of the plate. If the plate is 1 meter on a side, then

$$10 \log \frac{\sigma}{1m^2} = 41.5 \text{ dB}_{sm} \tag{33}$$

The loss for a square plate, $1m^2$ in physical area, is then

$$10 \log P^{sc}_{rcv}/P_o = -71 \text{ dB} + 41.5 \text{ dB} = -29.4 \text{ dB} \tag{34}$$

It is worthwhile to keep in mind that the far field distance of $2D^2/\lambda$ for such a plate is equal to 67m, or 214 ft.

IV. CW REFLECTIVITY RANGE

When making measurements on a CW reflectivity range, the procedure generally is (1) to null out the returned signal in the absence of a target, (2) to place the target in the incident field, and (3) to measure the returned signal with the target in place. The signal measured in Step 3 is assumed to be target return since the return from other objects was nulled out in Step 1. (There are some inherent errors due to shadowing of the background by the target and due to multipath return, but these errors generally are small and will not be discussed here.) One easily can see that the sensitivity of the range depends upon the depth and stability of the null which is achieved in Step 1. In other words, if one wishes to measure backscatter from a target with a cross section of -30 dBsm, then the null signal in Step 1 must be well below the signal returned from such a target. The conditions which produced the null in Step 1 must be maintained during the entire measurement procedure, and it usually is considered good practice to remove the target after the measurements in Step 3 and check the null to be sure it has not degraded too much since Step 1.

In a CW compact reflectivity range, there are three main signals that must be nulled out before placing a target on the support structure. They are (1) leakage from the transmitter to the receiver at the circulator (or hybrid), (2) reflections within the transmission lines, and (3) reflections from the room and surrounding equipment. The resultant of the three signals is nulled out by a fourth signal which can be coupled directly from the transmitter line as indicated in Figure 9.

It is convenient to reference the depth of the null to the signal that would be returned by a target having a cross section of one square meter, but let's first estimate the depth of the null which can be achieved.

The four signals indicated in Figure 19 are illustrated as phasors in Figure 10. Signals 1 and 2 are confined to the waveguide (or coax) plumbing. Their phases at the receiver can vary with frequency and temperature changes, but they will be insensitive to movements of range components. Signal 3, on the other hand, radiates into the test chamber, is scattered, and then is received again by the feed. The phase of Phasor 3 can be changed by small movements of the feed, of the range reflector, or of surfaces of the test chamber.

In a compact range within an anechoic chamber about 65 percent of the radiated energy strikes the reflector, is collimated, and then strikes the back wall. About 10 to 12 percent of the energy strikes the ceiling, and the remaininder is essentially divided equally among the two side walls and the floor.

In an absorber-lined chamber, for all practical purposes, one can consider that all energy leaving the feed eventually strikes absorber, so the magnitude of Phasor 3 essentially depends on back-scatter reflections from the absorber and upon the chamber size. In this respect, the signal represented by Phasor 3 can be called "room return" in a manner similar to the custom of using "target return" to refer to the signal returned from a target. Thus, in a well-designed chamber, the room-return power probably would be very low relative to the transmitter power.

One must keep in mind that Phasor 3 is actually the resultant of several phasors, each of which represents a contribution from a particular part of the chamber. For example, if the back wall moves while the rest of the chamber is stationary, then only the components representing reflections from the back wall will be changing phase.

Now the various parts of a compact reflectivity range will be considered to estimate the null-depth sensitivity to movements. The most sensitive part probably is the feed because all energy radiated and received passes through this device. The second most sensitive part is the range reflector because the largest portion of radiated energy strikes the reflector and because rays are reflected from it twice—once on transmission and once on reception. The third most sensitive part is the back wall because essentially all of the energy reflected off of the reflector strikes the back wall. Of lesser sensitivities will be the side walls and the floor.

Since the feed appears to be the most sensitive part, consider it first. The feed actually radiates energy in many directions, so when it moves in a particular direction, all reflected rays are not changing phase at the same rate. Most of the energy is radiated along the feed axis, however, so axial movements should cause the largest phase changes in Phasor 3.

To simplify the problem, consider that the feed is vibrating (moving) axially and that all energy within 45° of the axis is radiated and received axially. In typical compact-range feed horns, about 80% of the energy is radiated and received within a $\pm$ 45° cone; this represents a level only 1 dB below the total radiated power, so simply assume in this discussion that all energy is radiated and received axially.

Next, assume that the feed moves axially a distance $\Delta\ell$. Then the total round-trip (transmission and reception) path length change is $2\Delta\ell$, and the phase change of Phasor 3 is

$$\Delta\xi = \frac{2\pi}{\lambda}(2\Delta\ell) = 4\pi\frac{\Delta\ell}{\lambda} \quad . \tag{35}$$

Let E_3 be the field magnitude of Phasor 3. If under stationary conditions, the received signals have been nulled out, and then Phasor 3 changes phase by $\Delta\xi$, the null will be replaced by a signal of field magnitude ΔE as illustrated in Figure 11. The value of ΔE is

$$\Delta E = E_3 \, \Delta\xi$$

or

$$\Delta E = 4\pi\frac{\Delta\ell}{\lambda} E_3 \quad . \tag{36}$$

This is the field magnitude of the disturbing signal for axial vibrations of the feed having peak-to-peak excursions of $2\Delta\ell$. The power level in the disturbing signal is related to the power level in the room return by

$$\Delta P = (4\pi)^2\left(\frac{\Delta\ell}{\lambda}\right)^2 P_3 \quad , \tag{37}$$

where P_3 is the power represented by Phasor 3. This signal should be referenced to the signal returned from a target having a radar cross section of one square meter; the latter can be calculated from Equation 26.

It is instructive to perform some simple calculations. Assume that the compact range has about a 12-foot range reflector (height and width) and a 12-foot focal length, then the following values would be typical for X-band operation:

$$\begin{aligned} G &= 8 \\ \lambda &= 3 \text{ cm} \\ R &= 4\text{m} \quad , \end{aligned} \tag{38}$$

then from Equation 26, for a 1 m^2 target,

$$P_r = \frac{(8)^2 (3 \times 10^{-2})^2}{(4\pi)^3 (4)^4} P_t$$

or

$$P_r = 1.1 \times 10^{-7} P_t \quad . \tag{39}$$

This is the reference signal, that is, the power returned from a target having a radar cross section of one square meter.

Now refer to Equation 37, and assume the room-return power is -60 dB relative to P_t. Also, assume that the peak-to-peak axial vibration of the feed is only 0.002 inch. Then

$$\Delta P = (4\pi)^2 \left(\frac{10^{-3}}{1.18}\right)^2 10^{-6} P_t$$

or

$$\Delta P = 1.1 \times 10^{-10} P_t \quad . \tag{40}$$

From 39 and 40, it can be seen that the disturbing signal is about -30 dBsm for the assumed conditions, and this would be the limit of the achievable null depth. Similar calculations were made for other vibration magnitudes and for other values of room return, and the results are presented in Figure 12.

Although the null depth is sensitive to very small movements of range components, do not interpret this to be a weakness peculiar to CW compact ranges. A similar sensitivity applies to all CW ranges, but with reasonable care, good performance can be achieved with both compact and conventional ranges.

V. PULSED REFLECTIVITY RANGE

In a pulsed reflectivity range, the return from the target can be range-gated, and return from unwanted objects can be rejected. For example, assume that we have a compact range with a 12-foot focal length located in a room that is about 36 feet long, 20 feet wide, and 17 feet high. If a short-pulsed signal is transmitted and received by the feed, the returned signal would be as indicated in the symbolic A-scope display of Figure 13. Note that most return from the room occurs within the first 25 feet of range. The main exception is the back wall return at about 42 feet.

This is a very simple analysis of how a pulsed system would perform on a compact range, but it indicates that satisfactory performance probably could be obtained if the receiver recovers fast enough to detect the target about 30 to 40 feet beyond the feed horn. The analysis neglects multiple reflections within the room, but such reflections are expected to be small.

The receiver recovery problem is serious but probably within the state-of-the-art; both the skirts of energy on the transmitted pulse must be very low and the receiver must recover quickly. In the example above (see Figure 13), the target is about 35 feet beyond the feed horn. If we allow the equivalent of about 10 to 15 "free-space" feet of transmission line between the transmitter/receiver and the feed horn, the target return will be received about 90 to 100 nanoseconds after the transmitted pulse.

In the previous section, we estimated that the return power from a 1 m^2 target would be about -70 dB with respect to the transmitted power. If we want to measure a -40 dBsm target, the receiver sensitivity must be considerably better than -110 dB with respect to the transmitted pulse within 90 to 100 nanoseconds.

VI. CONCLUDING REMARKS

It has been demonstrated through approximate arguments that familiar far-field equations can be used with compact ranges. For coupling between the feed (source antenna) and the test antenna, we have the Friis equation:

$$P_{rcv} = \left(\frac{\lambda}{4\pi R_o}\right)^2 G_f \, G_{aut} \, P_o \quad . \tag{41}$$

For backscatter measurements, we have the radar equation

$$P_{rcv} = \frac{G_f^{\,2} \, \sigma\lambda^2}{(4\pi)^3 \, R_o^{\,4}} P_o \quad . \tag{42}$$

In the above equations, R_{rcv} is the received power, P_o is the power accepted by the feed horn, λ is the wavelength, σ is the radar cross-section, G_{aut} is the gain of the antenna under test, G_f is the gain of the feed horn i the direction of the principal ray that passes through the center of the test zone, and R_o is the distance from the feed horn to the point where the principal ray strikes the reflector.

In the course of the arguments, it was demonstrated that a compact range can be considered to be an angle filter. The solid angle passband $\Delta\Omega$ is

$$\Delta\Omega \equiv \left(\frac{a}{R_o}\right)^2 \quad , \tag{43}$$

where a is the dimension of the feed horn aperture and R_o is as defined above.

In a CW reflectivity range, the achievable null depth is sensitive to movements of range components. The most sensitive component should be the feed horn. Vibrations of the feed will introduce a disturbing (null filling) signal. For axial vibrations having a peak-to-peak excursion of 2 $\Delta\ell$, the power level in the disturbing signal is

$$\Delta P = (4\pi)^2 \left(\frac{\Delta\ell}{\lambda}\right)^2 P_3 \quad , \tag{44}$$

where P_3 is the power level of the room return.

In a pulsed reflectivity range, the return from unwanted objects can be rejected; however, there is a receiver recovery problem due to the short distances involved on a compact range. Although the recovery problem is serious, it is likely that a solution is available with current state-of-the-art components.

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APPENDIX I
IDEALIZED FEED HORN

In the course of discussion within this paper it is convenient to make certain oversimplifying assumptions about the feed horn of the compact range. This appendix develops the ideas that describe this impractical but useful concept.

In order to design a compact range one must break with past traditional practice followed in designing far field antennas. The ideal compact range would have a test zone just as large as the dimension of the offset reflector used. Furthermore, the power density of the planar wave would be uniform throughout the test zone. The requirement of having a large test zone with uniform illumination implies that one wishes to use a feed whose pattern is uniform over the sector of solid angle intercepted by the reflector at the focal point and which is zero outside this solid angle. In achieving this ideal configuration one finds in practice that the pattern of the feed horn has much less taper at the edges of the reflector than one would have if he were designing an offset reflector antenna for far field use. Neglecting the differential loss along the various ray paths from the focal point to the reflector, one may take then as his ideal feed, a horn with a pattern that is constant over a sector of solid angle, equal to its beamwidth.

The directivity of an antenna at the peak of its pattern is given by the ratio of the peak radiation intensity to the average radiation intensity for the entire pattern. For the sector shaped feed horn, then, the peak directivity is given by

$$D_f = \frac{\left(\frac{dP}{d\Omega}\right)_{pk}}{P_t/4\pi} = \frac{P_t/\Delta\theta\cdot\Delta\phi}{P_t/4\pi} \tag{I.1}$$

$$= \frac{4\pi}{\Delta\theta\cdot\Delta\phi}$$

In these expressions P_t is the total power transmitted and $\Delta\theta$ and $\Delta\phi$ are the beamwidths in θ and ϕ. When $\Delta\theta$ and $\Delta\phi$ are nearly equal the denominator is simply $(\Delta\theta)^2$. One may convert this into an expression for the gain of the feed horn by making the further idealization that the horn is lossless and therefore has an efficiency of unity. Therefore, for a feed horn with efficiency equal to unity and having equal beamwidths, the expression for the gain in

$$G_f = \frac{4\Pi}{(\Delta\theta)^2} \tag{I.2}$$

Now relate this expression to the dimensions at the aperture of the feed by utilizing the conventional relationship between gain and effective area and the conventional approximation for the beamwidth of an antenna.
The effective area is therefore

$$A^f_{eff} = (\lambda^2/4\pi)G_f = \frac{\lambda^2}{(\Delta\theta)^2} \tag{I.3}$$

The beamwidth is given by

$$\Delta\theta = \lambda/a \tag{I.4}$$

where a is the dimension of the feed. Therefore

$$A^f_{eff} = a^2 \tag{I.5}$$

for this idealized feed horn and its gain is

$$G_f = \frac{4\pi a^2}{\lambda^2} \quad . \tag{I.6}$$

APPENDIX II
COMPACT RANGE GEOMETRY

The design of compact ranges based on paraboloidal reflectors typically utilized an offset feed horn in order to achieve a clear aperture for illumination. A sample profile view of a range is illustrated in the accompanying Figure II.1. The primary parameters which control the operations of the reflector as a compact range are its focal length f and its outside dimension D. A secondary parameter is also very important for the discussion of this paper; it is the distance R_o from the focal point F to the point Q_o where the principal ray strikes the surface of the reflector.

The distance R_o can be easily computed from the geometry of the con-configuration. The equation of the paraboloidal surface is simply

$$4fx = r^2 \equiv (y^2 + z^2) \qquad \text{(II.1)}$$

where the x axis is taken to be the axis of the paraboloid and r is the cylindrical radius. The principal ray from the focus to the center of the test zone is drawn in the centerplane of the range, which may be taken to be the x-y plane. In this plane the equation of the surface is simply

$$y^2 = 4fx \qquad \text{(II.2)}$$

Devote the coordinates of the point Q_o in the xy plane as x_o, y_o). Assume that the test zone of the range is centered on the aperture of the offset paraboloid; this gives the result that $y_o = D/_2$

$$y_o = D/_2 \qquad \text{(II.3)}$$

The x-coordinate of the point Q_o is then given by

$$x_o = \frac{y_o^2}{4f} = \frac{(D/2)^2}{4f} = \frac{1}{16}\frac{D^2}{f} \qquad \text{(II.4)}$$

The distance R_o is also given by a simple application of analytical geometry;

from the Pythagorean Theorem one has

$$R_o = [y_o^2+(f-x_o)^2]^{1/2} \tag{II.5}$$

Substitution from II.2 and expression gives

$$\begin{aligned} R_o &= [4fx_o + (f^2 - 2fx_o + x_o^2)]^{1/2} \\ &= [f^2 + 2fx_o + x_o^2]^{1/2} \end{aligned} \tag{II.6}$$

$$R_o = f + x_o \tag{II.7}$$

Combining II.4 and II.7 then gives readily the following expression for the Secondary Parameter R_o in terms of f and D:

$$R_o = f + \frac{1}{16}\frac{D^2}{f} \tag{II.8}$$

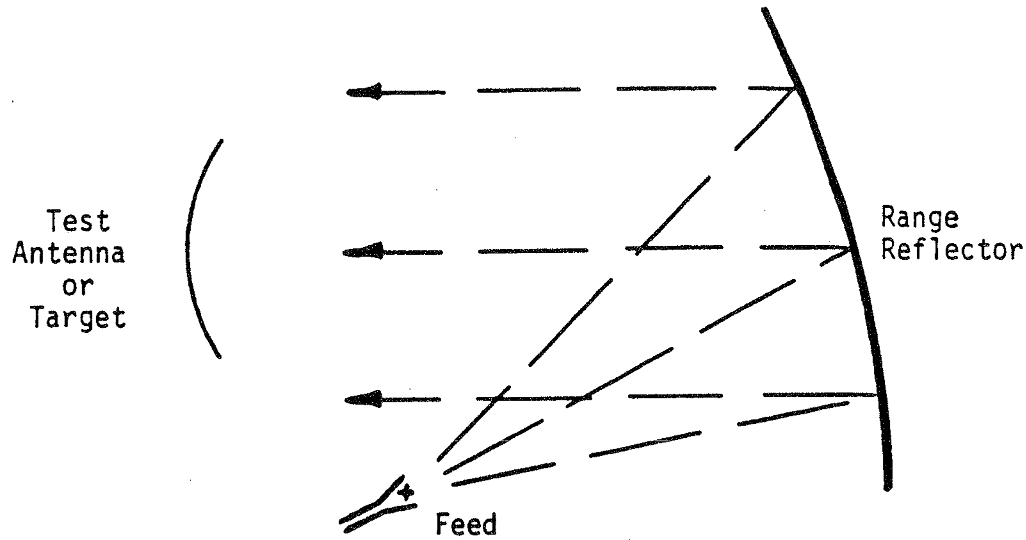

Figure 1. Schematic representation of a compact range employing a reflector and feed.

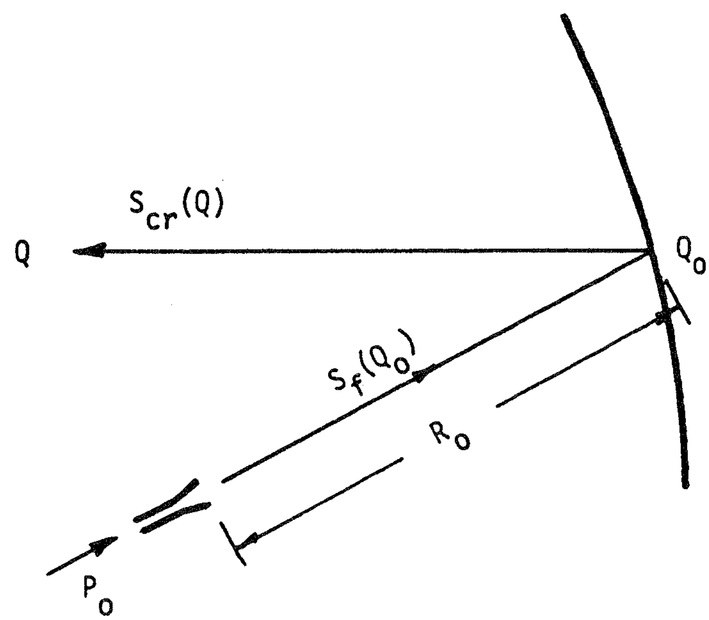

Figure 2. Power density relationships with the feed transmitting.

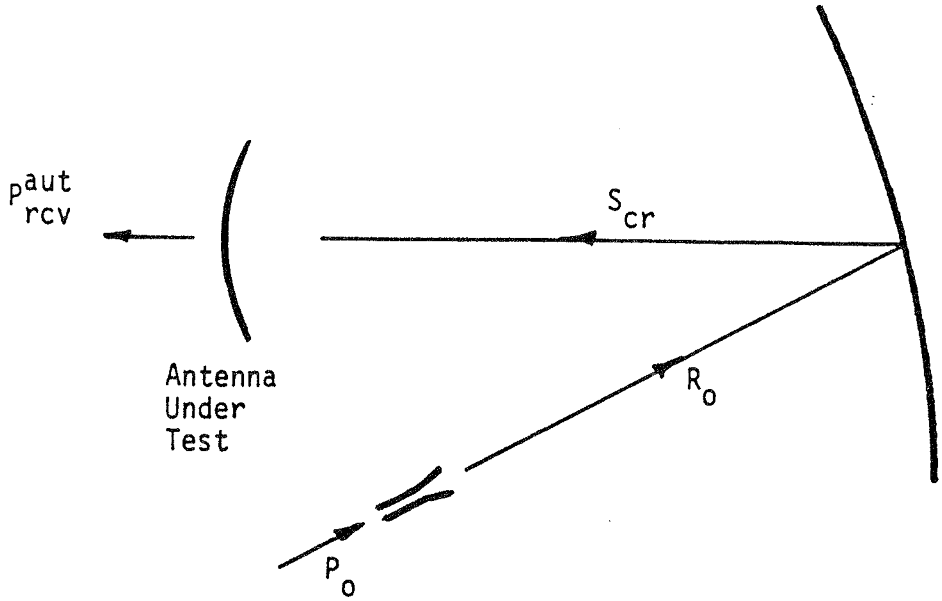

Figure 3. Port-to-port power transfer from feed to the antenna under test.

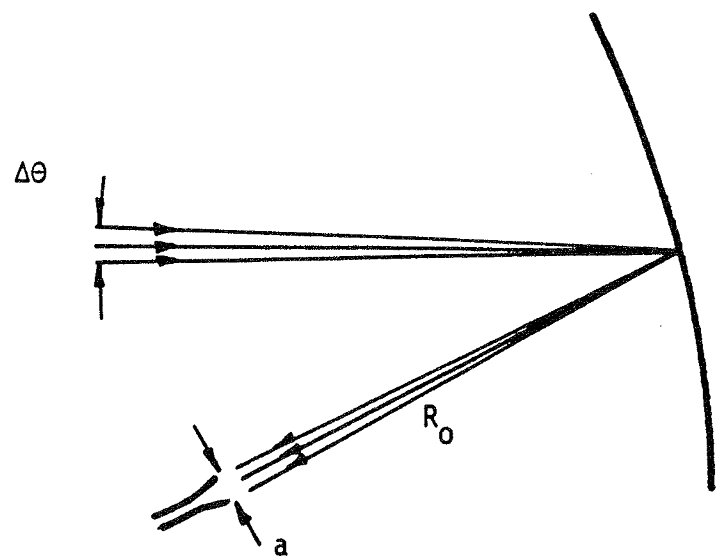

Figure 4. Ray diagram used to estimate the passband of the compact range angle filter.

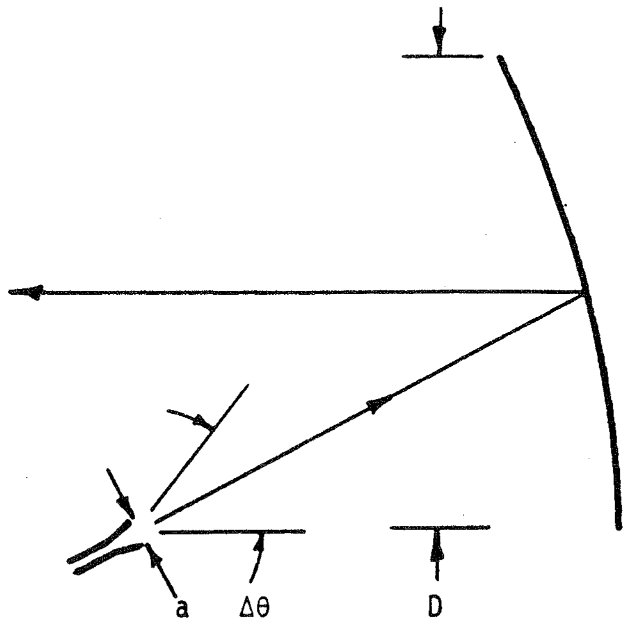

Figure 5. Feed-reflector antenna configuration used to estimate the angle filter passband from consideration of the beamwidth.

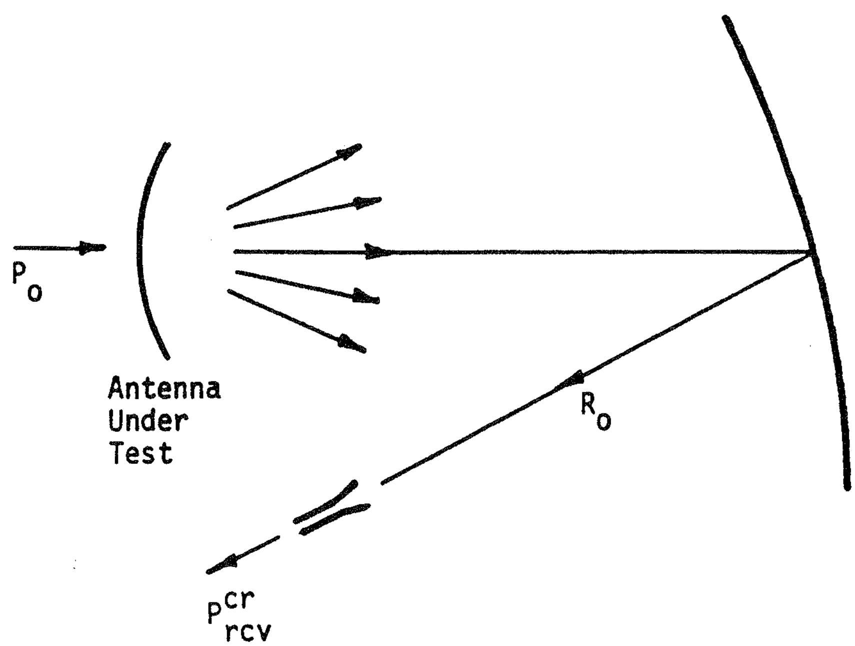

Figure 6. Port to port power transfer from the antenna under test to the compact range feed.

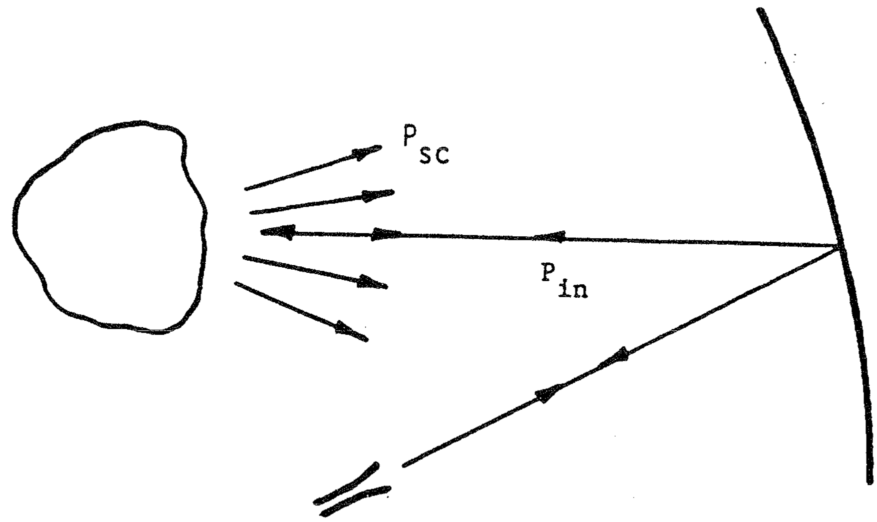

Figure 7. Schematic diagram of incident and scattered plane waves in a compact range radar cross section measurement.

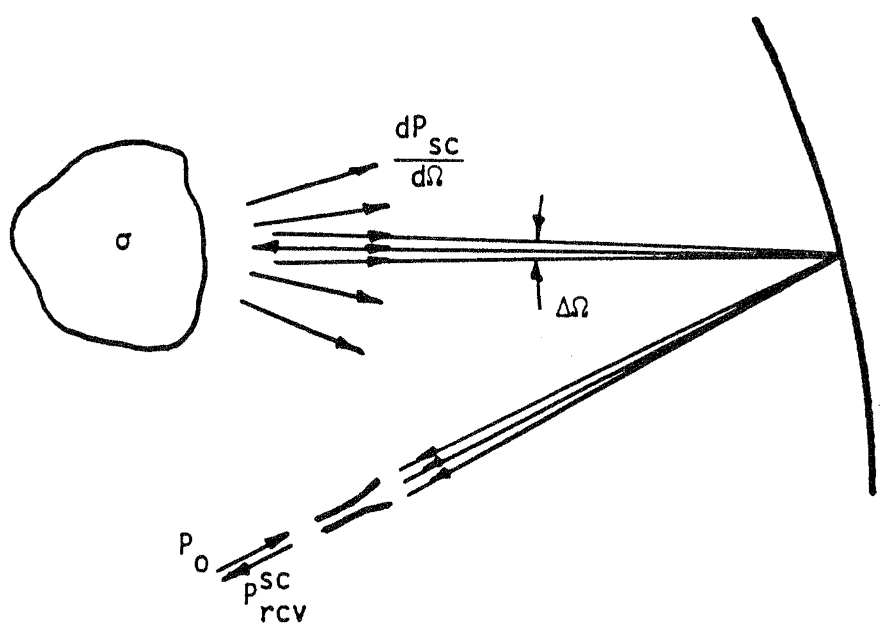

Figure 8. Schematic diagram of compact range angle filter operation in radar cross section measurement.

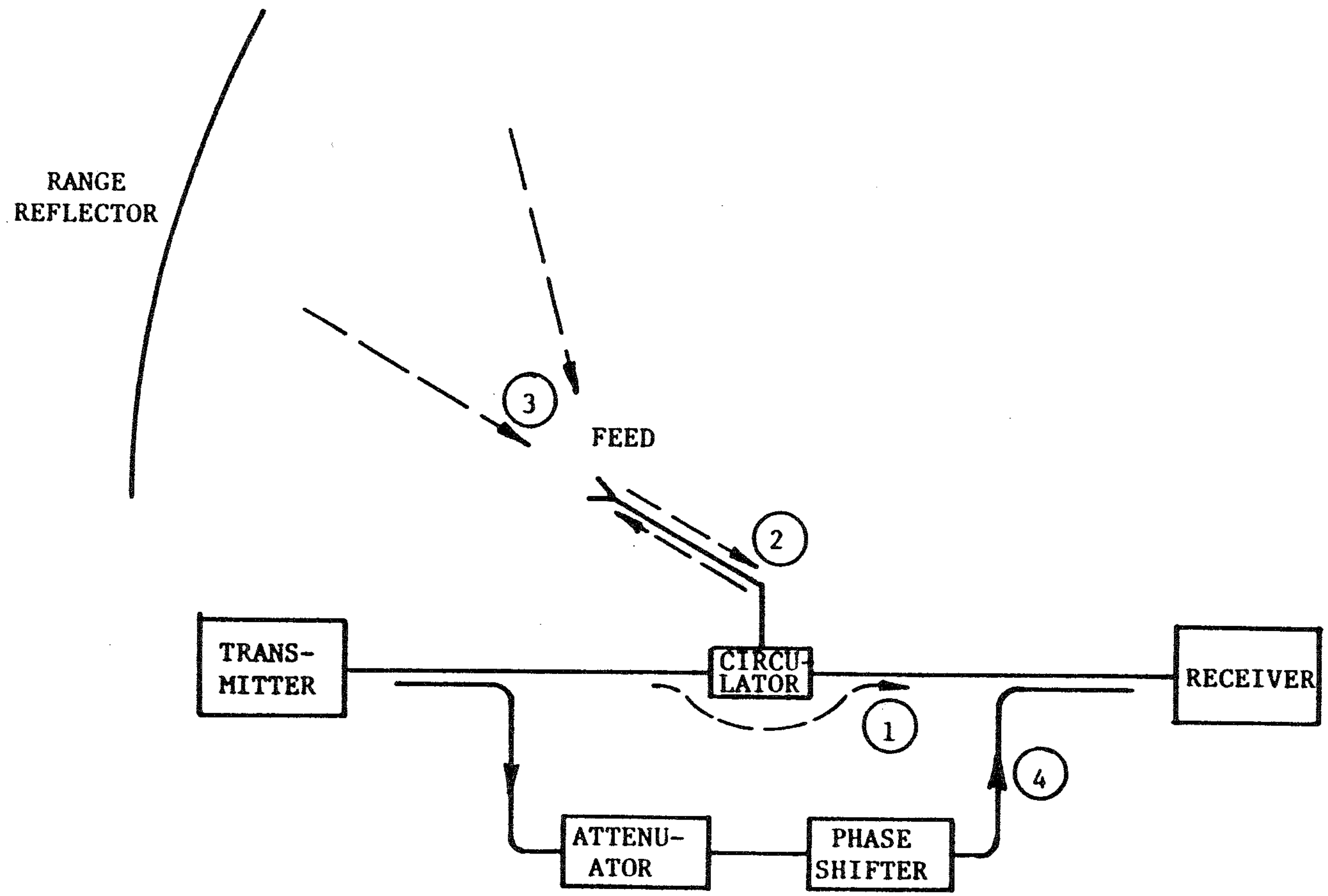

Figure 9. Schematic drawing of a CW compact reflectivity range illustrating the three main signals that must be nulled out by a fourth signal coupled directly from the transmitter.

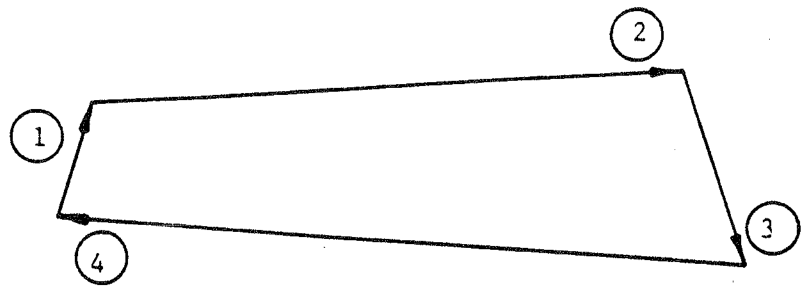

Figure 10. Phasor representation of the four signals indicated in Figure 9. The magnitudes and phases of the first three phasors have been chosen arbitrarily; the magnitude and phase of the fourth phasor is adjusted so that it terminates at the origin of phasor 1 (thus, producing a null signal).

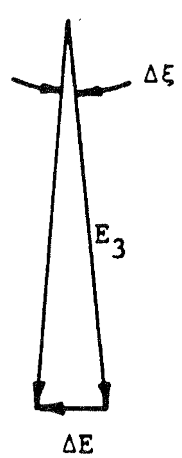

Figure 11. Phasor diagram illustrating a change of phase in the room-return signal and the generation of a disturbing signal which replaces the null.

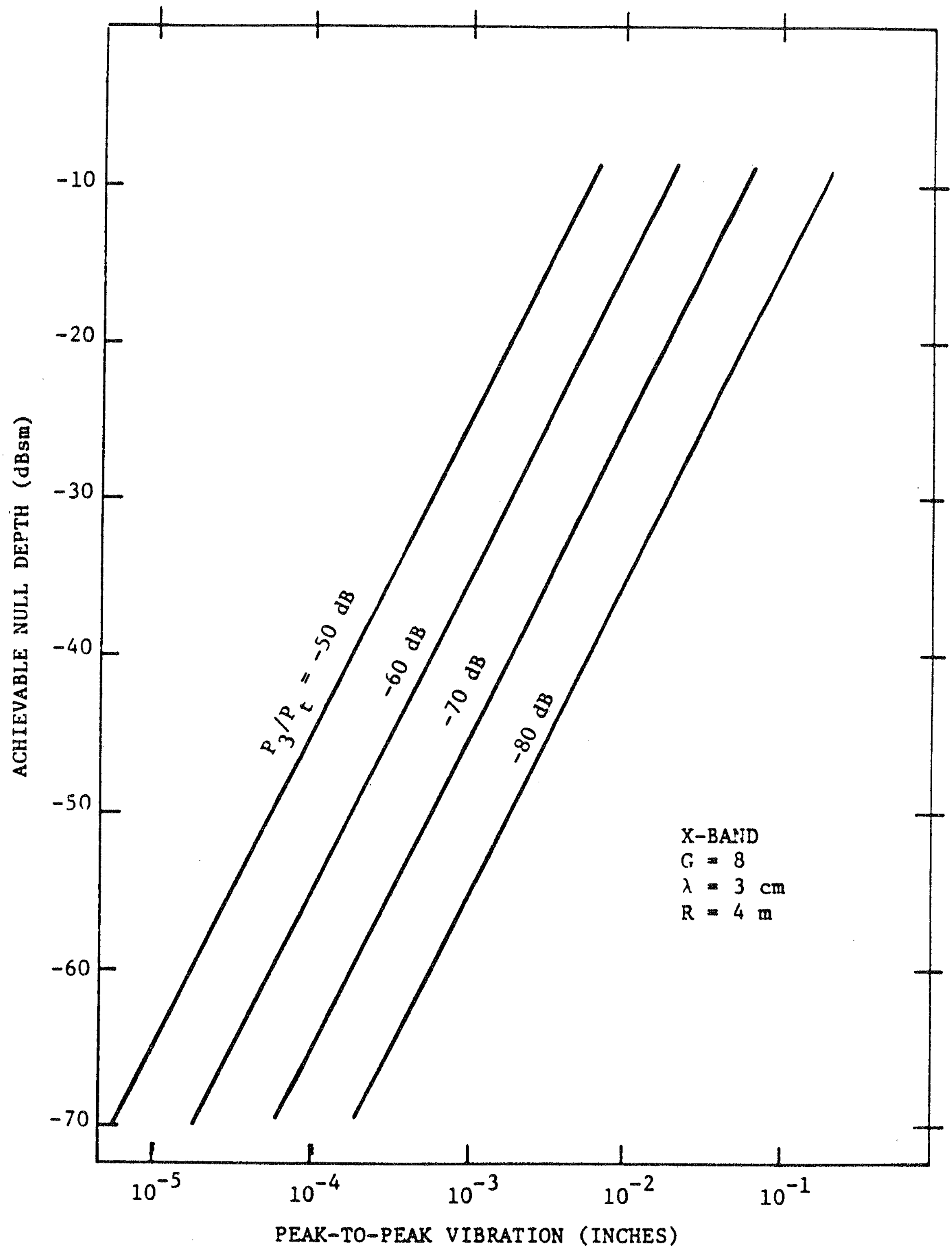

Figure 12. Achievable null depth versus peak-to-peak vibrations of the feed for various ratios of return to transmitted power at X-band.

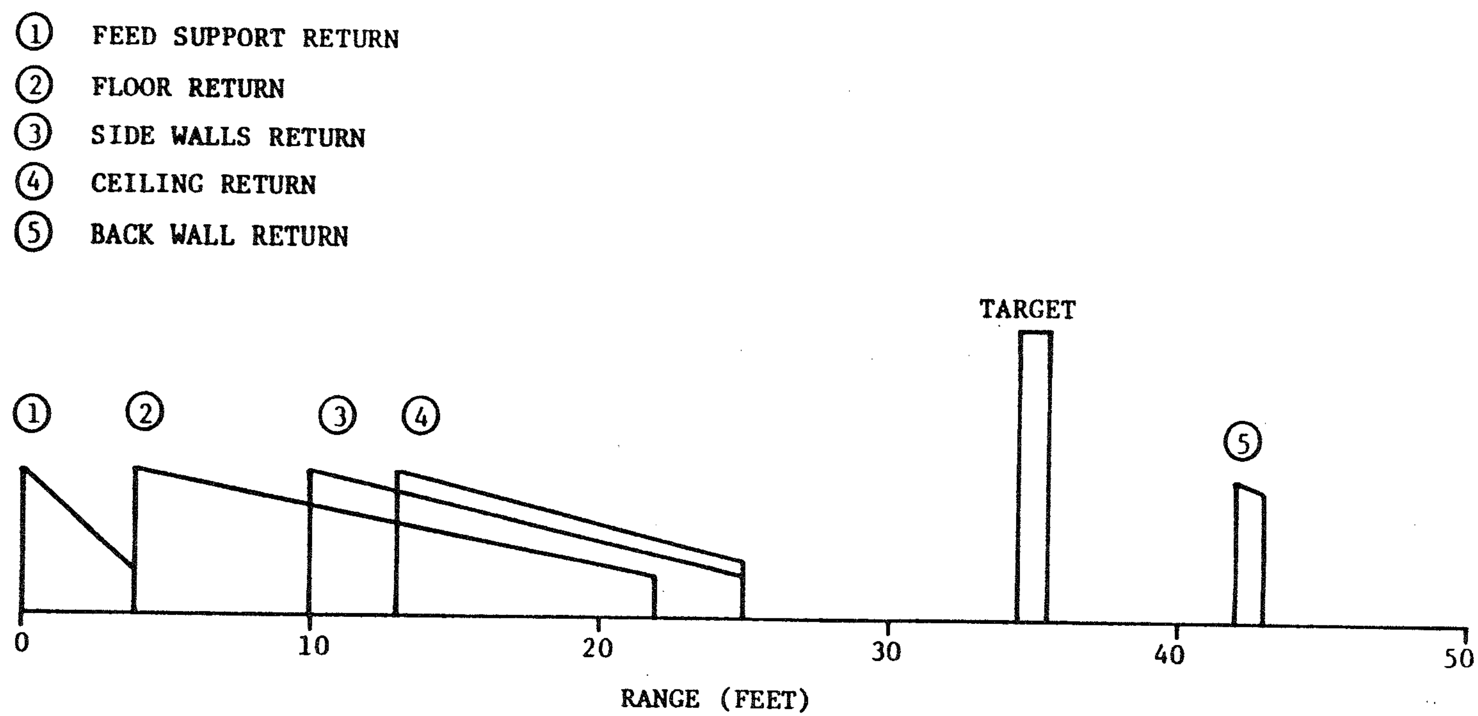

Figure 13. Symbolic A-scope display for a short-pulsed radar on a compact reflectivity range. Range is indicated as distance from the feed.